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DEVELOPING MATH TALENT

A Comprehensive Guide to Math Education for Gifted Students in Elementary and Middle School

Second Edition

Susan G. Assouline, Ph.D.,
& Ann Lupkowski-Shoplik, Ph.D.

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SECOND EDITION

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A COMPREHENSIVE GUIDE TO MATH EDUCATION FOR GIFTED STUDENTS IN ELEMENTARY AND MIDDLE SCHOOL

SUSAN G. ASSOULINE, Ph.D.
&
ANN LUPKOWSKI-SHOPLIK, Ph.D

PRUFROCK PRESS INC.
WACO, TEXAS

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Edited by Jennifer Robins

Cover and Layout Design by Marjorie Parker

ISBN-13: 978-1-59363-616-6

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Waco, TX 76714-8813
Phone: (800) 998-2208
Fax: (800) 240-0333
http://www.prufrock.com

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Dedication

To José, Jason, and Sonja—Thank you for your unwavering support, inspiration, and love.

—SGA

To Dad, Mike, Helena, and Anthony, with all my love.

—ALS

Foreword

BY LINDA BRODY

I am honored to have the opportunity to add my thoughts to those of Julian Stanley, whose Foreword to the first edition of this book follows. As Julian neared the end of his life and reflected on his legacy, he was most proud that the programs he had founded to serve academically talented students were not only continuing but flourishing. He was also comforted to know that the special efforts he had made to mentor and pass the torch, so to speak, to a new generation of leaders who would continue to serve academically talented students had been successful. Among those leaders are Susan G. Assouline and Ann Lupkowski-Shoplik. Julian would applaud the work that this newly revised volume represents and be particularly proud of the authors.

Julian Stanley founded the Study of Mathematically Precocious Youth (SMPY) in 1971. After working with the young Joseph Bates who successfully entered Johns Hopkins University full time at the age of 13 following Julian's intervention, Julian established SMPY to find other youths who reason exceptionally well mathematically and to provide them with opportunities to achieve their potential. The early years of SMPY were filled with a great deal of experimentation and efforts to validate options. Eventually a model of talent identification and development emerged that included systematic talent searches using above-grade-level assessments followed by academic accommodations utilizing appropriate accelerative strategies and challenging supplemental programs.

By the late 1980s, this model was well-established. The Johns Hopkins Center for Talented Youth (CTY) was running annual talent searches and summer programs, talent search programs had been established at other universities in the U.S., and SMPY had returned to its roots to focus on counseling the most mathematically precocious

students, continuing to find new ways to serve them, and conducting research. This is when I first met Ann and Susan, who joined me as postdoctoral fellows at SMPY.

Ann and Susan arrived with great enthusiasm and were very much dedicated to the task of helping mathematically talented students excel. It was an exciting time for all of us, as we worked collaboratively on a common mission, and were guided by Julian, who shared his theories, insights, and advice, and who inspired us to want to help exceptional math reasoners find resources to develop their talents.

Our work at SMPY, first and foremost, was to provide encouragement and advice about advanced academic options to the students who qualified for what was then referred to as the "700M group" (i.e., students who scored above 700 on SAT-Math before age 13). The most fun was when a family would visit SMPY, and we had the opportunity to get to know the student on a personal level. Other contacts were by phone and postal mail—it's hard to believe this was before e-mail. We also published a newsletter for the students—written in many ways like an informal, very long letter, full of information about challenging educational opportunities and resources, and signed by the whole staff. This was also before the Internet so the newsletter was an especially valuable and important resource for the families to learn about programmatic opportunities.

Although we sometimes encountered resistance to our efforts, we soon learned the power of information to overcome resistance. When schools were presented with evidence of a student's ability that came with above-level test results, they understood the need for subject acceleration. When parents were informed about programs that could benefit their children, they were eager to have them participate. When students learned about other students who had excelled in and loved a competition experience or a summer program, they were eager to enroll also. *Developing Math Talent* is a coordinated effort to provide much-needed and comprehensive information to those who are eager to help mathematically talented students develop their talents.

While they worked at SMPY, Susan and Ann became particularly interested in extending the talent search model to serve students at younger ages. Both of them went on to establish talent searches

for elementary school students at their respective institutions, Ann at Carnegie Mellon University and Susan at The University of Iowa. This is the third book on which they have collaborated, with each successive one benefitting from the additional insights they have gained over the years from developing and directing advanced academic programs and providing guidance to mathematically talented young learners and their families.

As the second edition of *Developing Math Talent* is about to go to press, we are also getting ready to celebrate, in 2011, the 40th anniversary of the founding of the Study of Mathematically Precocious Youth. This book is evidence that we will not just be celebrating the past, but also the present and the future. It is evidence that Julian's vision of having new leaders carrying on his work is a reality, and that future generations of mathematically gifted students will be assured of getting the accommodations they, in Julian's words, "sorely need and richly deserve."

Julian liked to incorporate quotations from poetry into his speeches and publications to make his points, and I know that Ann and Susan will join me in getting a bit misty as I reprint his paraphrasing of Browning to make my final point: "A mathematically precocious youth's reach should exceed his or her grasp, or what's an educational system for?" *Developing Math Talent* will help guide educators and parents as they work to extend both the reach and grasp of children who reason extremely well mathematically.

Dr. Linda Brody
Director, Study of Exceptional Talent
Center for Talented Youth
Johns Hopkins University
Baltimore, MD

Foreword to the First Edition of *Developing Math Talent*

BY JULIAN C. STANLEY (1918–2005)

In the early 1970s at Johns Hopkins University, I started annual talent searches and special, supplemental, accelerated, academic courses for seventh graders who reasoned exceptionally well mathematically. This kept me and a small staff, especially Lynn Fox, Daniel Keating, and William George, busy experimenting during that decade. We tried many identification and instructional procedures to determine which were most effective. These early efforts resulted in our first book, *Mathematical Talent*, published by The Johns Hopkins University Press in 1974.

In 1979, I "gave the shop away" by founding what is now the Center for Talented Youth and relinquishing control. This quickly led to similar regional programs at Duke University, Northwestern University, and the University of Denver. In the 1980s, I basked in the reflected glory of their great success, but was perhaps a bit at a loss to fill the work vacuum created by my no longer running the Study of Mathematically Precocious Youth (SMPY).

Then along came a brilliantly foresighted foundation executive, Raymond Handlan, who pointed out that I should be training post-doctoral students in our philosophy and methods. I gladly accepted his offer to fund three postdoctoral fellowships. How the field of giftedness has benefited from those! Two of the most outstanding learners and performers were Ann Lupkowski, who spent 3 years with us, and Susan Assouline, who spent 2. They are the coauthors of this remarkably helpful book. Until their coming, SMPY and its later offshoots were working chiefly with seventh and eighth graders, even though we realized that many mathematically and highly able boys and girls need

educational facilitation far earlier than that. We have become accustomed to finding even kindergartners who are far advanced beyond 1 + 1 = 2. One was doing number-theory research shortly after his fifth birthday! (We verified that.)

Ann and Susan developed a collaboration that has lasted over the years across considerable distance, Pittsburgh versus Iowa City. Their first contribution was a major innovation, the profoundly helpful but simply titled *Jane and Johnny Love Math: Recognizing and Encouraging Mathematical Talent in Elementary Students*. This current book grew out of that predecessor as their hands-on experience with many mathematically talented youth helped them revise and update their ideas and recommendations. The result should be widely useful to teachers, parents, educational administrators, methods-of-teaching courses, and even mathematically talented students themselves.

What happened to the other postdocs whom Mr. Handlan's foundation supported financially? After a fantastically successful research career at Iowa State University, Camilla Benbow is now dean of Peabody College of Vanderbilt University. Linda Brody heads the Study of Exceptional Talent (mathematical and/or verbal) in the Center for Talented Youth (CTY) of Johns Hopkins University. Elaine Kolitch is a math education professor at a state university in New York state. Each postdoc helps enrich the others.

The moral has become clear to us: Start as early as possible in helping intellectually talented boys and girls. Capitalize on their already-developed and fast-developing predispositions. Be sure they get a well-rounded, full education, but without holding them back in the areas where they have the greatest potential. If one of these areas is quantitative reasoning, this volume is essential. Probably no one else in the world is as well equipped as Drs. Assouline and Lupkowski-Shoplik to guide the needed supplementation of the mathematics curriculum for the early school years.

Dr. Julian C. Stanley
Founder, Center for Talented Youth
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Introduction

This book is about getting from Point A to Point B—not in the mathematical sense, but in the common, everyday sense of finding exceptional talent (Point A) and developing that talent (Point B). We have all learned that there are an infinite number of points along a line, and it's impossible to identify each and every one of them—but it has been our aspiration to identify the main topics for parents and educators who are ready to ensure that mathematically talented students are challenged.

We are confident that the readers of this book are sincere in their commitment to helping talented young students develop. We are equally confident that, although there are many excellent books about gifted children and gifted education, there is little material available that has the special focus of the development of mathematical talent in young students. Unfortunately, it is nearly as true today as it was several decades ago: The professional development of educators typically does not include information about teaching mathematically talented children. Parents, too, do not have an operating manual to assist them in their efforts. The consequence of this lack of information is uncertainty about the educational goals for mathematically gifted students. This uncertainty translates to a sense of helplessness and a need for guidance.

Of course, establishing appropriate academic goals—knowing about going from Point A to Point B—is relevant for all students. However, mathematically talented students have some unique needs (e.g., focused commitment of time and energy to keep engaged in their talent area). Nurturing their development will require an encouraging and supportive learning environment. Partnerships between educators and parents are necessary to ensure commitment and continuity of programming. Both parents and educators need to keep in mind that these talented students need appropriate encouragement to not only get to Point B, but to reach for points well beyond.

Readers who are familiar with the first edition of this book may be wondering how this new edition differs from the first. The structure of

the two editions is essentially the same; however, each chapter contains new and updated material from the previous edition. As with the first edition, the essence of our work is to provide a comprehensive guide for parents and educators alike who are committed to developing the talent of exceptional students.

Talent development is multifaceted and requires that parents and educators understand that their roles in developing students' mathematical talent are complementary. Although the chapters of this book are intended for both audiences, there are two chapters that have a specific emphasis: Chapter 2, "Advocacy," is aimed at parents, and Chapter 8, "Teaching Mathematically Talented Students," is aimed at educators. Nonetheless, we are confident that educators will benefit from reading the chapter on advocacy, and parents will benefit from reading the chapter on professional development.

We also considered it important that the scope of the book represent the broader educational picture. Consequently, throughout the book we briefly visit some important educational topics, including general issues around curriculum standards, educational reform, and international comparisons, as well as more specific issues such as working with students who are twice-exceptional (i.e., gifted and learning disabled). We provide references so that the reader who has additional interest in the topic can easily obtain more information.

As a special feature, we have drawn upon our clinical and practical experiences with mathematically talented students and interwoven these experiences with a solid research base to produce chapters that tell compelling stories about mathematically talented students. The cases and examples presented throughout will resonate with the experiences of parents and educators. These examples are an outcome of our involvement with students, their families, and educators through The Connie Belin & Jacqueline N. Blank International Center for Gifted Education and Talent Development at The University of Iowa and the Carnegie Mellon Institute for Talented Elementary and Secondary Students (C-MITES) at Carnegie Mellon University. These two centers provide opportunities for identification and programming for academically talented students, as well as resources and professional development opportunities for teachers.

Although our focus in this book is on the mathematically talented student, we incorporate a comprehensive view of the student as a "whole child." Even though most of our comments target specifically the student's development of mathematical aptitude, when relevant, we discuss the impact of our recommendations for the entire child.

Determining where to begin was challenging because all of the topics in each of the chapters are integrated. We decided to begin by addressing the excuses we've heard as an explanation for ineffective programming and/or placement decisions. Some excuses are simply expressions of inertia. Other excuses reflect inaccurate assumptions and attitudes that impede identification and programming for talented students. Chapter 1, "Excuses for Not Developing Mathematical Talent," lists 18 excuses, and we provide a response for each of the 18 excuses. The final excuse: "We aren't really sure what to do, so we think the best approach is to do nothing," offers a response that is the backdrop for the following chapters. When readers are finished with this book, they will no longer be able to use the excuse, "We're not sure what to do, so we do nothing."

Because parents typically are first to recognize the student's talent, we decided that the chapter to immediately follow Chapter 1 should guide parents in their efforts to advocate for their student in a school setting. For both Chapters 1 and 2, we distilled the available research and integrated that research with our experiences as an introduction to objective information useful in a variety of educational settings.

In our extensive experience with parents, we have found that the majority have doubts about the need or the way to advocate for change in their child's educational program. They are concerned that educators and other parents will view them as "pushy" parents. We always respond with a question: "If you don't advocate for your child, who will?"

Chapter 2, "Advocacy," views advocating for an appropriate program to challenge a gifted student as a process defined by a series of interactions between parents and educators. The parent-educator interactions are the focal point of Chapter 2, and a journey metaphor is applied extensively. The journey can be tedious because, for most people, the effort and activities needed to advocate are not familiar and require sustained attention. This chapter includes the presentation and

discussion of a detailed case in which a parent advocated for curriculum and program changes for her mathematically gifted student from kindergarten through middle school. Typically, parents will initiate the request. Very often, parents are also responsible for gathering objective information to be used in the decision-making process. They must then skillfully work with educators to implement a program based upon the information. Simple, but not easy.

Chapter 2 is the raison d'être for the book. The subsequent chapters are intended to facilitate the interactions between parents and educators by giving very specific, detailed information about assessment, curricula, programming, and resources.

Assessment and testing are the first steps to advocacy, and we devote three chapters (Chapters 3–5) to the topic. Both Chapters 3 and 4, "Using Academic Assessment to Make Informed Decisions About Mathematically Talented Students" and "The Diagnostic Testing→Prescriptive Instruction Model," respectively go into great depth regarding the process for conducting assessments of mathematically talented students and using that process to design individualized instruction for one student or a small group of students. In addition to detailing the process for an assessment, Chapter 3 provides a review of tests that are useful in working with mathematically talented students. An exhaustive review of tests was impossible; however, the review that is presented is thorough and comprehensive.

Chapter 3 presents a report of an assessment conducted for a mathematically talented student. The purpose of the report is to demonstrate how a range of tests serves various purposes when generating recommendations for programming. One purpose of testing may be placement. However, in our view, placement is secondary to diagnosis and programming. That is why the next chapter, Chapter 4, describes a system for precisely determining where mathematically talented students should begin instruction so that they are challenged by new material and not stifled by material they already know. The system for diagnosing where to begin instruction is based upon diagnostic testing followed by a specific prescription for instruction based upon the results of the diagnostic testing. This approach is applicable to both individual students and small groups of students. Parents and educators

will find this common sense approach to be very useful in establishing appropriately challenging programs for exceptionally talented students. This model is set up in such a way to prevent the "gaps" in a student's background that educators worry about when considering acceleration or a placement that is different from the "norm."

Parents and educators will find the contents of Chapter 4 very useful because a detailed, step-by-step explanation and several examples are provided. Combining Chapter 4's DT→PI model with the information in Chapters 3 and 5 yields a formula for implementing challenging math curricula and programming for individual students, small groups of students, or larger groups.

Chapter 5, "Talent Searches," is the third of the three chapters detailing the goals, purposes, and benefits of academic testing. Talent searches are an important part of the history of gifted education, and are the primary method of discovery of mathematically talented students. Although talent searches were founded with the seventh- or eighth-grade student in mind, we have been involved in the application of the talent search model with elementary students since the late 1980s. This chapter presents the results of more than two decades of research regarding the benefits of young students' participation in the above-level testing provided by talent searches. These benefits include educational diagnosis; educational programming and opportunities based upon talent search results; opportunities for scholarships, awards, and honors; and finding a true peer group.

Throughout Chapter 5, we offer a variety of examples in which talent search results are used to guide program placement and planning. This chapter uses charts and diagrams to emphasize that above-level testing is a powerful tool for distinguishing among bright students and identifying whether a curricular approach should be more accelerative or enriched. The results from the talent search emphasize to parents and educators the degree of academic aptitude among students. The talent search provides comparison information so that teachers and parents can recognize who, from among a group of bright students, needs an enriched curriculum and who needs a more accelerated curriculum.

Since their inception, talent searches have spurred new opportunities for students and have motivated teachers to regard their students

as individuals with unique needs for academic challenge. In its own way, the talent search is responsible for the multitude of students who participate in summer and academic-year programs that introduce them to advanced academic content. While reading about the talent search model, the reader is initiated into the discussion regarding program and curricular options, the major topics for the two chapters that follow.

Chapter 6, "Programming," contains information about programs currently available for mathematically talented students, ranging from enrichment in the regular classroom, to ability grouping, to individualized instruction and radical acceleration. No one option is the "right" choice for all mathematically talented students. In this chapter, and throughout the book, we emphasize that the goal is to find the "optimal match," in which the level and pace of the program are correctly matched to the abilities and achievements of the student. One of the options we discuss and advocate is acceleration, in which a student moves ahead in mathematics by taking a more advanced math class or by skipping a grade. Contrary to popular opinion, the long-term impact of acceleration is usually positive; research over the past 60 years has provided empirical support for acceleration as an appropriate and necessary program option for gifted youth. We also have described a number of exemplary programs to illustrate how different schools have provided appropriate educational opportunities for their mathematically talented students.

Chapter 7, "Curricula and Materials," provides an extensive review of curricula and materials. We argue that mathematically talented students should study a core curriculum presented in a systematic manner, not a random assortment of enrichment topics. Enrichment and acceleration should be used together to challenge mathematically talented youth. The curriculum offered in most schools is not challenging enough for mathematically talented youth, and the information provided in Chapter 7 provides guidance on how to supplement and differentiate that curriculum for individual students. We present a collection of resources and materials that teachers can use to differentiate the mathematics curriculum. Teachers can go beyond the material that is presented in the typical textbook by using manipulatives, math games, computer programs, and math contests.

Chapters 6 and 7 are the definite purview of teachers; thus, we use the next chapter, Chapter 8, "Teaching Mathematically Talented Students," to focus on issues confronting educators who are expected to work with mathematically talented students. In the same way that Chapter 2 had a specific target audience (parents), we wrote Chapter 8 for teachers. We had several goals in mind for this chapter, but we had no intention of providing a recipe or prescription for teaching mathematically talented students, nor did we intend to enhance the teacher's specific skill for teaching mathematics.

Rather, the purpose of Chapter 8 is to address both philosophical and pedagogical issues facing today's educators. In this chapter, we present our philosophy about the role of teachers in the lives of mathematically gifted students. We explore the teacher's evolving role as the student's grade and maturity level increase. We discuss the issue of quality of instruction versus quantity of instruction and explore this same issue with respect to teacher preparation. Although we do not provide a prescription to resolve these issues, we believe that, by highlighting them, we add an important voice to the discussion.

Chapter 9, "Case Studies," is the application of information presented in Chapters 1–8. Parents and educators alike seem to appreciate the lessons offered by the stories of the six cases presented, and we are grateful to the students and their families for their willingness to share their experiences and insights. Chapter 9 seems to be the favorite chapter for many readers of the first edition of this book; we have received comments about that chapter from both parents and educators. In this edition, we included some of the same case studies (with updates), but we also included some new cases. We describe mathematically talented students from preschool through graduate school. On the one hand, Elizabeth's parents knew that their daughter was exceptional, but they didn't want her to be perceived as different from her classmates. On the other hand, Arthur's parents wondered if their son was being sufficiently challenged. Zach, currently a graduate student, offers tremendous insight into the development of talent as he reflects on his K–12 experiences. Each case is unique, yet their stories have a common theme: The children love math, are energized by learning math, and are eager to be in an educational setting that

supports their extraordinary talents. Ensuring challenging experiences required cooperation between home and school, and each case provides an interesting story about advocacy and program implementation.

No situation was perfect; therefore, an important rationale for presenting a variety of experiences is to emphasize that students are resilient to some of the less-than-ideal programming and missed opportunities that have been part of the educational journey experience by each. We hope that parents and educators will see (a) the critical role of advocacy in providing appropriate programming and (b) that, even if students remain underchallenged for a (hopefully brief) period of their life, their abilities are not lost, although their enthusiasm may be jeopardized. The chapter concludes with some "lessons learned" that were brought to life by the case studies.

Developing Math Talent concludes with a Resources section and a Glossary. The Resources section provides suggested books, websites, programs, and the like, which should be helpful to parents and educators who are developing programs for mathematically talented students. The Glossary presents terminology related to gifted education and assessment.

♦♦♦

Most introductions are written *after* a book is completed, which was the case for both editions of *Developing Math Talent.* Writing an introduction gives the authors a chance to review the contents of the book and to reflect on the process. One thing we know unequivocally: We could not have written this book without each other. Our collaboration started in 1988 when Julian C. Stanley, founder of the Study of Mathematically Precocious Youth at Johns Hopkins University, brought us together as postdoctoral fellows. Dr. Stanley and his associate, Dr. Linda Brody, worked extensively with mathematically talented youth in seventh grade through college. As our mentors, their work inspired us to branch into the study of younger mathematically talented students.

Dr. Stanley had an important goal for his associates and postdoctoral fellows: furthering the program of research that would foster the identification and talent development of mathematically gifted students. Dr. Linda Brody is now director of the Study of Exceptional Talent

(SET) at Johns Hopkins University. SET evolved from Stanley's SMPY, which has moved to Peabody College at Vanderbilt University and is codirected by Dr. Camilla Benbow, Dr. Stanley's first postdoctoral fellow and current dean of Peabody College, and Dr. David Lubinski, a professor of psychology. The postdoctoral positions were funded by the Atlantic Philanthropies, and we are especially grateful for the personal interest and support that Keith Kennedy from the Atlantic Philanthropies gave us.

Our first collaboration resulted in the 1992 publication of *Jane and Johnny Love Math: Recognizing and Encouraging Mathematical Talent in Elementary Students*. *Jane and Johnny Love Math* emphasized the existence of mathematically talented students and delineated methods of addressing their needs. It was based upon our experiences with hundreds of talented elementary students. Our goal was to describe educational options that allow students to move systematically through the elementary mathematics curriculum while matching the curriculum to their abilities and achievements. That book has served its purpose well. However, since its 1992 publication there have been many significant developments in the field of gifted education; therefore, we wrote a new book about mathematically talented students for their parents and educators. Five years have passed since the 2005 publication of the first edition of *Developing Math Talent*, and once again it was time to update the material extensively.

Developing Math Talent offers the "why" for identification, placement, and programming for mathematically talented students. We have realized that the answer to "why" it is critical to discover and develop math talent begs another question: "how" to develop that talent. Therefore, our most recent collaboration is a hands-on book, *Building Gifted Programs for Math Talented Students: Tools for Teachers* (2011). *Building Gifted Programs* is a companion to *Developing Math Talent* and provides detailed options for educators who will implement programming for mathematically talented students. We affectionately refer to the *Building Gifted Programs* book as our "cookbook," because it provides comprehensive recipes for each step of the identification-placement-programming journey. In it, we share our expertise in how to develop programs for these exceptional students.

Our own journey began with the publication of *Jane and Johnny Love Math*, and we have learned much along the way. We look forward to the future for lessons to be learned from the latest edition of *Developing Math Talent* and its companion, *Building Gifted Programs*. We hope these books will enhance you professionally and personally.

Susan G. Assouline
Ann Lupkowski-Shoplik

Acknowledgements

Our current and former colleagues played an important role in the development of this book. The Belin-Blank Center's director, Nicholas Colangelo, provided valuable advice and resources that helped bridge the miles between Iowa City and Pittsburgh. We are also grateful for support and assistance from the other Belin-Blank Center administrators and clerical staff.

The Carnegie Mellon Institute for Talented Elementary and Secondary Students (C-MITES) staff members offered their support throughout the writing process. We are also grateful to C-MITES teacher Francy McTighe, who provided many insights during long discussions about what works best for mathematically talented students. Marty Hildebrandt and Anne Burgunder suggested some of the resources that are listed in this book.

Dr. Jennifer Robins provided helpful editorial comments and much support during the entire process. Also we appreciate Joel McIntosh's advice and support throughout the publication process of both editions. Joel and Jenny have been good colleagues.

We are appreciative of the friendship and support of Dr. Julian Stanley and Dr. Linda Brody. Their effective mentorship continues to be the star that we reach for.

CHAPTER 1

Excuses for Not Developing Mathematical Talent

> *An **excuse** is a reason which you give in order to explain why something has been done or has not been done, or in order to avoid doing something.*
>
> —Retrieved from Google Dictionary

In the late 1980s, when we first started working with mathematically talented students, there was limited material available for educators or parents that would assist them in understanding the need to provide academically talented students with appropriate challenges. Over the years, teachers, professors, administrators, and policy-makers have become more concerned about the status of both curriculum and programs for mathematically talented students. Simultaneously, new products and advanced technology have been introduced. New research and reports that have compiled old research (e.g., *A Nation Deceived* [Colangelo, Assouline, & Gross, 2004a, 2004b] and *Foundations for Success* [National Mathematics Advisory Panel, 2008]) have made salient the issues that now must be addressed. In this chapter, we take on these issues and, in some cases, the excuses that they have become to explain why some students are kept from progressing through an appropriate curriculum at a pace for which they are ready.

Excuses . . . Excuses

Excuse us. In the previous sentence, we are using *excuse* as a transitive verb and a pretext for our examination of the variety of excuses or justifications that often are presented as reasons for not doing a certain kind of assessment or for not using specialized curriculum and programming for mathematically talented students. In this chapter, we present some common excuses that we have heard over the years that negatively impact the development of math talent (see Table 1.1). You will see that some of the excuses are diametrically opposed to each other. This inconsistency in reasoning about mathematically talented students is one reason why having a rational response for each excuse is so important. Our goal is to provide the information and research to back up well-informed, balanced responses to any one of these "excuses" for not implementing appropriate programming for mathematically talented students. The responses to these excuses set the stage for the subsequent chapters.

Excuse 1: At the elementary level, the school's gifted program already meets the needs of *all* mathematically talented students.

Response: The names of many elementary gifted programs (e.g., ELP, or Extended Learning Programs) reveal their emphasis, which generally is on gifted behaviors and higher order thinking skills. These programs tend to be referred to as pull-out or resource programs because students will leave their regular classroom a couple of times a week for a period of time (45 to 60 minutes is typical) to participate in a preestablished enrichment curriculum. This enrichment can take a variety of forms, including activities unrelated to mathematics, problem-solving activities, or mathematically oriented enrichment activities (Lupkowski & Assouline, 1992). Unfortunately, it is a frequent practice in this country for gifted students to participate in pull-out programs where the topics they study are explicitly designed to be unrelated to the regular curriculum. For example, a mathematically talented student in a gifted program might be studying Shakespeare, growing plants for a science project, or participating in a community service

Table 1.1
Common Excuses That Negatively Impact the Development of Math Talent

1. At the elementary level, the school's gifted program already meets the needs of all mathematically talented students.
2. We already have a program for mathematically talented students.
3. Specialized programming is not necessary because enrichment is the "safest" way to challenge mathematically talented students and every math class has extension opportunities for students who finish their assignments early.
4. Teachers in our school are trained in differentiating the curriculum and can make adjustments for mathematically talented students within the regular classroom.
5. The National Council of Teachers of Mathematics (NCTM) standards recommend that prealgebra, algebra, and geometry be woven throughout the school years beginning in prekindergarten. This guarantees that mathematically talented students will have a differentiated curriculum and that prealgebra and algebra, in particular, should not be provided as separate courses for middle school students.
6. Mathematically talented students who are allowed to move through the curriculum quickly will run out of classes before they finish high school.
7. Mathematically talented students aren't ready to study algebra until eighth or ninth grade.
8. We shouldn't allow accelerated programming because students who are accelerated in mathematics will burn out before they reach college.
9. Accelerated math programs lead to gaps in students' mathematical foundation.
10. On the pretest, the student didn't earn 100%, so he or she doesn't qualify for advanced programming.
11. A student who makes mistakes in computation doesn't need advanced programming.
12. Too much time is devoted to standardized testing in our schools, so parents or teachers shouldn't ask for additional testing.
13. Our district doesn't accept any outside results; our district uses only nationally standardized grade-level tests to identify students for the gifted program.
14. Our district has stopped administering nationally standardized tests and uses only tests developed by the local district or building.

Table 1.1, Continued

15. Acceleration in math means taking math with students in the next higher grade and we have a policy against that.
16. The student has already skipped a grade in mathematics. That is sufficient acceleration and no additional changes are needed in the student's mathematics program.
17. There is no reason to do anything at the elementary or middle school level because mathematically talented students cannot be identified until high school.
18. We aren't really sure what to do, so we think the best approach is to do nothing.

activity. Although these are all valuable enrichment activities, they do not advance the student's understanding of mathematics.

Sometimes, there are opportunities for math extensions through these enrichment programs, but these are offered in addition to, rather than in place of, the grade-level math curriculum. Although gifted programming provides many opportunities, by its very nature it cannot provide the one thing that is needed to develop the talents of mathematically able students: systematic progression through challenging curriculum, which is part of a predetermined scope and sequence. Few gifted programs provide this type of structured progression through mathematics.

Additionally, the selection criteria for enrichment-based gifted programs are typically intended to identify students based upon their general ability. For example—and despite the recommendations against using composite cut-off scores (Lohman, Gambrell, & Lakin, 2008)—many programs have a set cut-off of 130. Thus, a student with a verbal score of 125, a quantitative score of 129, and a nonverbal score of 120 would *never* make the cut-off for gifted programming. Nevertheless, students with quantitative scores such as the student in the example above need to have further assessment to determine their readiness for advanced mathematics.

Excuse 2: We already have a program for mathematically talented students.

Response: Mathematically gifted students are a varied group with respect to their interests (Lupkowski-Shoplik & Assouline, 2001) and abilities (Colangelo, Assouline, & Lu, 1994; Lupkowski-Shoplik & Swiatek, 1999). The curriculum for these students should reflect that diversity of talent (Sheffield, 1999a), which is why we do not recommend just one curriculum or one program for all mathematically talented students.

For mathematically talented students, we advocate using resources that are readily available by adapting the school's mathematics curriculum. For example, we might use the curriculum from a higher grade level with younger talented students. Chapters 6 and 7 provide a range of ideas that will help school personnel devise the right level and pace of curriculum for talented students.

Excuse 3: Specialized programming is not necessary because enrichment is the "safest" way to challenge mathematically talented students and every math class has extension opportunities for students who finish their assignments early.

Response: When using this excuse, "safe" is typically a synonym for "effective." Fortunately, there is extensive research to dismiss this excuse. The most effective intervention for highly able students is acceleration (Rogers, 2004, 2007). On a broader note, it's more appropriate to recognize that not all mathematically talented students are alike and demonstrate a variety of needs based upon abilities. Thus, programming should be similarly varied, which would include both enrichment and accelerative opportunities.

A student may have the good fortune to be placed with a teacher who is capable of appropriately enriching the math curriculum. The teacher would assign material that is at a greater depth than what is presented to students in the regular mathematics education program and also would bring in new topics that are not a part of the regular curriculum. The difficulty with this approach is that it depends heavily on having a teacher who is well prepared in mathematics and has the time and inclination to differentiate to a radical degree for the mathematically talented student. Also there is the concern that the

student will no longer have the same type of mathematically enriched curriculum once he or she moves to the next grade level.

For mathematics, good acceleration includes much enrichment, and good enrichment is accelerative (J. C. Stanley, personal communication, May 1998). Mathematics builds upon itself so that, in reality, it is extremely difficult to "enrich" a student without actually accelerating his or her study of mathematics. All of these issues are discussed thoroughly in this book, especially in Chapters 4, 6, and 7.

Excuse 4: Teachers in our school are trained in differentiating the curriculum and can make adjustments for mathematically talented students within the regular classroom.

Response: Differentiation of curriculum is synonymous with Vygotsky's *zone of proximal development*, which is described by McGrew and Flanagan (1998) as the zone where there is an "optimal match." An optimal match is that ideal situation where learning opportunities are matched to a learner's needs and readiness. As a model, differentiation is rich and robust because it recognizes the individual differences inherent in classrooms and focuses on a systematic approach to tailor the curriculum and instruction to meeting the needs of the learner (Tomlinson & Jarvis, 2009). Therefore, it is logical to assume that educators who have received in-service training about differentiation would be able to handily meet the needs of gifted learners. There are two considerations about this assumption. First, "differentiation is just hard work. It requires more preparation, formal and informal assessment, and well-developed classroom-management skills. It cannot become a reality in a school or school district all at once by administrative fiat" (Borland, 2009, p. 115). The second point is aptly made by Hertberg-Davis (2009), "differentiation of instruction . . . like any approach to educating gifted students . . . functions best as a critical component within a spectrum of services provided for high-ability learners" (p. 253).

Excuse 5: The National Council of Teachers of Mathematics (NCTM) standards recommend that prealgebra, algebra, and geometry be woven throughout the school years beginning in

prekindergarten. This guarantees that mathematically talented students will have a differentiated curriculum and that prealgebra and algebra, in particular, should not be provided as separate courses for middle school students.

Response: This interpretation of the position of the National Council of Teachers of Mathematics (NCTM) is just that—an interpretation. In fact, in the NCTM *Principles and Standards* (2000), we found the following statement: "In recent years, the possibility and necessity of students' gaining facility in algebraic thinking have been widely recognized. Accordingly, these Standards propose a significant amount of algebra for the middle grades" (p. 211). It is also important to remember that NCTM's recommendations are generally intended for all students. Therefore, it is up to the educator and parent to work together to consider the ways in which the curriculum standards can be used to assist them in addressing the needs of their mathematically talented students.

Advocating that algebra and geometry be introduced to all students beginning with the primary grades is a positive shift. However, there are many students who are ready for actual coursework in algebra and geometry prior to 8th, 9th, or 10th grades when this coursework is typically introduced. Mathematically talented students still need a differentiated curriculum that allows them to move forward within the traditional sequence of courses. Because they are very able in math, they will be ready for an algebra class before eighth grade.

Excuse 6: Mathematically talented students who are allowed to move through the curriculum quickly will run out of classes to take before they finish high school.

Response: This pretext for not implementing advanced programming is usually mentioned when considering the possibility of acceleration, even at a young age. Of course, it is important to think about the long-term impact of acceleration, even when a student is in the early grades. It is true that if students skip one or more years of mathematics they might not have any more courses to take in their high school building when they reach 11th or 12th grade. However, there is always more mathematics to study (ask any high school math teacher

or college mathematics professor!). It's just a matter of figuring out how to get the right course to the student. Students, educators, and parents might need to be flexible and creative to solve this problem. For example, a student might study math with a mentor in 4th grade, study geometry with high school students while in 6th grade, and take a calculus course on a college campus while in 11th grade. With the increasing availability of computers and Internet-based courses, students in rural areas no longer have to be left out of these opportunities, and parents and educators do not have to be as concerned about the problems of transportation and scheduling. Chapters 5 and 7 provide information about the many options available for students.

Excuse 7: Mathematically talented students aren't ready to study algebra until eighth or ninth grade.

Response: This single statement implies multiple excuses, and we respond accordingly. First, in a traditional mathematics sequence, algebra is not formally introduced until ninth grade or, at best, eighth grade. In some school settings, educators react strongly to exceptions to the long-accepted sequence. In addition to the historical component to this excuse, there also has been a concern that students younger than grade 8 may not be developmentally ready for the formal, abstract reasoning required by algebraic principles. However, we have evidence that many fifth and sixth graders have successfully completed algebra or geometry and have moved into higher level math classes and have been very successful.

Furthermore, the work of Julian Stanley and his colleagues have discounted the developmental aspects of this excuse. For more than 40 years, Stanley and his colleagues (Lupkowski-Shoplik, Benbow, Assouline, & Brody, 2003) have been discovering students who are ready for algebra well before grade 9 and many who are ready as early as grades 5 or 6 (and a few who are ready well before grade 5!). Benbow and Lubinski's (1996) longitudinal research with the Study of Mathematically Precocious Youth (SMPY) at Vanderbilt University has had a far-reaching effect and, as a result, many students have been accelerated into algebra.

The report *Foundations for Success* (National Mathematics Advisory Panel, 2008) explicitly states that "all school districts should ensure that all prepared students have access to an authentic algebra course—and should prepare more students than present to enroll in such a course by Grade 8" (p. xviii). Logically, mathematically gifted students will need to have access to algebra earlier than their same-aged peers.

Excuse 8: We shouldn't allow accelerated programming because students who are accelerated in mathematics will burn out before they reach college.

Response: The issue of "burnout" seems to come up often, but our experience and research indicate that this is not the real problem. Rather, the real problem is the nondiscovery of students who are ready for a challenging curriculum and the fizzling out of their talent development. Mathematically talented students who are discovered at a young age and receive appropriate opportunities to develop their mathematical talent continue to excel in mathematics throughout their school careers and beyond. Waxman, Robinson, and Mukhopadhayay (1996a) found that talented students maintained their advantage: Children in their research group tested at the ages of 3–5 were retested at the end of first grade and again after second grade. As a group, they continued to score above their peers, and they made impressive gains over those years.

Julian Stanley, Camilla Benbow, and their colleagues have conducted a fascinating longitudinal study of exceptionally talented students whose academic talents were discovered at the age of 12 or 13 (Benbow & Lubinski, 1996; Benbow & Stanley, 1983; Lubinski, 2004). These students have been studied into graduate school and beyond. The results clearly show that not only can students be identified at a young age, but also their abilities and achievements continue at high levels into adulthood. For example, in one cohort of high-achieving graduate students participating in the most selective graduate programs in the sciences, many of them had been identified early, enrolled in special programs in elementary school and junior high, participated in accelerative opportunities through high school, maintained stellar academic records, and gained admittance into the most select undergraduate colleges and universities. Most of them had

taken advantage of research opportunities during their undergraduate years.

> Clearly, we have here a pattern of excellence begetting excellence. . . . They took advantage of educational opportunities presented to them and this began early. It appeared to have a snowball effect on their achievements. With each stage their academic credentials stood out more and more. (Benbow, Lubinski, & Sanjani, 1999, p. 66)

Exceptional talents that manifest themselves early do not disappear; when nurtured, they continue to develop.

Excuse 9: Accelerated math programs lead to gaps in students' mathematical foundation.

Response: Educators often are concerned that if students skip a grade or a section of mathematics they will never have an opportunity to study the material and will have gaps in their mathematics background. There is a simple solution to this problem: Students can be pretested to determine what they do not know. Then, they can work with a tutor or mentor on the topics they do not yet understand so they can "fill in" those gaps. This pretesting and follow-up instruction also prevent the unfortunate situation where mathematically talented students who have already learned the majority of the material to be presented in a course are still required to sit through an entire year of that mathematics course. An in-depth explanation of this Diagnostic Testing→Prescriptive Instruction (DT→PI) model is provided in Chapter 4.

Excuse 10: On the pretest, the student didn't earn 100%, so he or she doesn't qualify for advanced programming.

Response: Implicit in this excuse is the idea that mathematically talented students do not make errors—or if they do, then they are not truly mathematically talented. The student version of this "mastery = 100% correct" mindset is known as *perfectionism*. In fact, it is unreasonable to expect perfection from anyone!

Students can demonstrate excellent understanding of a topic or concept, yet still make simple mistakes. However, this begs the question: If we do not require 100% correct on a test to demonstrate mastery, what is the appropriate indicator for mastery?
According to Nitko (1996):

> A frequently used passing score is 80% (or as near as you can come to this with the number of items you have for assessing a learning target). There is no educational justification for 80%, however. The important point is not the exact value of the passing score or passing percentage. Rather, it is the *minimum level of knowledge* [emphasis added] a student needs to demonstrate with respect to each learning target to benefit from further instruction. This may vary from one learning target to the next. Use your own judgment, remembering that setting a standard too low or too high results in misclassifying students as masters or non-masters. (p. 291)

Generally, setting a pretest mastery level at 85% or 90% correct ensures the student has indeed mastered the material, yet some room has been allowed for minor errors. The purpose of pretesting should always be diagnostic in nature. A pretest should help educators determine where instruction should begin. If a student earns 100% on a pretest, the educator needs to move on rapidly to a new topic. Pretesting should continue until the student has missed enough problems to have further instruction, but is not frustrated by the difficulty level of the assessment. For problems missed on the pretest, it is helpful if the teacher asks the student to rework the problem, showing all of his or her work. In this way, the teacher can determine if the student answered the question incorrectly because he or she did not understand the concept or because of a careless error.

Excuse 11: A student who makes mistakes in computation doesn't need advanced programming.

Response: Research has shown that many mathematically talented students have an excellent understanding of advanced mathematical

concepts while simultaneously having relatively less developed computational skills (Lupkowski-Shoplik, Sayler, & Assouline, 1994; Rotigel, 2000). This means that their computation skills may lag significantly behind their understanding of mathematics, resulting in a students who understand abstract mathematical ideas (such as algebraic variables), yet often gets the wrong answer when multiplying fractions. Rotigel and Lupkowski-Shoplik (1999) suggested that students show this weakness because:

1. they have demonstrated competence with a computational skill, but are not allowed to move on to another topic, so they make careless mistakes;
2. they may prefer to do computations in their heads as a way to stimulate themselves intellectually, because the material they study in school is not interesting or challenging;
3. some underlying cognitive construct may cause the difference;
4. direct instruction is generally required for students to learn computational skills, while students may develop a conceptual understanding on their own; or
5. teachers are impressed by these students' exceptional abilities and may assume that drill or practice on routine computation tasks is unnecessary.

Unfortunately, the typical elementary school math curriculum doesn't allow these talented youngsters to demonstrate their reasoning skills. Nobody knows how good their conceptual skills are because so much time in the elementary curriculum is devoted to practicing rote tasks.

Excuse 12: Too much time is devoted to standardized testing in our schools, so parents or teachers shouldn't ask for additional testing.

Response: School personnel are sensitive to taking too much instructional time away for testing. Many schools have eliminated testing students with nationally standardized tests and instead test students only with the state-level assessments required by the No Child Left Behind (NCLB, 2001) legislation. Many parents have expressed frustration because the only information they have from

their school about their mathematically talented child is the basic state-level assessment, which measures whether or not students are making "adequate yearly progress" (AYP). These state assessments are designed to help schools determine if the school population is attaining the academic standards established for that grade level. The state assessments are not designed to measure whether or not an individual student has learned something new that school year or what that student has yet to learn. Therefore, the state assessments do not provide enough information to develop a challenging program for a mathematically talented student. What is needed is specific testing that measures mathematics ability and achievement.

In the short term, there is the perception that testing takes away time from instruction. Let's do the math: If there are 1,080 hours in an academic year and the amount of standardized testing is 10 hours, we're talking about less than 1% of the total academic year being used in testing. With this modest investment of a few hours of time, in the long-term, appropriate assessment actually helps us to be more efficient and effective in programming.

Excuse 13: Our district doesn't accept any outside results; our district uses only nationally standardized grade-level tests to identify students for the gifted program.

Response: Although grade-level testing information may be a useful component of the screening process, the results are not sufficiently informative to be used to create a program that addresses the needs of the student(s). Using only grade-level tests to assess exceptionally talented students provides very limited information. For example, two students might earn identical scores on a grade-level test (such as a 99th percentile on the mathematics total section of a basic skills battery), but in reality demonstrate very different abilities in mathematics; their classroom teacher might notice that one student still seems much more capable than the other, in spite of test scores that look the same. If the students were given an above-level test (one designed for older students), they might earn very different scores, thus demonstrating different abilities. An above-level test measures the students' abilities more accurately and allows them to showcase their abilities. The more

specific information we have now obtained is much more helpful in making programming decisions (see Benbow, 1992a, for a discussion of how individual differences in above-level test scores in seventh and eighth grades predicted achievements 10 years later). The concept of above-level testing is the foundation upon which the talent search model was built (Chapters 3 and 5 provide more detailed information about above-level testing and talent searches). Additionally, much of the research used to dispel the excuses presented in this chapter derives from talent search data.

Excuse 14: Our district has stopped administering nationally standardized tests and uses only tests developed by the local district or building.

Response: Curriculum specialists often have developed tests that are based upon the curriculum used by the district. Information from this type of test can be very helpful in the actual determination of a proper sequence of curriculum. However, these data cannot replace the information from the comparisons of normative groups that are provided by scores from standardized tests, especially when it comes to understanding the need for an accelerated pace. Therefore, locally developed assessments are necessary but not sufficient.

Excuse 15: Acceleration in math means taking math with students in the next higher grade and we have a policy against that.

Response: There are two parts to this excuse. The first part implies that acceleration is the *only* option available. Of course, that is not necessarily the case. In fact, the drawback of taking math with older students, aside from the scheduling problems that might occur, is that the math is still being taught for the average students in the class. Although accelerated students may at first enjoy the higher level of mathematics, soon they will notice that the math is still being taught at the same slow pace as before, and they might again begin asking for more challenges in math.

The second part of this excuse moves the issue from the individual level to the larger policy level. According to the 2008–2009 *State of the States in Gifted Education* (National Association for Gifted Children

[NAGC], 2009), only eight states had a policy regarding acceleration. In 2009, a work group comprised of the Belin-Blank Center's Institute for Research and Policy on Acceleration, the National Association for Gifted Children, and the Council of State Directors of Programs for the Gifted developed *Guidelines for Developing an Academic Acceleration Policy*. This helpful document is available online at no cost at: http://www.nagc.org or http://www.accelerationinstitute.org. In addition, more than 50 years of research regarding acceleration was summarized in *A Nation Deceived: How Schools Hold Back America's Brightest Students* (Colangelo et al., 2004a, 2004b).

Excuse 16: The student has already skipped a grade in mathematics. That is sufficient acceleration, and no additional changes are needed in the student's mathematics program.

Response: We have long known the benefits of academic acceleration and *A Nation Deceived* (Colangelo et al., 2004a, 2004b) has made this information accessible to every educator. In that report, 18 forms of acceleration were identified (Southern & Jones, 2004) and most of them are *not mutually exclusive*. Thus, a grade-skip may be an important step to ensuring challenge, but as the student progresses, and as other interests emerge, it may be necessary to accelerate the student again, or to provide for additional challenge such as a locating a mentor.

Excuse 17: There is no reason to do anything at the elementary or middle school level because mathematically talented students cannot be identified until high school.

Response: The general nature of the elementary mathematics curriculum might lead one to believe that it is only when students are at the secondary level that they are ready to be recognized as mathematically talented. However, it is necessary to identify mathematically talented students well before high school so that adjustments can be made in their educational programs.

Using above-level standardized tests, students as young as third grade have been identified successfully for challenging programs in mathematics that are offered on university campuses such as Carnegie

Mellon University and The University of Iowa. These students were identified through the Elementary Student Talent Search testing program described in Chapter 5. An above-level testing program has been effective in selecting students who will perform well in fast-paced, high-level summer classes for academically talented students (Swiatek & Lupkowski-Shoplik, 2000b). In many cases, the test results confirm what the parents have long suspected: that their child has a talent in mathematics. Much anecdotal evidence also is available concerning mathematically talented children younger than age 8. Information provided by parents in these cases is especially helpful. Nancy Robinson and her colleagues (Waxman et al., 1996b) have systematically identified preschoolers and kindergartners for special programs in mathematics at the University of Washington. Students in the Waxman et al. (1996b) study were first identified by their parents, and individual achievement tests confirmed the parent nominations.

Excuse 18: We aren't really sure what to do, so we think the best approach is to do nothing.

Response: Doing nothing is still an intervention. Some educators are concerned about the social-emotional effects of alternative programming for talented students. For that reason, they have made the decision that "safe is better than sorry." What is missing from this reasoning is the fact that choosing to not implement a program is still an intervention and the research demonstrates that when there is no programming, there is no growth.

Conclusion: What Should Be Done?

In this chapter, we have briefly addressed the major excuses that we hear most often as reasons not to provide specialized assessment, curriculum, and/or programming for mathematically talented youth. Broadly speaking, these excuses reflect a naïve perspective that ignores individual differences among students. We offered responses to these excuses and introduced facts to refute the excuses. The remaining chapters of this book will elaborate on these topics.

The one overarching excuse that we find most disturbing is the last one: It is best not to do anything different from the regular programming already offered in school. For parents of mathematically talented students, this can be a particularly frustrating situation that poses a dilemma for them regarding whether or not they should advocate for their child. Therefore, advocacy is the subject of the following chapter.

CHAPTER 2

Advocacy

KEY POINTS

- Parents are a child's primary advocate.
- Advocating for an appropriately challenging program is not the same thing as pushing a child.
- Often, advocacy will result in some form of academic acceleration.
- Effective advocates are informed about the characteristics and needs of mathematically talented students.
- Objective information that can be matched with the available resources within a school is critical for effective advocacy.
- Effective advocates maintain a positive attitude of cooperation with the school throughout the process.

> *Advocacy: "active support of an idea or cause; especially the act of pleading or arguing for something"*
>
> —Retrieved from Google Dictionary

Two key words in this definition—*support* and *pleading*—capture our motivation for devoting an entire chapter to this topic. We continuously receive pleas, typically from parents, for help and support in obtaining challenging curriculum for their students. Thus, a primary focus of this chapter is the parental role as an advocate for talented students. One of our specific goals is to guide parents in seeking useful information and to offer advice about approaching their children's educators to generate a supportive partnership between home and school. On the one hand, parents are well informed about

their children's out-of-school learning experiences, personality, and general level of social-emotional development—in other words, they know their children best and are in the best position to notify educators about their children's academic interests and needs. On the other hand, educators are in the best position to provide curriculum and instruction. Although many teachers initiate and advocate for a building- or district-level change in student programming, our focus here is on the parent who is advocating for an accommodation that will result in more challenging programming and curricula for a specific child. Chapters 6 (Programming), 7 (Curricula and Materials), and 8 (Teaching Mathematically Talented Students) provide information for educators.

Early in the chapter, we recount advocacy journeys by two sets of parents (both mothers). Alexis's mother provides extensive documentation about her experiences in advocating for change for her daughter, beginning with Alexis's entry to kindergarten and continuing through the early years of high school. The journey experienced by Charlie's mother is somewhat shorter (at this point in the journey, Charlie is only in fifth grade) and slightly smoother. Nevertheless, to ensure that he is challenged, Charlie's parents will have to continue to advocate for change.

This chapter concludes with discussions on a variety of topics related to advocacy, including the relationship between gifted education and educating mathematically talented students, advocacy and legal issues, and parent-teacher conferences. Several lists and examples provide the reader with easy access to this information.

Why Should Parents Advocate for Change in Their Child's Educational Program?

Recent research in the field of gifted and talented education supports previous "best practice" findings that engagement in the talent area is critical for the development of talent (see Bloom, 1985; Csikszentmihalyi, Rathunde, & Whalen, 1993; Rogers, 2007). Indeed, Rogers's (2007) synthesis of research highlights that "consistent practice

at progressively more difficult levels in skill, coupled with the talented learner's natural ability to link new knowledge to prior knowledge and skill, accounts for what ultimately is perceived as expert performance" (p. 383). Lack of challenge leads to disengagement—boredom—and little learning takes place when students are bored (Csikszentmihalyi et al., 1993).

To ensure challenge, change is sometimes necessary, which is where the concept of advocacy comes into play. The individuals involved in making decisions about changing a student's educational program may have very different ideas about what the student needs and what resources should be made available to him or her. Sometimes there is even a philosophical discrepancy about the role of either parent or school in terms of how active one or the other should be in advocating for or implementing change. The purpose of this chapter is to help parents and educators understand the advocacy process with respect to attitudes, interactions, and information.

Upon whom does the responsibility of finding appropriate programs with adequate challenges for mathematically talented students fall? Should it be the sole responsibility of the school? We propose that the responsibility ultimately falls on the parents. In fact, Rogers (2002) culled hundreds of research articles, summarized dozens of her own interactions with parents, and overtly deduced,

> it is unlikely that the schools will suggest a uniquely individualized educational plan. So, if a plan is going to be implemented, you must be the one to initiate it, to collect the data, and to design the initial plan. This isn't all bad; after all, you are the one who knows your child best. (p. 8)

Two Journeys: The Long and the Short Roads to Program Change

The first journey is told by Alexis's mother. Her detailed account is found below and is set off.

Case 1: Alexis

Kindergarten: The path from mutual cooperation to a passive perspective of natural development. When she entered kindergarten, Alexis was the only student in her class able to read. Fortunately, her teacher was very supportive and gave Alexis every opportunity to read and write on her own. At the end of the year, a few additional students were able to read and write, and the teacher shared with me her philosophy that there is a wide window in which children develop reading and writing skills and other children will "catch up."

Parents of gifted children often are told that other children will catch up. Of course, the vast majority of students do master reading and writing by grade 2. However, the individual differences that are apparent among students at the beginning of kindergarten remain even as children mature as part of the natural process of development. Average students do not catch up with precocious students. In fact, research supports our observation that the gaps between gifted and average children observed during the primary grades only get wider as children progress in school (Waxman et al., 1996a).

Alexis was both verbally and mathematically talented. She was first identified as being exceptional because of her talent in the verbal area. Although this is something for which her parents will continue to advocate, the struggle for curricular change focused on mathematics because Alexis's educators were very resistant to any proposed change in this area.

Grade 1: The bumpy road from passive natural development to parent-school conflict. When Alexis entered first grade, her reading skills far exceeded the other students in her class, and she was dreadfully bored during the reading period. I got the first indication of the difficulties to come when I consulted with Alexis's teacher. I suggested that she give Alexis a writing assignment to occupy her time and to help her hone her writing skills. I was informed that the teacher was not permitted to give anything but a reading assignment during the required reading period. Fortunately,

she agreed to allow Alexis to choose a book to read when she had completed the class assignment. The degree of difference between Alexis's reading ability and that of her peers was still not clearly apparent to us. In retrospect, one indicator of the difference was that her classmates did not recognize Alexis's Halloween costume when she dressed as Pippi Longstocking.

When I picked Alexis up on the last day of first grade, her teacher told me that she had asked each student to set a goal for second grade and that Alexis had said that she wanted to learn division. Her teacher commented, "You know, she already knows multiplication." Well, I hadn't known. "Should I teach her division?" I asked. "No," her teacher responded. "If you teach her division, she will be bored when she learns it in third grade."

Educators often suggest that gifted students not study a new topic (even though the student is ready for the topic) because they anticipate boredom when it is formally introduced in a higher grade or they are concerned that future teachers "won't have anything to teach them." Many teachers do not realize that gifted students have often learned a topic well before they arrived in the classroom and are ready for even more advanced topics (Reis et al., 1993).

Grade 2: On the road of parent-school conflict the exit to cooperation is missed. Even though students were ability-grouped for math and reading and some students were in a talented and gifted (TAG) program, which met weekly for 2 hours, things were actually worse. The high-ability groups in reading and math were operating at a level that was lower than what was needed to keep Alexis engaged. When I suggested to Alexis's reading teacher that Alexis might need some enrichment, she responded that she taught the highest level; therefore, Alexis could not possibly require enrichment. In addition, the teacher suggested that Alexis's immaturity might be the reason for her dissatisfaction with the curriculum. But, I wasn't to worry, she told me. Alexis is young for the class and will eventually catch up. I observed Alexis's "immaturity" on parent observation day. There, in the presence of her classmates' parents, as well as her classmates and teacher, Alexis spent the entire lesson

balancing pencils on her ears. Alexis's behavior was very surprising to me because she was typically more attentive than most children her age (e.g., she could sit through long programs of classical music).

Alexis allied with another bright student and, together, they walked slowly to reading so they would miss part of the class. Alexis complained bitterly about school. The only day she did not resist going to school was on the day she went to the TAG program. By May, Alexis requested that I speak with her principal about her level of unhappiness in school. However, I thought that perhaps Alexis was just having a difficult year and maybe there was a teacher-student mismatch. I decided to wait until third grade.

Concerns about social-emotional development often are cited as a reason for holding students back academically (Assouline, Colangelo, Lupkowski-Shoplik, Lipscomb, & Forstadt, 2009; Colangelo et al., 2004a). Unfortunately, students often act out when they are bored, and these behaviors are perceived as immature. In reality, this is a teacher's way of abdicating the responsibility of addressing the academic reasons for the bright student's acting-out behaviors. Immaturity is a pretext for a "wait and see" approach. Later, parents regret the lost time because the situation rarely improves independently of intervention.

Grade 3: The smooth road of cooperation gives way to conflict and detours. Because Alexis had an absolutely wonderful teacher whom she adored, the year started out great. This upbeat beginning obscured from us the difficulties we would encounter as we traveled through a fog of passivity on the now-rough road of educational advocacy. The first sign of a need to advocate appeared during the parent-teacher conference when Alexis's teacher indicated that Alexis's intellectual ability was far beyond other students in her class. She commented that Alexis had many interesting things to say, but most went over her classmates' heads. Around the same time, I began to notice that Alexis was having a much easier time of her schoolwork than other bright children, even other children in her

gifted class. She was selecting books to read that other children in her high-level reading group couldn't read and would not have selected.

Even though Alexis was very advanced verbally, most of her complaints were directed toward the math class. However, when I discussed these concerns with her teacher, she refused to give Alexis additional work on the pretext that it would be perceived as special treatment. However, she encouraged Alexis to do the "challenges" that she provided for any student who had finished his or her work. Furthermore, she suggested that I send additional work for Alexis, as long as it did not go beyond the material being covered in class. However, Alexis told me she didn't want to do the challenges or the work being sent from home because it was just more of the same stuff she was bored with in the first place. She said she would rather daydream. But, she also stated she still wanted to learn division and wanted to know why she hadn't learned it since she had been told it would be taught in third grade.

Alexis's teacher felt she had done all she could and we agreed that the next step was to talk to the principal about the situation. The principal was unavailable, but suggested that we meet with the assistant principal. Little did we suspect that this was part of an administrative strategy to interfere with our attempts to advocate. At that meeting, the assistant principal seemed very understanding. My husband and I suggested a number of possible resolutions, but all were politely declined for one reason or another. The assistant principal recognized that Alexis was very bright, but told us that many children in the school were very bright. She suggested that Alexis had a behavioral problem stemming from an attitude of entitlement and that she felt superior to the other students. This could not have been farther from our (or her teacher's) observation of Alexis. In fact, Alexis had many friends at all levels of ability. We perceived that Alexis was well liked and highly regarded by her friends and that they all knew she was smart—or it was not important to them. At that point, we decided to get a professional opinion.

Detour for the evaluation. The idea of seeking a professional opinion from a psychologist who specialized in evaluating gifted students was reinforced by an e-mail discussion with other parents of gifted students. Essentially, I had stumbled upon an e-mail list for parents of gifted students called TAGFAM (see http://www.tagfam.org). Until discovering this list, I was unaware that there were different

levels of giftedness and that there were different approaches to teaching gifted students. My brief participation on this list made me consider that perhaps my daughter needed more than her high-ability reading and math grouping or the TAG pull-out class could provide and that a licensed psychologist could help in obtaining this information.

My first encounter with Dr. Lance, the psychologist specializing in working with gifted students, was via telephone. I recall how shy I was about using the word "gifted" and how I was even more reluctant to assign this description to my child. I told Dr. Lance that I was not sure that I needed to have Alexis evaluated, but was considering the possibility that Alexis's problems in school might stem from her needing more academic challenges, not from the possibility that Alexis had emotional problems. After I revealed more about Alexis's school environment, the psychologist suggested that further evaluation would be helpful.

The first part of the evaluation consisted of a series of surveys to complete, an interview with me and my husband, and a portfolio assessment of Alexis's work. Dr. Lance then administered to Alexis an IQ test and an achievement test. From these results, Dr. Lance suggested an additional IQ test, which was administered at a later date. The results of the tests and evaluation were compiled into a report that was shared with the school. Among other things, the achievement tests indicated that, as a third grader, Alexis was already performing computational math at a sixth-grade level (her lowest achievement test score). So, she apparently had learned division, although it had never been taught to her formally.

Yield for a meeting with the school. After sending the psychologist's report to the school, we scheduled another meeting with the principal. When we arrived, we were told that the principal had, unfortunately, been called away. The assistant principal greeted my husband and me and politely suggested that the psychologist, who had accompanied us, wait in the hall during the meeting. In our city, a parent can bring anyone he or she likes to a meeting with the school. I insisted that Dr. Lance was necessary if there was anything in the report that needed explaining. The assistant principal indicated that there would not be a need for an explanation and we were on

"her turf" now. However, Dr. Lance insisted on being present and was included in the meeting.

Our goal for this first meeting was to improve Alexis's attitude toward school. Even with an exciting, stimulating teacher, Alexis was growing increasingly dissatisfied. She was reluctant to go to school and overtly expressed that she was not learning anything in school and that she could barely tolerate sitting through class. In addition, her work began to show signs of tuning out; for example, sometimes she could not answer homework questions because she had daydreamed through a lesson. The assistant principal suggested that these problems were due to behavioral/emotional problems, not academic problems. She went on to say that the school was one of the best in the city with some of the brightest students, thus the school was already providing for academically gifted students. It was suggested that perhaps we were not doing enough for Alexis outside of school.

During that meeting, every suggestion we generated was shot down for one reason or another. In particular, we suggested that Alexis move to a higher grade for math. We were told that was not possible because then she would have nowhere to go when she got to sixth grade. However, the assistant principal agreed to excuse Alexis from math class. This decision was based upon evidence that Alexis had perfect or near-perfect scores on several tests that the class had been taking as practice for an upcoming standardized test. So, Alexis was excused from math and would instead work with the TAG teacher on a writing assignment. Everyone was in agreement—the assistant principal, the classroom teacher, the TAG teacher, Alexis, and us (her parents). After Alexis worked on the writing assignment for 3 days, I got a call from the principal. She indicated that I had misunderstood and that Alexis couldn't be excused from math because Alexis, like all of the other students, needed to practice for the standardized math test. This was very demoralizing for Alexis. In fact, after this incident, Alexis became a reluctant writer. She had gotten the message that her writing was "bad," mostly because the TAG teacher wouldn't work with her on the project.

The principal also indicated that she would not discuss adjustments for Alexis until receiving Alexis's standardized test scores.

She reasoned that those scores, more than the scores from the psychologist's evaluation, would indicate Alexis's ability.

Further meetings with the principal are dead-ended. One day before Alexis's test scores came back, the principal called me at work to express her concern that I had provided Alexis with inappropriate reading material. I was shocked and asked for clarification. The principal had observed Alexis's class when the students were completing an assignment. For the assignment, students were to choose a play and select a scene to perform in class. The principal observed Alexis perform Amanda's monologue from Tennessee William's ***The Glass Menagerie***, a play she deemed inappropriate for a third grader. I responded that my recollection of the play did not include sexual content, foul language, or violence. I also made it clear that Alexis had selected the play herself. "But," the principal replied, "her presentation to the students will not endear her to them." I used the opportunity to suggest that perhaps now she understood the problem and might work with us to adjust Alexis's education appropriately. Instead, she suggested that I read David Elkind's book, ***The Hurried Child***, although she acknowledged that she had not read it.

The second meeting, which was to have been scheduled after the district-administered test results were in, was never scheduled.

The end of the road. We had, by this point in time, decided that Alexis could not return to this school in September because it was a "toxic" environment for her. Even if we were able to get the school to agree to some sort of academic adjustment, we had the impression that the environment would be too unpleasant, difficult, or both for Alexis to continue in that school. So, we started the application process to private schools. In addition, Alexis had gotten some relief from the imposed academic boredom by enrolling in the Education Program for Gifted Youth (EPGY) through Stanford University (see the Resources section for more information about this distance-learning opportunity).

However, I was very angry. It did not sit well with me that I was supposedly sending Alexis to the best public school in our city, but that her needs couldn't be met there because she was too smart.

So, while we considered private school, I continued to advocate for Alexis in public school.

Alexis had been in this school for almost 4 years (K–3). My husband and I were very active members of the Parents' Association and we participated in school activities in ways that were helpful. We thought we had developed positive relationships with parents, teachers, the assistant principal, and the principal. It seemed to us that Alexis's "profile" was ideal. She was a socially well-adjusted child who was liked by other children and her teachers. She was generally well-behaved and performed well academically. In our perception, there were really no negative aspects to her behavioral, academic, or social record in school. So, it was especially surprising to us that the school was so recalcitrant to making adjustments and so antagonistic to us in general. I will add that we never demanded anything in particular. We regarded the faculty and administrators of the school as experts in education and treated them with respect. What we really wanted, initially, was for them to advise us on how to best accommodate Alexis so that she would remain happy in this "excellent" school.

The roadblocks described above are, regrettably, not uncommon. Neither are excuses such as, "But, there are other bright students in this school," or "If we do it for your child, we'll have to do it for all of the children." One of the most dangerous roadblocks is found in the reluctance of school officials to recognize the results from tests other than school-administered tests. Parents must keep in mind that they are making a reasonable request: match the curriculum to the abilities and achievements of the child. In order to do this, Alexis's family continued to gather and document:

- specific information about Alexis's abilities,
- expert opinions about Alexis's educational needs,
- research pertinent to Alexis's situation, and
- information about the experiences of other families of gifted children.

Grade 4: The road of conflict is full of legal potholes.

Throughout that year, I tried to meet with the district superintendent, who did not return my calls. I also researched the educational laws,

statutes, and policies for Alexis's school, school district, city school system, and state. My goal was to find written indications of policies that would help in our quest to get Alexis's needs met. I found out that it was not immediately obvious where one might find help. Surprisingly, I got the most sympathy and information from people who had dealt with the educational rights of students with physical, mental, and learning disabilities. These individuals seemed to know the laws and how to implement them within the system. However, the laws that apply to students with disabilities do not apply to gifted students in my state.

I consulted three lawyers, all of whom told me that there were no legal provisions for gifted students in my state and city and, therefore, they could not help me. I called every educational administrator I could think of, including my district office, the chancellor's office, and the state Department of Education office. I called the education editors of the major newspapers in my city. I called the offices of my local, state, and federal representatives. None could help, although many suggested that they would "look into the matter" by calling the district superintendent. Several called me after talking with someone at the district. So, at the least, the district superintendent was getting calls from these various officials or their representatives.

Although most students in our city attend their zoned middle school after elementary school, there is a special magnet middle school in our city that draws students from a number of zones. I knew of this middle school and was planning for Alexis to apply there when the time came. One of the big attractions for us was that this school has a philosophy of attending to the individual needs of all its students. It occurred to me that I could resolve this by grade acceleration to fifth grade so that Alexis could begin middle school at this particular school.

I started by suggesting this to the principal of Alexis's elementary school, who made it clear that she would never approve of grade acceleration for Alexis, even though it would mean getting rid of us! So, I tried to bypass the principal. Although the receiving middle school was favorably disposed to this idea and was willing to consider Alexis as a "special case," they could not make the grade acceleration occur. Another individual in a high-level position in the district wrote a letter on Alexis's behalf, indicating the district's support; however, she could not make the grade acceleration occur either. In fact, the only

person with the authority to accelerate Alexis was the one person who refused to consider it: the principal of Alexis's school.

The scores on the standardized tests taken in school indicated that Alexis received perfect scores on all four tests (city reading and math; state reading and math). I called to remind the principal that a meeting was to be scheduled upon receipt of the standardized test scores. Not only did she refuse to meet with us, but she was verbally abusive to us. I immediately called the district office to report our conversation. About 10 minutes later, I received an apologetic phone call from the principal. Nonetheless, she made it clear that, because she would not grade accelerate Alexis, there was no reason for her to meet with us and she would not take the time to do so. I sent a letter to the school chancellor, documenting our experience.

At the end of the school year, the district superintendent called to say he would meet with us to discuss our situation as well as to consider a transfer of Alexis to a different school. I invited the district official who had been sympathetic to our situation and the psychologist to attend that meeting. I asked Alexis's teacher to write a letter (to me) describing how she experienced Alexis in her classroom. Much to my chagrin, the principal's views of Alexis had influenced the teacher, and her letter stated things diametrically opposed to her previous comments and observations. Among other things, the teacher's letter to us indicated that Alexis refused to do the class "math challenges." In my opinion, the teacher had not accurately (or honestly) described Alexis.

We prepared carefully for the meeting with the superintendent. I had documentation of my meetings with the assistant principal at Alexis's school, which included written correspondence. I brought a few articles about acceleration and gifted children and all of the official documentation of educational policy that applied to Alexis's situation. I also carefully selected samples of Alexis's work that showed high ability and indicated her boredom with school. I had the tests from the EPGY work Alexis was doing. I also included evaluations of Alexis and her work in the teacher's handwriting. In addition, I brought some of the "math challenges" the teacher provided and similar work Alexis had done at home. I met with the psychologist in advance of the meeting to plan how we would make our presentation. I was concerned because it had come up in meetings with the assistant principal that Alexis's short stature would become an issue. Fortunately, I was able

to obtain a letter from a parent on TAGFAM describing the successful acceleration of her short-statured son.

The district superintendent was receptive to what we had to say, although he expressed concern about Alexis's emotional health. My husband made it clear to him that we also were concerned with Alexis's emotional health and our only reason for seeking an adjustment to her academic situation was to ensure her emotional well-being. The district superintendent expressed concern about Alexis's short stature. I produced the letter from the TAGFAM parent. The district superintendent expressed concern about the fact that Alexis refused the more challenging work the teacher had provided. I showed him the "challenges," then I showed him similar, but more complex work Alexis was doing at home and told him that Alexis refused the "challenges" because they were not interesting to her. When he looked at the work Alexis was doing at home (they were complex logic puzzles), he expressed surprise and understanding. We left the meeting with an agreement that he would help place Alexis in a different school with a more receptive principal who would give Alexis a trial period in the fifth grade—a one-grade acceleration.

The results of the standardized testing were informative, but so were specific examples of the student's work and evaluations of Alexis's work by the teacher. The decision to accelerate Alexis took place before the availability of the *Iowa Acceleration Scale* (Assouline, Colangelo, Lupkowski-Shoplik, Lipscomb, & Forstadt, 2009), which would have been a significant aid in the acceleration decision-making process.

Grade 5: The smooth ride on the road of cooperation.

The decision to skip grade 4 was made after the close of the school year, so we were unable to meet with the principal of the new school. We decided to take a chance on this arrangement, with the idea that, if it didn't work out, we would still have the option of private school.

About a week before the start of school in September, I was able to bring Alexis to visit the school and meet with the principal. The difference in the principal's attitude, compared to the principal of the previous school, was astounding. This principal talked directly to Alexis. He told her that it was important to him to see that she

was happy in school because only then would she become a lifelong learner. He called her prospective classroom teacher into his office, and she brought Alexis to the classroom for a visit. During Alexis's visit to the classroom, we talked with the principal about our concerns regarding the acceleration; we wanted him to remain flexible with academic adjustments for Alexis without penalty. We wanted the ability to opt out if, for any reason, we felt it was not right for Alexis. Moreover, we wanted the school to do everything possible to ensure the success of the acceleration. I had known of students who were accelerated, only to have the situation undermined by a difficult teacher or administrator or by a lack of sensitivity to problems that might ensue. Both the principal and the teacher were very agreeable.

The acceleration went without a hitch. In fact, there were even fewer problems than I had anticipated. Alexis made friends rather easily, performed well academically and was liked by her classmates and teachers. By October, she was able to invite as many friends from her new school as from her previous school to her birthday party.

At the beginning of the academic year, I tried to convince Alexis that she didn't need to continue the EPGY distance-learning program for math. My reasoning was that she would be in a math class more closely matched to her ability, and I was reluctant for her to have the extra burden of an additional math assignment each day. But, Alexis insisted on continuing EPGY, although she didn't want anyone to know about it.

A few months into the fall, Alexis decided that she didn't want the extra work and stopped the EPGY program. However, by March, she was asking to do EPGY again. This time, however, she wanted to do EPGY instead of math in school and instead of the math homework assigned by the teacher. I suggested this at the spring parent-teacher conference and was absolutely flabbergasted when the teacher responded that it would be okay and all that was needed was to do this without jeopardizing Alexis's relationships with her classmates. I asked if she needed to talk to the principal about this. She indicated that she did, but it would not be a problem. After working out a few details, Alexis started EPGY, this time through Johns Hopkins University. We decided that the best way to do this was for Alexis to do the computer-based lessons at home and bring the paper-based homework to do in the classroom during the scheduled math lesson. We also agreed that, if for any reason the teacher felt it necessary

for Alexis to participate in a math class or do some of her assigned homework, Alexis would. In addition, Alexis would have the option of choosing to participate in the math class and do the assigned homework any time she wanted to. For the remainder of the year, there were only three homework assignments the teacher asked Alexis to do. I understood from Alexis and the teacher that Alexis participated in the math class about once a week, usually when they did some sort of group work. Also, I sent the EPGY reports to the teacher so that she could monitor Alexis's progress and performance.

The transfer to this school, within the same district, seemed to make all of the difference in the world. I am firmly convinced that finding a school that is the right fit for your child is more important than any one thing a school could provide. In our case, we needed a school whose teachers and administrators recognized that Alexis's giftedness required some academic adjustments. In addition, we needed Alexis to be in a school that would be willing to exercise flexibility in applying those academic adjustments. An indication that this school would be flexible came when the principal indicated at our first meeting that, if grade acceleration didn't work out, we would try something else.

Even when things are going smoothly, attention to the child's program is still required. There is still a need to monitor progress and plan for the future.

Grade 6: At the crossroads of decisions. Acceleration from grade 3 to grade 5 was successful and included additional subject acceleration in math through the distance-learning program. However, as fifth grade was nearing conclusion, I knew I had to explore an appropriate placement for middle school. We had a couple of options for public school: (a) Alexis could remain at her current school through sixth grade and go to the middle school it fed into for seventh grade; or (b) she could go the special magnet middle school starting in sixth grade. We discussed our options and concerns with the principal of Alexis's school, who urged us to keep Alexis at his school. He also suggested we speak with the principal of the middle school she would go to as a seventh grader. Because the principals

knew each other well, Alexis's current principal was willing to explain the situation and discuss the need for accommodation and flexibility.

However, the response of the middle school principal was discouraging. He did not want to talk with me because it conflicted with his policy of not talking to parents until after the child is enrolled in his school. But, I was insistent. I explained that the principal had urged me to call him to discuss our "special case" and that I needed to talk with him in order to make a decision. He reluctantly let me explain Alexis's situation to him over the phone, after which he told me that it appeared as though I had a very smart daughter who would likely be placed in honors classes just like every other bright student. I inquired as to how math would be handled, as Alexis had already completed seventh-grade math in fifth grade and was likely to progress more in sixth grade. "She will be placed in honors seventh-grade math," he said. He then explained that they didn't differentiate the curriculum for their students in seventh grade, no matter what. High-ability students were just placed in honors classes. In eighth grade, however, high-ability students could be subject-accelerated one grade in math and science (this was a state requirement).

I then decided to call the special magnet middle school and was connected to one of the guidance counselors. I received a similarly lukewarm response. "You say your child is gifted," the counselor said. "Every year I get calls from parents who tell me they have a gifted child and want to know if their child will be getting enough homework." He made it clear that the school was not a school for gifted children. They had heterogeneously grouped classes, no gifted programs, and no honors classes. I told him that I was aware of the mission and philosophy of the school and that I would be more than happy if Alexis never got any homework. However, she was grade-accelerated one year and subject-accelerated in math by several years. I wanted the opportunity to show him Alexis's evaluation and to discuss whether they could accommodate her.

After reviewing Alexis's evaluation, the counselor indicated that he believed Alexis could be happy in his school as long as I wasn't looking for special gifted programs or honors classes. He said that he would make Alexis's teachers aware of her abilities. As far as math was concerned, he specified his understanding that Alexis could not go into a regular sixth-grade math class without having some adjustments

made. Although he did not know the specifics, he assured us there would be an accommodation based upon her needs.

We decided that Alexis would go to this school. I must admit that I was a bit nervous leaving that meeting without a specific commitment. On the other hand, I knew of too many parents who pushed for specific accommodations for their child only to have their child locked into an untenable situation they had to make work, much to the detriment of the child. The truth of the matter was that I didn't even know at that time what to ask for. I couldn't anticipate what grade level of math Alexis would be ready for at the start of the next academic year. I also didn't know if Alexis would be better suited to—or if the school could better accommodate—single-subject acceleration, grade acceleration, EPGY instead of math in school, or a differentiated curriculum.

With each school year, Alexis and her family had several options for school placement. In this case, living in a large city was an advantage for them. However, for some families, there will be geographic constraints. The choices may include making changes within the regular classroom, accelerating the student, or removing the student from the class for distance learning or homeschooling.

Grades 6–9: Back on the road of cooperation. Before the start of sixth grade, I met again with the guidance counselor to review the need for math accommodations. The counselor showed some of the math teachers Alexis's EPGY work. Also, I conferred with the people at EPGY with regard to what they thought appropriate for Alexis. We discussed possibilities including subject acceleration and the use of EPGY instead of classroom math. But, Alexis felt strongly that she wanted to be in a classroom.

Within the first few weeks of school, the guidance counselor discussed with Alexis her math placement, and she was placed in Sequential I Math, the subject-accelerated math class for qualifying eighth graders (this is normally a ninth-grade class). Their rationale was that starting Alexis out at the highest level would give them the

option to move her down if it didn't work out. They believed this would be easier than moving her up to an advanced class.

We impressed upon the guidance counselor that we wanted this to be a trial; if Alexis were uncomfortable with this arrangement for any reason, she could opt out. As in her previous school, however, we wanted this situation optimized so that it had every chance of success. We expressed our concerns about a number of factors: ease of leaving and returning to class; level of support of the home teacher; and, most importantly, would Alexis be accepted in a class of students several years older and much larger than she? Everyone involved was positive about this arrangement, and it ended up working out quite well.

The math placement was close to perfect for Alexis. The most difficult problem for Alexis was getting used to showing her work. This was not a skill she had honed through the distance-learning math. She did many of the steps in her head, and she didn't have a good sense of what parts of the work she needed to show in order for the teacher to glean her work process and in order to prevent careless errors.

With the appropriate math placement, the mismatch between Alexis's ability level and the curriculum in the other subjects was less important. Even Alexis expressed that she was not concerned when she was not challenged, but only when she was bored. Moreover, the other classes were interesting for the most part, and Alexis appeared to be receiving instruction at an adequate level. In areas in which she excelled, she did more on her own and there was a lot of room for that kind of flexibility. This school had heterogeneously grouped classes that spanned a rather wide spectrum in terms of student abilities. That first year, in sixth grade, Alexis's class included a girl who read at the first-grade level; Alexis, who read above the 12th-grade level; and students at every level in between. Yet, Alexis's academic needs seemed to be well satisfied. I know that other parents of students at all ability levels felt their child's needs were satisfied as well.

There also didn't seem to be any envy that we experienced at the first elementary school. There are usually several parents who resent that you are seeking a special service for your child or that your child is getting special treatment. Those attitudes were nonexistent at this school, probably because most parents felt their own child was getting what he or she needed. Indeed, the treatment of Alexis—that she was put into the advanced math class—was not really viewed as a special

service. The guidance counselor told me that, when he invited Alexis to participate on the math team (which only included eighth graders), he got some questions from other students and parents. But, he said he handled it by explaining that Alexis was qualified for the team.

As the end of the year approached, I knew we had to work out a new solution for the following year. The school did not offer a math course beyond Sequential I Math. Alexis's math teacher offered to teach Sequential II Math to Alexis one-on-one. In addition, I gathered information about distance-learning classes. But, Alexis really wanted to be in a classroom. I spoke with the guidance counselor, and he suggested that I call high schools to see if any offered the appropriate class at a time Alexis could attend and whether they would allow her in the class. I called the math department chairperson at the three high schools closest to our home or the middle school. The only math chairperson who was receptive to the idea was the one at a specialized school for high-ability students. The school would offer the appropriate class early in the morning, and Alexis was welcome to attend on a trial basis. She would have to miss homeroom and part of first period at her middle school 4 days a week. Also, I would have to provide transportation. The guidance counselor at Alexis's middle school was very supportive. He was not concerned that she would miss part of her first period 4 days a week. He expressed that she could easily make up the work. So, we went ahead with this arrangement and found that it worked out quite well.

A major consideration in selecting a high school was whether there would be enough math courses to keep Alexis challenged for 4 years. When she entered ninth grade, she had completed Sequential III Math and was ready for precalculus. At the secondary level, more educators, the school counselor, department chairs, teachers, and Alexis herself were involved in the decision-making process.

Case 2: Charlie

The efforts by Charlie's mother varied somewhat from those of Alexis's mother. Generally, the early attitudes of Charlie's educators

were very positive and there was little need for either of Charlie's parents to actively advocate for a change in his program.

Preschool to fall semester of third grade: A smooth ride. Charlie started kindergarten when he was 4 years old. Although his birth date was before the state's required cut-off date, his parents considered the possibility of holding him back one year because he was so young. Eventually, they decided to send him to kindergarten, even though he was one month shy of his fifth birthday. This is because they consulted extensively with Charlie's preschool teachers who emphasized his strong academic aptitude; thus, Charlie's parents felt comfortable sending him to kindergarten.

Charlie thrived in the primary grades of kindergarten, first, and second. In second grade, he was identified as being "superior in cognitive ability" and also was designated as "all around gifted." This meant that he was exposed to enrichment activities in his second-grade classroom and for the first part of his third-grade year.

Grade 3: The smooth ride is slowed by roadblocks. In the middle of third grade, Charlie's parents moved the family to a small, rural town in a different state. As is sometimes the case, his "gifted" label did not automatically transfer from the first school district to the new one. Complicating the efforts to have him reidentified as gifted was the fact that in his new school students took grade-level standardized tests—a first step in the gifted programming identification process—in the fall. However, in the previous school, the grade-level tests were administered in the spring. Thus, he did not have any grade-level standardized testing in third grade.

Nevertheless, Charlie's mother had excellent documentation of the testing from his former school, which she used to advocate for him. As she so aptly described the situation: "It all comes down to the documentation and statistics." Once identified as gifted, Charlie was eligible for the pull-out program in the new district. In this district, there are three pull-out sessions per year. Each pull-out session is 3 weeks in duration, and students are involved in gifted programming for 2 hours per week. During these sessions, he had an opportunity to work on math puzzles and games, and he also was exposed to an algebra enrichment program.

Grade 4: Roadblocks and detours continue. Interestingly, Charlie's math scores on one of the grade-level standardized tests administered by his school remained fairly static, which meant that his percentile ranking (the number that indicates how he performs relative to other students in that grade) went down slightly. Charlie's mom attributed this decline to the complete lack of challenge during the spring semester of third grade. Although there is no way to prove this assumption, we maintain that the lack of challenge is never a positive when it comes to academic development.

During the first semester of fourth grade, Charlie's parents were informed about the opportunity to participate in one of the university-based regional talent searches (see Chapter 5 for an extensive discussion of the talent search model). He took an above-level test, EXPLORE (a test for eighth graders), and did very well on all of the subtests, which include English, mathematics, reading, and science. Participation in the talent search gave his parents information about his performance on above-level tests and gave him access to advanced programming at that university during the summer between grades 4 and 5.

Grade 5: Slow down to one lane due to road construction. During the course of the fourth-grade year, Charlie's math scores were essentially static. As mentioned above, in the summer between fourth and fifth grade, Charlie took two commuter classes: "Advanced Math" and "Fractions"; he also attended a one-week residential program and took a prealgebra class. In the fall of fifth grade, his standardized math scores "shot up." Again, there is no proof, but his mother attributed this improvement to the summer coursework.

Two things happened during the first semester of fifth grade. First, Charlie participated in a fall talent search, which provided new above-level test scores. Second, Charlie's parents referred him to a private psychologist for additional assessment. After receiving the results of the private assessment, his parents met with school personnel to address their concerns about the program and to plan for the immediate and long-term future.

Charlie's mother described the results of the meeting as "mixed." The negative outcome was the decision to have no immediate change. The three reasons given for this decision were (a) scheduling problems,

(b) concerns about other students, and (c) concerns about careless errors on some of his calculations. In essence, Charlie would remain in fifth-grade mathematics and would continue to work with a mentor outside of the school setting.

The positive outcome of the meeting was that in the spring semester of fifth grade, Charlie would be allowed to test out of the sixth-grade curriculum and the prealgebra content and, if he obtained an 80% on each test, he would be placed into algebra at the beginning of sixth grade. This change was actually a change in policy because previously no fifth grader had been allowed to test out of both sixth-grade math and prealgebra. Therefore, even though Charlie's parents' goal was not to change the system's policies, that was one of the outcomes.

Despite the above-level testing from the talent search, the extensive private assessment, and an extended meeting with school personnel, all of whom were committed to better understanding the degree to which Charlie needed academic challenge, Charlie finished fifth grade in the same way he started—no change! At this point, the summer after grade 5, he will go to a 3-week residential summer program where he will take algebra. When he starts school as a sixth grader, he will presumably test out of both sixth-grade math and prealgebra and will be allowed to take algebra as a sixth grader. It is not an ideal solution, especially because he is not receiving a challenging curriculum from the school, but as we discuss below, not all programming will be ideal.

Roadblocks to Advocacy

Both Alexis's and Charlie's cases illustrate some important attitudinal and policy barriers parents encounter as they advocate for their children. Table 2.1 summarizes these obstacles, which often translate into physical roadblocks preventing action on behalf of the learner. The adjacent column suggests responses to eliminate these roadblocks.

Table 2.1
Roadblocks and Ways to Avoid Them

Roadblocks	Response
There is not a clear indicator as to which educator—the teacher, principal, or superintendent—should be approached first.	Begin at the source. If there is a situation in the classroom that needs to be addressed, talk to the teacher. If you are making a placement decision, talk to the principal. If you have exhausted both of these options, talk to the superintendent.
Key administrators cannot (or do not) attend scheduled meetings.	Call in advance to make sure the principal and other necessary members of the decision-making team (e.g., the current and receiving teachers, school psychologist, and school counselor, if possible) will be present. After-school meetings are preferable to before-school meetings.
Educators are concerned that, "If a student works on advanced material in this grade, the teacher in the next grade won't have anything new to teach."	A student who needs advanced material in one grade will continue to need advanced material in subsequent grades.
The classroom teacher suggests, "The student is immature and doesn't seem ready for more advanced material."	Educators often use social-emotional concerns as a reason for holding students back academically. Unfortunately, students often exhibit immature behaviors when they are bored.
The administrator indicates, "There are other bright students in this school, and they all need enrichment."	Enrichment might be appropriate for some students, but don't let this concern divert the focus on the individual child's needs as determined by performance on advanced material and tests. Document, record, and save work samples.
The parents' concerns are dismissed as a natural desire for all parents to think their child is gifted.	All children are special, but not all children have the same academic needs.
Professional assessments are not considered valid and therefore are not used in program planning.	It is important to communicate with the child's teacher prior to the assessment. When the assessment is complete, review with the psychologist the best way to disseminate the results.

Table 2.1, Continued

Some educators equate parents' efforts on behalf of their gifted child with the "pushy" parent syndrome.	Over a period of several years, the Belin-Blank Center distributed questionnaires to students in its talent search. One question asked gifted students about their parents' involvement in their education and more than 90% of respondents perceived their parents' involvement in their education was "about right." If anything, students want to see their parents more involved.
There is either an overt or a subtle attitude that students who finish assignments more rapidly than their peers can help their classmates.	Students who are having trouble with an assignment need special assistance from a teacher, not from a fellow student. Only the teacher has the appropriate pedagogical background to help students who don't understand a concept.

How Are Gifted Education Programs and Programs for Mathematically Talented Students Related?

Programming for mathematically talented students is strongly connected to gifted education. In fact, for mathematically talented elementary students, it not uncommon for the gifted education teacher to be the professional who is responsible for ensuring that they receive appropriate challenges. Therefore, when parents realize that they need to advocate for a change in their child's math program, they often look for support from the gifted education teacher or coordinator and any associated gifted education policy.

Unfortunately, there is no federal policy for gifted education. However, there is a federal definition, which was first published in 1972 in the Marland Report. In 1988, when Congress reauthorized the Elementary and Secondary Education Act, they included the Jacob K. Javits Gifted and Talented Students Education Act, which represents the only federal program for gifted and talented students.[1] Regrettably,

[1] The act used the following definition: "The term gifted and talented student means children and youths who give evidence of higher performance capability in such areas as intellectual, creative, artistic, or leadership capacity, or in specific academic fields, and who require services or activities not ordinarily provided by the schools in order to develop such capabilities fully."

neither the 1972 document nor the 1988 Jacob K. Javits Gifted and Talented Students Education Act are federal government mandates. A federal mandate would have required school districts to provide special programs for gifted students (Karnes & Marquardt, 2003) and, thus, there is no actual federal gifted education "policy."

Despite the lack of a federal policy, there are multiple approaches to providing services for gifted students throughout the individual states. For example, some states have a mandate for the provision of gifted education, whereas others only "permit" gifted education. That is, the school district may provide gifted education, but the district is not required to accommodate the gifted child. To determine if their state has a statutory mandate requiring special instruction for the gifted, parents can ask an official in their state's Department of Education directly, or they can refer to a publication from the Council of State Directors of Programs for the Gifted, *State of the Nation in Gifted Education: How States Regulate and Support Programs and Services for Gifted and Talented Students*, which is published on a biennial basis and available through the National Association for Gifted Children website (http://www.nagc.org). Parents who attempt a legal solution from a state that does not have a mandate have little chance of legal recourse.

Parents who enter the legal process in a state with a statutory mandate for the provision of gifted education have *somewhat* of a chance in the legal system. Even so, as parents and educators from Pennsylvania—a state that mandates gifted education—discovered, the litigation process can be time-consuming and expensive. *Centennial School District v. Commonwealth Department of Education* (1988) demonstrated what happens when parents are forced to go to court (Karnes & Marquardt, 2003; see also Karnes & Marquardt, 1988). In this case, parents of a boy identified as intellectually gifted and having outstanding abilities in mathematics and reading requested that the school district provide instruction to serve his special talents in the regular classroom (Karnes & Marquardt, 1988). At the time, the student was receiving gifted education services through a resource approach—a pull-out program—which allocated 90 minutes per week. The district claimed that this was the only economically feasible approach. The family requested a due

process hearing, and an impartial officer ruled in favor of the student. This ruling was upheld through two appeals by the school district. Ultimately, the school district appealed to the Pennsylvania Supreme Court, who also ruled in favor of the student by determining that the resource program did not provide the individualized instruction required under state law (i.e., an Individualized Education Program [IEP] that tailors the instruction to the student). However, the court went on to say that the individualized instruction need not "maximize" the child's abilities. Thus, the school district was required to provide an individualized, appropriate curriculum to its gifted children, but it did not have to allocate additional resources to do so.

School districts in Pennsylvania were therefore required to serve the needs of their gifted students using instructional staff and curricula the school district already had in place. "The case provides a judicial precedent that gifted education proponents can use to support the demand for individualized, appropriate education for gifted students, particularly in states statutorily mandating that districts provide gifted education" (Karnes & Marquardt, 2003, p. 597).

The tendency of educators to resist change can be extremely frustrating, and the seemingly intractable position of schools may even result in the pursuit of legal recourse by both parties (e.g., the Centennial School District appealed three times a ruling that favored individualized programming for a student in the district). However, this choice of action is not expedient, and informal or quasiformal resolutions are preferable to those that involve the courts (Karnes & Marquardt, 2003). In fact, Karnes and Marquardt (2003) recommended that the resolution of educational issues begin with the "source" (i.e., teacher or gifted education coordinator) and, if necessary, proceed through negotiation, mediation, due process, administrative review, and, as a last recourse, the courts.

Figure 2.1 illustrates the process of resolving discord between parents and schools about a child's educational program. The process begins with informal means, including parent-teacher conferences, the child study team, and negotiations within the school. If the interactions with the teacher, gifted education coordinator, or principal do not result in the desired change, parents might move on to quasiformal

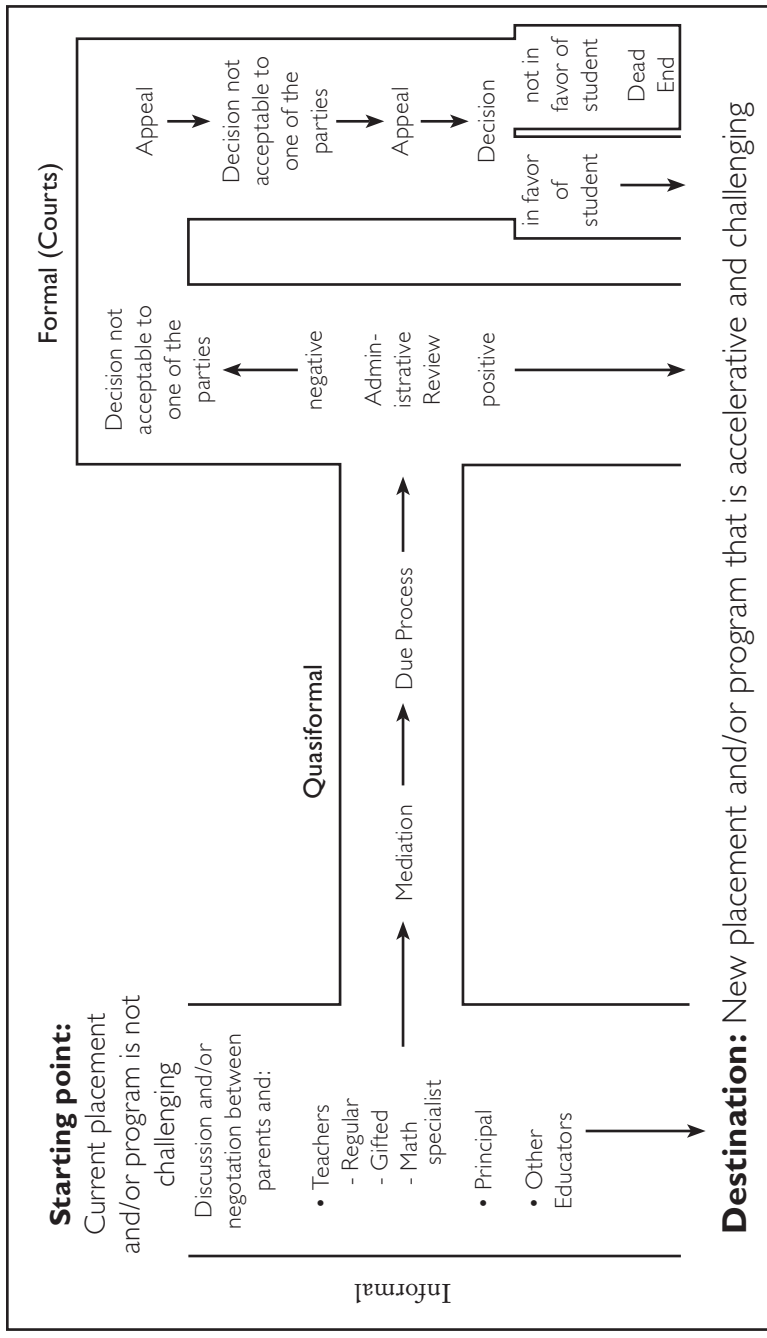

Figure 2.1. Resolution of educational issues.

means, including mediation, administrative review, and due process, in which the parents present the situation to a representative from the state's Department of Education. The exact procedures may vary from one state to another. Again, if the results aren't satisfactory, parents may pursue the court system, with its many roadblocks and delays. Obviously, we recommend that differences be resolved in the informal manner, so that changes can be made quickly and efficiently for the child. The final option, the courts, is the least desirable option for every single party involved.

What Do Parent-Advocates Need to Know?

Advocacy is hard work. The need to advocate often occurs when parents propose curricular or programmatic changes to accommodate a learner's advanced needs but the school/teacher rejects or resists the proposal. Because change is so difficult, advocacy requires sustained effort to obtain the desired result. In the best of situations, the parents' efforts result in an immediate program accommodation that never requires another meeting with the school. At the other end of the spectrum of possible situations is the scenario that requires many parent-teacher conferences, educational assessment, consistent and continual awareness of the curriculum, and annual meetings with teachers and administrators, as detailed in Alexis's case. Most situations will fall somewhere in between, as was the case with Charlie. The pointers below should be helpful to parents as they prepare to advocate for their children.

1. *Become an informed advocate.* Find out what options might be appropriate for your child now and in the future. What resources are available to you? Become informed about organizations, individuals, books, and programs that focus on mathematically talented youth. Reading this book is a great start! Recognize that you may have greater expertise with respect to the needs of mathematically talented students than your child's classroom teacher or principal, but also understand that

you cannot use this expertise to demand change. A parent's knowledge is best used to plan a program cooperatively.

2. *Obtain an objective assessment of your child's abilities and achievements.* This objective assessment will be essential to help you discern what types of programs and curricula will be appropriate for your child and what local resources you have. This assessment might be completed by your school district's psychologist or by a private psychologist in your community. Chapter 3 will be helpful as you begin this process.
3. *Set the tone for a positive partnership between parent and teacher.* At the beginning of the school year, think about what you can do as a parent to ensure your child has a good year in school. Get to know your child's teacher early in the school year and create a spirit of positive cooperation. Regularly attend parent-teacher organization meetings, open house nights, and parent-teacher conferences. Volunteer in your school. Help your child to solve problems in class him- or herself, rather than always calling the teacher. Go to the teacher first if there is a problem, rather than approaching the principal or superintendent (DeVries, 1996).
4. *As it becomes apparent that you need to request a change, approach the activity optimistically, as well as realistically.* Optimistically, you should assume that your child's teachers and administrators have the best interests of the student at heart. They are fundamentally concerned with their students; therefore, they are vitally interested in your child and all of the children in their classes. They believe they are doing the best job they can with their students—and most teachers are. Realistically, most suggestions for change will result in a defensive stance from the educator. When educators respond with "No" to your suggestion, they are operating under the assumption that they have good reasons to defend the status quo. Do not react to this defensiveness.

   Recognize that the policy supporting the teacher's negative response to your request is unjust for your child, but that you will not single-handedly implement a policy change to benefit him

or her. Concentrate on providing information that demonstrates the degree to which your child is underchallenged. Gently inform your child's educators about current research evidence regarding the harm of an underchallenging curriculum. We find it is most effective for parents to approach the school with an attitude that "We're all in this together," not the adversarial approach of "You should be doing this for my child!"

5. *Include your child in the decision-making process*. Find out what the options are and discuss them with your child. Of course, the younger the child, the more you will need to make the decisions for him or her. However, by the time the student is in sixth or seventh grade, he or she can be a fairly active participant in decision making. When the child is involved in these discussions, it is imperative to present a positive attitude toward the school. The student doesn't need to hear you saying, "Andrew's teacher doesn't know anything about math!" Parents can model advocacy by providing an open environment where children feel comfortable coming to them to address concerns. Parents can show their children how to approach their teachers and begin the process of shifting from parent-led advocacy efforts to student-initiated advocacy.
6. *Make requests in writing and keep good records*. Take good notes, keep copies of letters, and maintain a record of what has happened and the conversations and agreements you have had. Keep records of test results (standardized and teacher-made tests) and also save samples of your child's work (dated, with your child's age).
7. *Consider that your efforts to make changes for your own child may have a long-term beneficial impact on younger students*. You want to take care of your child's needs for appropriate challenge as expediently as possible. At the same time, consider that your efforts may improve the programs, curricula, and teacher training for future students. As we have already discussed, it is easier to effect change in a specific class with a specific teacher. It is much more complicated to make a systemwide change.

For example, Arthur's parents advocated conscientiously for their son to be sure that he would be challenged in mathematics. They paid for a mentor to work with him, and they spent many hours discussing his case with school personnel. Several years after Arthur had moved on to middle school, his parents could look back and see that the elementary school was now making adjustments for students who came after Arthur. They realized that their efforts had resulted in positive changes for many other students for whom the school was more willing to make accommodations. Arthur's parents paid for an individualized program that eventually became an accelerated class for mathematically talented students.

8. *Walk the fine line between not being a nuisance and not waiting too long to intervene*. Be concerned about earning the reputation as the parent who is always unhappy with the school, but balance it with advocating appropriately for your child. One parent realized that her mathematically precocious daughter hadn't learned much over the past year while she was waiting for school personnel to respond to her many requests for changes in her daughter's educational program. The mother regretfully reported that, because her daughter had mastered so much of the grade-level material, she was allowed to leave the classroom to go to the library where she did most of the data entry to change the library from a card-catalog system to a computer-catalog system. The mother reflected, "Well, she did learn a lot about computers this year." However, she also expressed regret that she hadn't been more vocal in articulating that the program accommodation should have matched the student's academic strengths.
9. *If all else fails, find out about your options for due process*. What are the proper channels in your district and your state for requesting a change to your child's educational program? When requesting a change, start with your child's teacher and principal. If you are not satisfied, contact the school district superintendent and state Department of Education with your questions. Your state may have a system in which parents can

request a meeting with a third party (a mediator) who will assist the parents and school personnel in coming to an agreement about an appropriate program for the student.

10. *Contact outside experts for assistance*. Again, the state Department of Education may be helpful. Also find out about local, state, and national gifted education advocacy groups (see the Resources). They generally have members who have "been there" and can give you valuable advice and information about specific resources. Also seek information from universities that offer programs for gifted students (e.g., Carnegie Mellon University or The University of Iowa).
11. *If you choose to change schools, write a letter to the school board explaining your reasons*. The central administrators of a district need to be aware of your concerns and actions.
12. *Be willing to compromise*. This is also known as "choose your battles." You won't get everything you want right away for your child.
13. *Sometimes, things won't go your way*. Sorry to say, but we often don't get exactly what we think we want when we want it. For a variety of reasons, students may not be stimulated or engaged by the curriculum every minute of every math period. Parents will not be able to make everything perfect for their child in school, and it will not harm the child if parents do not react every time a child says he or she had a boring day in school. Decide where and when it's important for you to intervene and when it's acceptable that your child is in a less-than-ideal situation.
14. *Find and recognize positive developments*. One family we know reacted to a parent workshop we gave by writing the letter in Figure 2.2. We appreciate their recognition of a positive situation, and we're sure their school personnel did, too.
15. *Don't give up*.

Dear Superintendent D.:

Recently, we attended a workshop entitled, "Parenting Mathematically Talented Youth" at Carnegie Mellon University. Dr. Ann Lupkowski-Shoplik, Director of C-MITES (Carnegie Mellon Institute for Talented Elementary and Secondary Students) presented the program. Our son "Logan" meets the criteria as mathematically talented via school psychological testing and the results of the Iowa Tests of Basic Skills administered in the second grade. We entered the workshop wondering whether we as a family and his educational team were addressing and meeting his needs. We were delighted to conclude the afternoon realizing that our school district has addressed our concerns and is well on its way to meeting the unique academic needs of our son.

As Dr. Lupkowski-Shoplik outlined many options and discussed the positive and negative consequences, it became clear that our school district has taken the steps to allow Logan's advancement and success. As we listened to horror stories from other families, which ran the gamut from poor curriculum to a student given the math book for the next grade and directed to a desk in the hall, we realized how truly blessed we have been. We would like to commend Logan's entire team for its insight and willingness to push for Logan's advancement.

- Mrs. V. M. recognized his talents and "started the ball rolling."
- Ms. C. T. performed initial testing and coordinated the effort.
- Mrs. S. F. performed diagnostic testing and pushed for a unique educational approach.
- Mrs. M. S. consulted and provided her insight for working with students with special needs.
- Mr. M. K. implemented the DT→PI (Diagnostic Testing→Prescriptive Instruction) Model using our district's choice of curriculum to allow Logan to accelerate to a level that better meets his needs.
- Mrs. M. B. and Mrs. C. S. cooperated on scheduling and are helping to meet his educational and social needs.
- Mr. A. B. recognized our concerns as parents and worked to make the entire process possible.
- Mrs. L. Y. assured a curriculum that is intentional and challenging and addresses both computation and theory.

More challenges await Logan's education and our attempts to assure the best possible programming, but we are comforted knowing that we are on the right track and surrounded by competent and caring professionals. Thank you for supporting the faculty and staff of the school district in their efforts to meet the needs of mathematically talented students.

Sincerely,
Mr. and Mrs. L.

Figure 2.2. Letter to superintendent.

How Does Advocacy Compare With the Regularly Scheduled Parent-Teacher Conferences?

Most schools have regularly scheduled parent-teacher conferences in which the teacher shares information about the child's progress with the parent. These conferences are fairly circumscribed, and the information typically flows from teacher to parent. During the conference, parents may receive information precipitating the need to advocate for a change in their child's curriculum, but time constraints of the parent-teacher conference usually mean that a separate meeting will need to be scheduled.

The following suggestions (adapted from DeVries, 1996) will help parents and teachers make the most effective use of the conference time. The goal is to achieve a positive partnership between home and school.

- On the day of the conference or meeting, arrive on time. Enter the conference confidently and positively. As you speak with the teacher, be specific about your concerns and give specific examples of your child's behavior. Explain what you have tried at home. Ask for suggestions on ways "we" (parent and teacher) can work together to improve the situation. When the teacher speaks, listen actively. Show through your comments and your body language that you are listening to what the teacher has to say. During the conference, try to achieve consensus on a plan of action.
- If you are uncomfortable or dissatisfied with a suggestion, tell the teacher that you would like time to reflect and think about the implications. Send a follow-up letter, make a phone call, or have a second conference.
- After the conference, send a thank-you note to the teacher for taking the time to discuss your child's educational progress with you. Make arrangements for a follow-up phone call or another conference, if needed.

Meetings that are scheduled as a part of a parent's efforts to advocate often have educators and administrators in attendance and may have

the information flowing from parent to educator as the parent explains the need for a change in the curriculum. Before the meeting, parents need to prepare themselves (see Figure 2.3). Make a list of the points you would like to discuss. Consider practicing what you will say. Gather samples of your child's work to take with you. You might consider inviting someone else to come with you, such as another family member.

What General Gifted Education Resources Can Help Parents Be Better Advocates?

There are several resources that can assist parents in getting to the juncture between needing a change in a child's academic program and implementing the change that will result in appropriate challenge in the academic setting. In this age of technological access, the amount of information available is almost overwhelming. For that reason, we have focused on three main sources about which we think parents should be informed. An annotated list of resources is included in the Resources.

National Association for Gifted Children (NAGC)

NAGC's website (http://www.nagc.org) is extremely comprehensive. By going to the home page, parents or educators can spend hours finding information about giftedness, resources, parenting, and the like. There is a page specially designed for parents that includes:

1. the ABCs of gifted;
2. practical tools for parents, including web resources and publications;
3. lists of summer camps and programs; and
4. discussion topics and tools (e.g., how to become an e-advocate for gifted education).

1. Bring samples of the work your child has done and be prepared to discuss why these samples demonstrate a need for more advanced material. Include work your child has done in school and outside of school. Consider bringing:
   - examples of recreational math books your child has used at home,
   - computer software or websites your child has used at home,
   - information about outside-of-school programs he or she has attended, and
   - information about other outside-of-school activities in which he or she has participated.

2. Gather test results and be prepared to discuss why the test results demonstrate a need for more advanced material.
3. You might need to discuss the child's attendance in school (e.g., child has excellent attendance) and how that might have an impact upon programming/curricular decisions.
4. Be prepared to keep the tone of the meeting positive. For example, start off the meeting with a positive comment: "My daughter really enjoyed the group activity in math class last week."
5. Make a list of points you would like to discuss at the meeting. For example:
   - "I'm concerned that my son isn't working up to his potential in your class. For example, in the summer program he attended this past year, he loved the prealgebra work he was doing."
   - "My daughter is reluctant to go to school in the morning because she is frustrated with the repetition of simple computations."
   - "I have found some really interesting math materials that my child might enjoy working on. How can we fit them into his program?"
   - "In what ways might we work together to keep him challenged?"

Figure 2.3. Getting ready to attend the conference or team meeting.

A Nation Deceived: How Schools Hold Back America's Brightest Students

This two-volume publication (Colangelo et al., 2004a, 2004b), which was funded by the John Templeton Foundation, highlighted the disparity between the solidly positive research on academic acceleration and the overtly negative beliefs about acceleration, which run counter to the research.

Both volumes of the report are available for download at http://www.accelerationinstitute.org/nation_deceived. Just a few short years after the publication of *A Nation Deceived,* the John Templeton Foundation funded the Institute for Research and Policy on Acceleration (IRPA; http://www.accelerationinstitute.org). This website has multiple resources about academic acceleration, all of which are completely free to the user. One resource that should be helpful for those who are interested in policy is the *Guidelines for Developing an Academic Acceleration Policy*. This document is the product of a three-way collaboration between the IRPA, NAGC, and the Council of State Directors of Programs for the Gifted. Even though our advice to parents is to focus their advocacy efforts on their child, the user of the *Guidelines* is likely to find useful information about policies in general.

Foundations for Success

Foundations for Success is a national report generated in 2008 by the National Mathematics Advisory Panel. As was true for *A Nation Deceived*, this report is a detailed analysis of the research about mathematics in the United States. However, there are important differences between this report and *A Nation Deceived*. First, *Foundations for Success* is a product of a Presidential Executive Order to establish a National Mathematics Advisory Panel within the Department of Education. Their directive was to report to the President on strengthening mathematics education. Whereas we find the entire report to be extremely useful, there is one recommendation that we think has the most relevance for advocacy for mathematically talented students. The panel recommends that:

> All school districts should ensure that all prepared students have access to an authentic algebra course—and should prepare more students than at present to enroll in such a course by Grade 8. The word "authentic" is used here as a descriptor of a course that addresses algebra consistently with the Major Topics of School Algebra . . . Students must be prepared with the mathematical prerequisite for this course according to the Critical Foundations of Algebra. . . and the Benchmarks for the Critical Foundations. (National Mathematics Advisory Panel, 2008, p. 23)

Again, our advice to parents is to focus on your child. Nevertheless, it is important to know the general educational terrain; in this case, the new norm for algebra is leaning toward teaching it in eighth grade, not ninth grade. Parents of the mathematically gifted student should be thoroughly familiar with the general recommendations for mathematics education because they have implications for programming for mathematically talented students who will need access to prealgebra and algebra well *before* grade 8. The National Mathematics Advisory Panel's concrete recommendations about general mathematics education have laid the foundation for thinking about the needs of mathematically gifted students.

Conclusion: How Can Parents and Educators Cooperate to Ensure Appropriate Programming?

The content of this chapter has been directed mainly at parents because they are the primary advocates for their children. Parents have the right and the obligation to pursue those educational avenues that will result in a differentiated curriculum for their mathematically talented child. Thorough and objective information regarding the child's progress is essential in advocating appropriately for children. Indeed, accuracy and relevancy of information are perhaps the most important aspects of advocacy.

Advocacy is heavily dependent on objective information gained through appropriate assessment (see Chapter 3), including test scores, portfolios, and specific information about strengths and weaknesses in mathematics, as well as other subject areas (e.g., writing and reading). This process yields information about the learner that can be matched to the resources within a school or district.

The goal should be the development of a plan to challenge the student in a consistent, systematic manner. One of the best ways to do that is through a program such as the one described in Chapter 4, Diagnostic Testing→Prescriptive Instruction (DT→PI). But, how can DT→PI, or any other program, be implemented and what is the interaction between parent and school? There are at least four possibilities, which are listed in reverse order of preference for what we think is best for the student and the school:

1. *Bypass the school.* The child works privately with a mentor. The arrangement is between parent and mentor.
2. *Bypass the school.* The student works privately with a mentor under the administrative expertise of a third party (e.g., university-based programs).
3. *Inform the school.* The student works privately with a mentor outside of the school day; however, the school is advised of the activity so the student can receive credit for this work. This communication also promotes long-term planning for the student's programming throughout his or her school years.
4. *Involve the school.* For example, the school administers a program such as DT→PI (described in Chapter 4) and the student receives credit and placement. Any resources or expenses are not the responsibility of the parents.

Although all four scenarios are possible, the third and fourth are the most conducive to a cooperative relationship between parents and school. These options require program and curricular changes for an individual child or small groups of very talented students. Change at the individual or small-group level is easier to implement than the system-wide change that is often desired by parents; however, such change is sometimes restricted by law or policy.

Change is neither easy nor immediate. Often, there is tension between a parent's request and the school or district policy. We advise parents to concentrate on finding ways to have their child's needs met rather than trying to overhaul the school system. However, parents should realize that in the long term their efforts at advocacy for an individual child may have a positive impact on programs provided for mathematically talented students in future years.

CHAPTER 3

Using Academic Assessment to Make Informed Decisions About Mathematically Talented Students

KEY POINTS

- An academic assessment can help in the placement of and programming for the mathematically gifted student.
- An academic assessment may include (a) testing, (b) interview, (c) observation, and (d) review of prior testing and classroom performance.
- Selecting the components needed for an assessment should be based upon the primary questions to be answered, which are usually concerned with placement and/or programming.
- The testing industry is vast and complex and volumes have been written related to measurement theory and practice. This chapter addresses the measurement issues most closely connected to mathematically talented students.
- A sample report highlights the major points of the chapter.

Assessment is a way of understanding a child in order to make informed decisions about the child.

—Sattler, 2008, p. 4

The mathematically talented student is the subject of this book and, not surprisingly, numbers are an important part of the development of the mathematically talented student. However, in this chapter it is not the student's talent with numbers that concerns us; rather, our subject is the educator's use of numbers—from assessments—to better understand how to nurture the talent development of mathematically gifted students.

Assessments and Tests: Aren't They the Same?

A succinct definition of assessment, provided in the introductory quote, goes straight to the purpose of this chapter: making informed decisions. Sattler (2008, p. 4) continued his discussion of assessment by describing the four pillars of assessment:

1. norm-referenced measures,
2. interviews,
3. behavioral observations, and
4. informal procedures.

Thus, for Sattler (2008), as well as for us, a norm-referenced measure—a test—is one component of an assessment. This distinction is important because making informed decisions should be based on more than a test score; however, a test score is a good place to begin. This begs the question: What test(s) should be used? The response to that question is presented over the next several pages.

Considerations for Choosing a Test

It sounds fairly simple to say that a test should be selected based upon its purpose. For example, an intelligence test should be used to measure intelligence, and a math test should be used to measure math achievement. But, it's more complex than that, partly because the testing industry offers so many choices that an educator can feel bombarded with options (the *Tests in Print* website provides reviews

or information for more than 3,500 tests!). In this chapter, we provide some basic information that will help you in the process of test selection and score use. The final decisions about *what test*, however, should be based upon careful consideration of the kind of information (e.g., help in making decisions about placement, programming, and/or pacing) needed as well as the overall availability of the options. Additionally, each test should include a technical manual, which provides relevant information about the development of the test as well as administration and scoring procedures, all of which may enter into the decision-making process regarding which test(s) to use.

Another concern in testing is *who* should conduct the assessment. Most educators have extensive training in curriculum and programming, but less exposure to measurement and assessment. Nevertheless, there are multiple tests and surveys that educators can administer to obtain information useful in the decision-making process. Some professionals, such as school psychologists or clinical psychologists, have received extensive training in assessment, especially relative to cognitive ability, yet they may have less familiarity with curriculum and programming, which makes it difficult to translate clinical results into educational applications. Of course, it need not be one or the other; in fact, one purpose of this chapter is to present concepts that will help bridge the gap between psychological test data and educational implementation. When teachers are more familiar with the role of individualized intelligence tests in describing the cognitive profile of a student, they can combine that information with specific achievement and aptitude results to make an appropriate programming decision.

Although not necessarily a consideration for selecting a test, we think it is important to address the issue of *ambivalence surrounding testing*. This ambivalence is understandable because, on the one hand, there are parents and/or educators whose experiences with testing have been negative, and they may have little confidence in the information available through a test. On the other hand, promoting the use of tests and testing in general can sometimes put the professional or parent at odds with the district philosophy regarding the use of standardized tests. The issues are complex and merit consideration; however, we assert that although standardized tests are not perfect

(what is?), they provide information that is critical to making an informed decision about programming. Indeed, standardized tests are actually a means to reduce the subjectivity and bias that often are components of feelings of ambivalence. Some professionals promote the large-scale implementation of alternatives to testing (e.g., use of portfolios or presentations or reliance on curriculum-based measures). Such alternatives might provide valuable information when used in conjunction with standardized tests, but for our purposes, they are not satisfactory alternatives for obtaining information about mathematically talented students.

Ability Tests

Tests that measure general cognitive ability go by multiple names, including intelligence tests, which imply that these tests actually measure intelligence. However, accomplishing this is nearly impossible because there are numerous definitions of intelligence, and these definitions often are based upon theories, which continue to evolve (Flanagan, McGrew, & Ortiz, 2000; Sattler, 2008). For our purposes, we suggest that tests of cognitive ability measure aspects of behavior that are oftentimes associated with human intelligence, such as verbal comprehension, abstract reasoning, problem solving, working memory, ability to put puzzles together, and general knowledge. Cognitive ability tests take on many forms, including the classifications of individually administered and group administered. In this section, we will discuss six tests of general ability beginning with individually administered tests.

Individually Administered Ability (Intelligence) Tests

Two familiar intelligence (ability) tests are the Stanford-Binet tests and the Wechsler tests. The Stanford-Binet tests were named for one of the authors, Henri Binet, of the Binet-Simon Scale, and the institution (Stanford University) where Lewis Terman developed revisions of the original Binet-Simon Scale. The Wechsler family of tests is named for

their original author, David Wechsler. Since the Stanford-Binet and Wechsler tests were first introduced in the late 19th and early 20th centuries, they have dominated the area of psychology concerned with mental measurement.

Now, in the 21st century, both of these giants still have a place in the field of intellectual assessment, but their status, relative to other testing measures as well as to each other, has shifted somewhat, primarily due to historical events in the education world. The Stanford-Binet has historically been connected with the field of gifted education, which is understandable for two reasons. First, Terman is regarded as the father of gifted education because of his significant contribution to the field by developing the Stanford-Binet Intelligence Scale. Second, he launched a historic longitudinal study based upon administration of the Stanford-Binet Intelligence Scale to 1,528 individuals. This landmark study, published in four volumes called *Genetic Studies of Genius* (Burks, Terman, & Jensen, 1930; Terman, 1925; Terman & Oden, 1947, 1959) put to rest many of the negative myths associated with giftedness.

However, decades later, in the latter part of the 20th century, special educators "made it clear that multiscore IQ tests were essential for proper diagnosis and remediation and that the one-score Binet was as outmoded as the Model T Ford" (Flanagan et al., 2000, p. xiii.). The Wechsler scales, which included three IQs and multiple subscales, were positioned to become central to the testing field. This is most likely why many psychologists today find that the "clinical profile" available through the administration of the most recent revision of the Wechsler Intelligence Scale for Children (WISC; the test used with individuals 6 to 16 years old) is desirable as a component of the assessment.

The relative newcomer to the field of individualized intelligence testing is the Woodcock-Johnson Tests of Cognitive Abilities. This test is a "newcomer" because it was first published in 1977. As was true with the earlier versions, the most current version, the Woodcock-Johnson III Tests of Cognitive Abilities–Normative Update (WJ III COG-NU; Woodcock, Shrank, McGrew, & Mather, 2007), is an individualized measure of general ability whose development was based upon a psychological model known as the information-processing

model. Many psychologists find that it provides diagnostic information, as well as "classification," or "placement" information.

The final individually administered test that we present is the Slosson Intelligence Test–Revised (Slosson Intelligence Test–Revised [SIT-R3]; Slosson, Nicholson, & Hibpshman, 2002). Many educators of gifted students have this test on their shelves and find it to be handy because it takes less than half an hour to administer and requires minimal training. However, these two facts—minimal training and short administration time—reveal something important: This test is only appropriate as a screening test. Furthermore, in searching for a comprehensive review of the SIT-R3, we discovered that there are no reviews for the 2002 edition and, as of this publication date, none were anticipated. The only available reviews are for the 1991 edition, the Slosson Intelligence Test–Revised (SIT-R); therefore, we reiterate Sattler's (2008) comments about the SIT-R, which merit careful consideration:

> The Slosson Intelligence Test–Revised might have some merit as a screening device but its shortcomings need to be recognized. . . . more information is needed about the stability and validity of the test . . . The SIT-R should not be used as a substitute for the Stanford-Binet: V, the WISC-IV, WPPSI-III, or DAS-II, especially in assessing children with disabilities. (p. 690)

Other individually administered ability tests exist, including the Kaufman Assessment Battery for Children–Second Edition (K-ABC-II; Kaufman & Kaufman, 2004) or the Raven's Standard Progressive Matrices (Raven, Raven, & Court, 1998). The purpose of this chapter is not to provide an exhaustive discussion of ability tests, or the history and debates surrounding the various theories that underlie these tests. Rather, we aim to introduce the reader to the major tests that are used in the field and to equip you with information that will help in understanding tests and test scores. Space precludes us from addressing some of the many interesting issues associated with testing; however, Sattler's (2008) *Assessment of Children* is a superb resource

for further examination of these topics as evidenced by the multiple citations.

Group-Administered Ability Tests

The second category of intelligence tests takes the form of a group-administered test. For special education placement and programming, individual intelligence tests are usually mandated. However, for broad educational purposes—especially when educators are screening for general purposes—group-administered tests, which by definition can be administered by educators to groups of students, are an important component of educational decision making. Equally important is the fact that specialized training as a psychologist is not needed to administer group tests; thus, they are typically more accessible to general educators. Two tests widely administered to groups of students are the Cognitive Abilities Test (CogAT) and the Otis-Lennon School Ability Test (OLSAT).

The Cognitive Abilities Test Form 6 (CogAT-6; Lohman & Hagen, 2001) is a psychometrically, psychologically, and educationally excellent group-administered test. It measures important reasoning abilities in three domains: verbal, quantitative, and nonverbal (or figural). Lohman and Hagen (2001) have argued persuasively that measuring reasoning abilities (especially inductive reasoning abilities) remains central to the identification and nurturing of gifted youngsters. Nevertheless, Lohman (2009) recommended combining scores from different tests, rather than relying on a single score.

A review by Rogers (2005) highlights one of the most useful features of the CogAT-6, the "Ability Profile," which is reported for each student. The authors argue that the profile of reasoning abilities is more indicative of *how* to teach than prescriptive of *what* to teach. Information about a student's performance on the CogAT-6, when combined with other information (e.g., results of an achievement test such as the Iowa Tests of Basic Skills [ITBS] or the Iowa Tests of Educational Development [ITED]), can help educators discover the unique learning needs of their students and can help inform decisions about instruction. This information is provided both in the manuals

that accompany the test and on the Internet for teachers who do not have access to the interpretive guides.

Also unique to Form 6 of the CogAT is the computation of confidence intervals for each score that are based on the consistency of the student's responses to the items and subtests that make up the score. This is particularly helpful when testing young students who might be confused about the directions for one of the subtests or have difficulty using a machine-readable answer sheet when tested above level. Finally, tables reported in the research handbook (Lohman & Hagen, 2001) give explicit guidance on how to use expected errors of measurement at different score levels to select the best level of the test for above-level testing of gifted children.

All of these features, plus a few others, appear in Form 7 of the CogAT (normed during the 2010–1011 academic year and released during the 2011–2012 academic year). For example, the primary battery, developed for students in K–2, has a reduced language load, with only one of the nine subtests requiring comprehension of oral language. This subtest can be administered in English or Spanish, or omitted altogether. Most notably, Form 7 of the CogAT is available online with directions in multiple languages (D. Lohman, personal communication, July 20, 2010).

The eighth edition of the Otis-Lennon School Ability Test was published in 2003 and is the most recent in the Otis series of ability tests that was first published in 1918. The OLSAT-8, like its predecessors, was designed for use as a group-administered ability test and yields a total score, a verbal score, and a nonverbal score. The OLSAT-8 was concurrently normed with the Stanford Achievement Test 10. According to Maddux (2004), "the OLSAT-8 may be useful as a quick screening instrument to locate those who might profit from more extensive testing" (para. 10); however, the major limitation to the OLSAT-8 is its relative lack of validity data demonstrating the relationship between its scores and school grades.

Results from group-administered tests can help educators understand the general nature of the ability levels of the students in their class. However, by their nature, group tests of intelligence require limited interaction between the tester and student. Group

ability tests are most appropriately used for initial screening purposes, and the results should not be used in isolation to make a programming decision about an individual student. When making placement or classification decisions, results from a group-administered test should be supplemented by an individually administered test. Also, additional specific tests of aptitude and achievement should be used when determining programming.

Making Sense of Numbers and Scores From Ability Tests

When discussing performance on a test, an initial concern is how many items were answered correctly. The number of correctly answered items is referred to as a *raw score*. Raw scores alone have very little meaning to anyone—even to the individual completing the test. Such scores only become meaningful when they are converted or transformed so they have a predetermined mean (average) and standard deviation. This permits them to be used comparatively, sort of like comparing apples with apples, rather than apples with oranges. One way is to statistically convert the scores so that they can be compared to a normative group; scores can then be reported as a *percentile ranking*. Percentile rankings are sometimes confused with the notion of percentage correct.

Although the concept of percentage enters into the discussion of percentile ranking, it is not related to percent correct; rather, percentile rank refers to the individual's "rank" in the comparison group. Another way to state this is that it refers to the number of individuals who earned scores as well as or better than the individual's score. Therefore, a score at the 50th percentile means that individual scored as well as or better than 50% of the individuals being "ranked" on the measured performance. When discussing high-ability students, the percentile rankings are often very high (e.g., 97th, 98th, or 99th). If we use the terminology of "as well as or better," we quickly realize that we are not talking about large numbers of individuals, because a 98th percentile rank means that the individual performed as well as or better than 98% of the comparison group; thus, only 2% would have earned a higher

score. Two percent is a very small number of individuals in any one school or classroom.

Scores obtained from ability tests have long been referred to as an *intelligence quotient* (IQ). However, we suspect that the use of the terminology most likely exceeds the understanding of the underlying basis for the term. Sattler (2008) attributed Terman with the adoption of the term. As cited in Sattler (2008), Terman renamed Louis Stern's "mental quotient," which was obtained by dividing mental age by chronological age. In addition to renaming it intelligence quotient, he also multiplied the number by 100 to eliminate decimals. Shortly thereafter, it was realized that IQs, as they were originally conceptualized, were not comparable for different ages; that is, the ratio of mental age to chronological age has a different meaning for different ages. This problem was solved by introducing the deviation IQ. A discussion about deviation IQ is well beyond the scope of this book. However, there are two important facts for the reader to know:

1. The average standard score of 100 and the standard deviation of 15 is the defining characteristic of the deviation IQ. Most ability tests use this standard, which permits comparisons. In other words, a standard score of 115 will always mean that the individual's score was one standard deviation from the mean—no matter what ability test was used or when it was administered.
2. Another main point is that 100, which represent the average standard score on most ability tests (and many other kinds of tests as well), is equivalent to the 50th percentile. Therefore, *any* test that has an average score of 100 will always have 100 as the 50th percentile.

Our point in presenting this terminology is to emphasize that the score obtained from an intelligence test is sophisticated and has significant implications about a student's general ability to succeed in a school setting. An IQ also is informative to educators and parents when it has been statistically converted to a percentile ranking. A percentile ranking allows educators and parents to understand the position of the

student's score relative to the scores from the other students who were part of the testing sample; this is important information.

A general intelligence test is most effective in determining the pacing of curricular material. For example, we have known since the 1930s (Hollingworth, 1942) that students with an IQ at the 98th percentile need class work that is at least 2 years beyond the work typically given to their age-mates. Because the 98th percentile is equivalent to a standard score of 132, we can determine that such a score is a little more than 2 standard deviations above the average standard score of 100; hence, the correspondence between 2 standard deviations and 2 years. Also, by recognizing that this score is at the 98th percentile, we acknowledge that *very few* (2% or less) students will need similarly high levels of challenge. As the number of standard deviations beyond the mean increase, the percentile ranking gets higher and higher, and the challenge needed relative to students' grade-mates becomes greater.

However, percentile rankings can be misleading for the upper range of scores. For example, all scores between 133 and 138 on the WISC-IV are at the 99th percentile. A score of 139 is equivalent to the 99.53 percentile, and a score of 155, which is the highest score that can be earned on the WISC-IV, is equivalent to the 99.99th percentile. Although 99 and 99.99 seem fairly close, a child with an IQ of 133 has different needs for challenge compared to a child with IQ of 155. Also, 155 is the highest IQ that can be earned on the WISC-IV; however, it may not accurately reflect just how exceptionally capable some students are. It is always important to go beyond the IQ and look at how the child performed on specific subtests, as well as the pattern of those scores. For specific information about programming in mathematics, one needs to go directly to assessments of mathematics achievement and aptitude.

Specific Aptitude Measures

Testing terminology can be very confusing. For example, how can a classroom teacher or parent distinguish between ability and

aptitude when measurement experts cannot do so and often use the terms synonymously? For our purposes, we distinguish aptitude tests from general ability (intelligence) tests in the following way (although there are surely multiple ways to distinguish ability and aptitude tests, we think this is the most useful). Primarily, the distinction revolves around the extent to which specific content is being measured. The items on an aptitude test are typically more content-specific, whereas general ability tests are designed to measure an individual's ability to do general problem solving that is not specifically related to school learning. Therefore, even though an aptitude test is similar to a general ability test in that they both measure problem solving, the problem-solving activities on an aptitude test also may be associated with school-related content—although it is not expected that the student has been exposed to that content. This latter comment is relevant to the distinction between aptitude and achievement tests (see pp. 76-80 for the discussion about achievement tests).

Aptitude tests are different from achievement tests, which are dependent upon mastery of specific content to which the student was exposed in the past. Unlike achievement tests, which measure *past* exposure to content, aptitude tests measure the potential for *future* performance. Some aptitude tests are designed to measure one specific aptitude (e.g., mechanical reasoning, reading comprehension, mathematical problem solving), whereas other aptitude tests measure multiple areas and take a general approach (e.g., verbal aptitude is a more general concept compared to reading comprehension). When measuring the aptitude of extraordinarily talented students, it often is necessary to use a test that was designed for older, more advanced students. In the subsequent sections, we first address aptitude tests that are designed uniquely to measure aptitude in mathematics; these tests are followed by aptitude tests that measure multiple aptitudes (e.g., verbal and mathematical).

Aptitude Tests That Measure Mathematical Aptitude

Iowa Algebra Aptitude Test (IAAT). The Iowa Algebra Aptitude Test published by Riverside Publishing was developed in 1931 by the

University of Iowa Testing Program (ITP). Its purpose is to help educators make informed decisions about the placement of seventh- and eighth-grade students into algebra, which is recognized as the first course of mathematics at the secondary level of curriculum. Since 1931, the IAAT has gone through four revisions, the most recent of which was in 2006 (Schoen & Ansley, 2006). Just as the previous edition was based upon the National Council of Teachers of Mathematics (NCTM) *Standards for School Mathematics*, so too the fifth edition was based upon the organization's *Principles and Standards for School Mathematics* (NCTM, 2000).

The updates to the fifth edition were both necessary and welcome. The material covered has remained fairly constant when compared to previous editions, although the updated names of the subtests reflect more recent approaches to mathematics education. The four subtests are:

1. Pre-Algebraic Number Skills and Concepts,
2. Interpreting Mathematical Information,
3. Representing Relationships, and
4. Using Symbols.

Administration of the test requires no special training and takes about 50 minutes overall to administer. The test can be machine- or hand-scored and includes two separate, yet parallel forms, which make if very useful for both pre- and posttesting.

Morse's (2007) and B. G. Rogers's (2007) reviews of the fifth edition generally concur that this is a very useful instrument for predicting success in algebra. B. G. Rogers (2007), in particular, stated that "The quality of this test is such that it may be viewed, in the future, as the criterion toward which other mathematics test will be compared" (para. 18). We agree, although we also recognize that the IAAT was not normed for students younger than grades 7 and 8, which is important to keep in mind when using it above level with mathematically talented younger students.

KeyMath-3 Diagnostic Assessment. The KeyMath-3 Diagnostic Assessment (KeyMath-3DA; Connolly, 2007), published by Pearson, is familiar to some educators because it is often used to diagnose students who have a specific learning disability in mathematics. It is

an individually administered test and recommended by reviewers as especially useful when assessing students who have learning difficulties (Graham, 2010).

This most recent edition has two important improvements to previous editions. First, the *Principles and Standards for School Mathematics* (NCTM, 2000) document was used to generate items for the KeyMath-3DA (Connolly, 2007). Second, there are parallel testing forms. However, this instrument has some important shortcomings if and when used with mathematically gifted students. Even though it is individually administered and is designed to provide a broad picture of a student's understanding and application of important mathematics concepts and skills, the material covered is generally considered too broad to provide information that is specific enough to develop a program for the mathematically gifted child. For example, it can indicate mastery of basic computation operations (i.e., addition, multiplication, subtraction, and division), but it cannot provide more substantial programming information. As well, it has been noted that there is a ceiling effect to the test, which makes it very difficult to use with mathematically talented children.

Orleans-Hanna Algebra Prognosis Test. The most current version of the Orleans-Hanna Algebra Prognosis Test (Hanna, 1998) evolved from the 1928 Orleans Algebra Prognosis Test, which was developed to predict the success of students in a first-year algebra course. It is similar to the Iowa Algebra Aptitude Test (Schoen & Ansley, 2006) in that it was developed to measure readiness to learn algebra. It is a 50-item test with the most recent standardization being in 1996. As a measurement instrument, it has an interesting design: There are five "lessons," each of which is followed by six test questions based upon the lesson. This design is dynamic in that it allows for some insight into the student's ability to learn basic algebra concepts during testing. The final section of the test is comprised of 20 review items that include topics linked to the NCTM (1989) *Curriculum and Evaluation Standards for School Mathematics* as they relate to the middle school mathematics curriculum (see the *Mental Measurement Yearbook* review by Ciechalski, 2007). However, given that the NCTM standards were updated in 2000, this places the Orleans-Hanna at a disadvantage in terms of being current.

As reported by Ciechalski (2007), the comprehensive manual for the 1996 edition includes complete descriptions of the validation studies of the redesigned new edition, and the interested reader is directed to that manual for more details.

Test of Early Mathematics Ability–Third Edition (TEMA-3). The TEMA-3 (Ginsburg & Baroody, 2003) is an individually administered test and, as was the case with its predecessors, was developed to assist educators in identifying learning disabilities or the likelihood of learning disabilities. Therefore, its applicability to mathematically gifted students is most likely limited.

Test of Mathematical Abilities–Second Edition (TOMA-2). The TOMA-2 (Brown, Cronin, & McEntire, 1998), includes four core subtests: Vocabulary, Computation, General Information, and Story Problems. It was developed for use with students ages 8–18. Sattler's (2008) review indicates that the TOMA-2 has limited use. This is true for the general student population, but even more so for the mathematically gifted student population.

Test of Mathematical Abilities for Gifted Students (TOMAGS). The Test of Mathematical Abilities for Gifted Students (TOMAGS; Ryser & Johnsen, 1998) was developed in alignment with the National Council of Teachers of Mathematics standards (NCTM, 1989). The descriptive information for TOMAGS states specifically that this test is not intended for diagnostic purposes. Rather, it serves as a screening instrument for the assessment of mathematical problem solving and reasoning of mathematical talent in children ages 6–12 years.

Aptitude Tests That Measure a Range of Aptitudes

Comprehensive Testing Program 4 (CTP 4). CTP tests, designed primarily for members of the Educational Records Bureau (ERB) and marketed by ERB, have been available since 1992, with the most recent version, the CTP 4, published in 2004. This battery of tests measures achievement for grades 1–11 in vocabulary, reading, writing, and mathematics. The CTP 4 battery also measures verbal and quantitative reasoning ability for grades 3–10, and there is a measure of quantitative reasoning ability for grade 3. The usefulness of the CTP 4 tests for the

talent search population has not yet been well documented. However, the tests were intentionally designed for students in schools where achievement is above average; therefore, it may be useful in curricular planning for students who typically score in the upper end on grade-level standardized tests of achievement. Additionally, ERB publishes three separate guidance manuals to assist teachers and administrators in the interpretation and application of the information available.

School and College Ability Test (SCAT). Despite the fact that the word "ability" appears in the title, the School and College Ability Test (SCAT, n.d.) is a scholastic aptitude test. Although developed in the mid-1950s by the Educational Testing Service (ETS), it is now used exclusively by the Johns Hopkins University Center for Talented Youth (CTY) as part of its talent search opportunities for grades 2–6 (see http://cty.jhu.edu/ts/index.html for extensive information about CTY and testing opportunities available through that organization; see also Chapter 5 in this book for a comprehensive discussion of the talent search model). Schools must purchase a licensing agreement from CTY, which allows them to then purchase the multiple levels and either form of the test. The tests are scored by a teacher.

The origins of SCAT with ETS are reflected in the two areas of aptitude measured by SCAT: verbal and quantitative. There are three levels: elementary, intermediate, and advanced, and norms available from grades 4 through 10. Specific information about the appropriate level to use with gifted students is available through CTY.

Achievement Tests

Achievement tests differ from aptitude and intelligence tests by specifically measuring what a student has learned. This seemingly benign statement is, in fact, a loaded statement because it includes one problematic assumption, which is that the student learned the material in the recent past, from a teacher whose goal it was to teach that material. We know that oftentimes gifted students can easily earn very high scores on achievement tests, but what we do not know is: (a) when did they learn that material, and (b) how did they learn it? In

other words, there is no guarantee that high scores reflect achievement of newly presented material in the current classroom setting—the achievement may indeed be the result of learning that didn't take place in a traditional setting. There is one primary reason that we bring this up. Mainly, an achievement test may indicate a level of mastery, but it is imperative that there be a follow up to this information to determine what the student is ready to learn next.

Familiarity with the major achievement tests that are available in schools will help educators and parents understand the degree of information available through results obtained from these tests. These results can be used to compare a student's progress to others and can serve as a baseline when measuring future progress. Sometimes, achievement testing is used for placement into programs. In addition to informing parents and educators regarding placement decisions, achievement tests also may be useful in a diagnostic capacity, as their results may play a critical role in helping teachers and parents plan the student's academic program. We will describe two achievement tests that are generally administered to groups of students, two individually administered achievement tests, and one test that is a computer-based, adaptive test.

Group-Administered Achievement Tests

Iowa Tests of Basic Skills (ITBS). The ITBS (Hoover et al., 2003) is an achievement battery of tests for students in grades K–8. This battery of tests was developed at The University of Iowa's well-known Iowa Testing Program (ITP) and published by Riverside Publishing. Although the precursor to the ITBS dates back to the mid-1930s, the ITBS publication dates begin with 1955, with the most current edition published in 2003. The ITBS is exactly what it says: a test that measures achievement of basic skills in the content areas taught in school, including vocabulary, reading comprehension, language, mathematics, social studies, science, and sources of information. The grade level of a test is intended to measure the skills at that grade and has questions that are representative for that grade. When given above grade level, the scores from the ITBS can be used as a way to measure

aptitude (i.e., potential to learn the material, as the material has not yet been taught).

The ITP also has developed a similar achievement battery, the Iowa Tests of Educational Development (ITED; Forsyth, Ansley, Feldt, & Alnot, 2006) for students in grades 9–12. The ITBS battery is more relevant for our purposes; however, we think it is important to mention the fact that the Iowa Testing Program is comprehensive for grades K–12.

Stanford Achievement Tests, 10th Edition (Stanford 10). The Stanford 10 (Harcourt Assessment, 2003) is a highly respected achievement battery developed for students in grades K–12 and published by Pearson. The first edition was published in 1923, and the most recent edition was published 80 years later, in 2003. As expected, the areas assessed include reading, mathematics, spelling, language, science, social science, and listening. The 10th edition was co-normed with the Otis-Lennon School Ability Test (OLSAT-8; Pearson, 2003).

The ITBS and Stanford 10 are nationally recognized and highly valued group-administered achievement tests and both have a distinguished history within the testing movement. They are effective at broadly measuring the achievement of students for a particular grade level. However, when students have attained scores at the highest levels, testing them above level can sometimes provide insight into a student's need for academic challenge.

Individually Administered Achievement Tests

Sequential Tests of Education Progress (STEP). The STEP (n.d.) battery of tests measures broad, global achievement in a variety of content areas including English expression, mathematics computation, mathematics basic concepts, mechanics of writing, and reading. STEP is available exclusively through the Johns Hopkins Center for Talented Youth (CTY) as a component of their comprehensive assessment services (see discussion on p. 76 about the SCAT; see also http://cty.jhu.edu/ts/index). STEP and SCAT are two prominent features of CTY's assessment program. Similar to SCAT, the STEP tests can be

scored by the teacher, and there are two parallel forms and multiple levels. Users must obtain a license prior to purchasing the materials.

Woodcock-Johnson III Tests of Achievement. The Woodcock-Johnson III Tests of Achievement (Woodcock, McGrew, Mather, & Schrank, 2001) were developed and conormed with the Woodcock-Johnson III Cognitive Abilities Test. In 2007, both tests underwent a normative update and now the tests are simply referred to as the Woodcock-Johnson III Normative Update (Woodcock et al., 2007). The achievement battery is also individually administered and was developed for use with individuals between the ages of 2 and 90. In all, there are 22 tests, and various clusters of the tests measure the general content areas of reading, language, writing, and mathematics. Many psychologists rely on the Woodcock-Johnson as an indication of an achievement/ability discrepancy representing a specific learning disability in reading, mathematics, writing, or any combination. As was the case with the Cognitive Battery, the Achievement Battery can only be scored by using a computer. Although it is an excellent achievement test battery, especially for diagnosing learning disabilities, the information from the achievement tests of this battery is typically not specific enough to develop programs for mathematically gifted students.

The Wechsler Individual Achievement Test–3rd edition. The Wechsler Individual Achievement Test–3rd edition (WIAT-III; Psychological Corporation, 2009) is an individually administered achievement test for prekindergarten through college. In the standardization process, part of the sample was administered the age-appropriate Wechsler Intelligence Scales. Composite and subtest scores on the WIAT-II are available for reading, mathematics, written language, and oral language. Because it is co-normed with the Wechsler Intelligence Scales, it can be particularly useful in evaluating students who are being considered for acceleration.

Computer-Adaptive Testing

Measure of Academic Progress. Compared to ITBS and Stanford 10, the Measure of Academic Progress (MAP) is relatively young (MAP;

Northwest Evaluation Association, 2005). The Northwest Evaluation Association (NWEA) was incorporated in 1977; NWEA produced its MAP assessment as a computerized-adaptive test for students in grades 2–10. MAP items are designed to be in alignment with state standards. MAP assessments are available in reading, mathematics, and language. Additionally, students can be tested up to four times a year, which differs substantially from the annually administered ITBS or Stanford 10. The concept of item difficulty adjusting to the student's responses holds promise for learners who are well above the mean for a grade level. That being said, we have some anecdotal information that in some instances, despite its adaptive nature, the test may not have enough difficult items for extremely talented students.

21st-Century Education Legislation That Impacts Testing

Advocacy and legislative issues as they pertain to gifted and talented students are addressed in Chapter 2; however, we include a small section in this chapter that relates to testing issues. We specifically consider No Child Left Behind (NCLB, 2001) and the Individuals with Disabilities Education Improvement Act (IDEA, 1990). Extensive treatments of both can be found elsewhere. Our focus is on the mathematically talented student within the context of each legislative act.

No Child Left Behind

In 1969, Congress passed the Elementary and Secondary Education Act (ESEA), which included a scheduled update at the beginning of the 21st century. When ESEA was updated, it was renamed No Child Left Behind (NCLB, 2001). This legislation has been the source of significant activity within schools, most of it related to assessment, which has impacted the movement to implement standards in all states and has influenced the level of accountability of districts. It is an understatement to say that much has been written about NCLB, and we are sure that readers of this book are among those who are very familiar

with NCLB. (See the NAGC website for an excellent explanation of NCLB: http://www.nagc.org/index2.aspx?id=999&terms=nclb.)

The explicit purpose of NCLB was to close the achievement gap, and through the system of accountability imposed on districts, that gap has narrowed. But what has happened is that low-achieving students have made gains (although they remain far behind) and high-achieving students have been, as termed by Finn and Petrelli (2008), "benignly neglected." Finn and Petrelli, in the foreword to the 2008 Thomas B. Fordham Institute Report, *High-Achieving Students in the Era of NCLB*, suggested that

> Loveless [primary author of the National Assessment of Educational Progress (NAEP) data analysis presented in the report] performs a unique analysis of NAEP data that shows big gains for low performers and stagnation for top performers associated not just with NCLB but with standards-based reform in general. States that adopted their own testing and accountability reforms in the 1990s witnessed similar trends. And for good reason: most state accountability systems, like NCLB's, put pressure on schools to get students over a fairly low bar. That meant helping low achievers. And voila. (p. 10)

Additional data in the report indicated that teachers are conflicted with respect to which students need their focus. On the one hand, teachers recognize the need to focus on low-achieving students. On the other hand, teachers also recognize that the curriculum for high-achieving students is not sufficiently challenging.

Finally, an unintended consequence of NCLB has been the abdication, by many states, of group-administered, grade-level standardized tests. This seems to be because the general consensus is that there is already "enough testing" as a result of the testing required by NCLB. Many schools have replaced tests such as the Iowa Tests of Basic Skills with state-level proficiency tests, with designations such as "proficient" and "advanced"; by design, there are no percentile rankings. This is quite likely due to the fact that schools now report their adequate yearly progress (AYP) relative to proficiency. As an example, one state

reported that the percentage of third graders scoring "advanced" in mathematics was 42.5%. This is especially challenging for parents and teachers who are trying to efficiently screen for students who would benefit from above-level testing (see Chapter 4 for more information).

Individuals with Disabilities Education Act and Response to Intervention

A decade after ESEA was passed, Congress turned its attention to individuals with disabilities. The passage in 1975 of PL 94-142, the Education for All Handicapped Children Act, was a watershed in education because it required that children with disabilities have a free and appropriate public education (FAPE). PL 94-142 was reauthorized and renamed IDEA in 1990 and it has been reauthorized and amended multiple times. The most recent in 2004 was the very first to introduce language relevant to gifted learners and the need to consider that gifted students also may have a disability. Gifted students with a disability are commonly referred to as twice-exceptional (see the NAGC position statement on twice-exceptionality at http://www.nagc.org/index.aspx?id=5094 for more information about twice-exceptionality).

IDEA identifies 13 disability categories, and the three that are most relevant to mathematically gifted students are: (1) specific learning disabilities (SLD); (2) autism spectrum disorders (ASD); and (3) other health impairments, which includes Attention Deficit/Hyperactivity Disorder (ADHD). Although we cannot cover the extensive information relative to IDEA, we want to make the point that disability and giftedness may overlap and the one thing that both have in common is the need for a comprehensive assessment to determine diagnosis and/or the level of intervention.

Today's educators entered the profession well after 1975 and thus are already familiar with IDEA. Over the course of the decades, very little has changed with respect to the definitions for each category; however, in 2004, in addition to adding language about gifted and talented learners, additional language regarding assessment for SLD was added and broadened the ways in which students could be identified by including a procedure known as Response to Intervention (RtI). This

has been a positive force for nongifted students who struggle early on with reading. However, for gifted students who may also have a disability, it has been extremely problematic. Some educators have interpreted the regulations to mean that comprehensive assessments are no longer necessary. This is not an accurate interpretation, but has nevertheless gained popularity because of the potential to reduce the amount of testing in schools. Simultaneously, it has veiled the understanding and appreciation for the individual differences among students. Using information from standardized tests to place students into programs and to develop specific program objectives has been replaced with a focus on classroom assessments, specifically curriculum-based assessments (CBA), as a way to implement RtI.

Curriculum-Based Assessment

The tests described above were norm-referenced. In other words, a student's performance is compared to the performance of other students on the same test. Measurement of the mastery of a particular set of skills from specific curriculum materials is referred to as curriculum-based assessment (CBA). Oftentimes, the publisher of a textbook series has done this for the teacher in the form of chapter and unit tests, or the teacher may choose to generate the test items based upon the specific curriculum being employed.

In contrast to norm-referenced assessments, which are typically used for classification purposes, curriculum-based assessments *may* provide a stronger connection between what is tested and what is taught. There are two advantages to this method. First, it gives the instructor a baseline against which progress can be measured. Second, because the items are derived from the curriculum, instruction is directly linked to the results. Thus, the risk of omitting instruction in a certain area is avoided; the teacher and student need not worry about gaps in the student's background.

Although CBA is not new, it has the appeal of a "novel" approach to assessment and, at first blush, may seem more relevant to a specific learning environment. However, a major caveat regarding CBA is the

tendency by some educators to develop items that emphasize lower order thinking (e.g., basic facts), rather than higher order thinking (e.g., mathematical applications or problem solving). Also, for some educators, CBA has become the answer to the perceived "competition" associated with norm-referenced measures. In reality, students in a class know almost intuitively who the more able students are. The best use of CBA is as a complement to the standardized measures that comprise an assessment. The same is also true of the standardized measures presented above. Results from standardized tests are most useful when combined with other information about the student, including curriculum-based information.

The Mathematically Talented Twice-Exceptional Student

When discussing the possibility that students who are mathematically talented may also have one or more disabilities, the most common question seems to be one of prevalence: How many students are we talking about? This is a very difficult question for a couple of reasons. First, although prevalence rates of students with disabilities are documented through federal legislation, no comparable legislation exists regarding gifted students. This raises the following questions:

- How many gifted students might have a disability?
- What disability is most common?
- How many students with a disability might be academically or artistically gifted?
- In what domains of talent are we most likely to see students with a disability?
- Are certain disabilities more likely to be associated with specific domains of talent (e.g., are writers more likely to suffer from bipolar disorder?)?

We would assert that specific answers to these questions are less relevant than having a basic acceptance and understanding of the concept

of twice-exceptionality. It is important for educators to recognize that sometimes students are not allowed to progress into an accelerated class if they are displaying a disability, especially if the disability is manifest as a severe social impairment. The reason often given is that the student needs to "mature." However, some students have such severe social impairments that they require special interventions to address their weak social skills. This should not, however, preclude them from being placed in classes that are appropriately challenging. In other cases, some students might be correctly placed in programs that provide sufficient challenge, but their inappropriate behaviors are interpreted as being primarily associated with an extreme level of giftedness when, in fact, there is an impairment that needs to be diagnosed and addressed. Clearly, this is a more complicated issue than we can address in this section. The purpose for discussing this topic is to highlight the importance of assessment for identifying both strengths and weaknesses.

The Psychoeducational Report and Recommendations for Accommodations

The law mandates formulation of an Individual Education Program (IEP) for students who qualify for special educational services. Although there is not a similar federal mandate for students who are identified for gifted and talented programs, the trend in many states is to create such a plan. The psychoeducational report, typically written by a psychologist who has administered the tests and thus is reporting the results, is pivotal in developing a student's personalized plan for programming regardless of a student's status as a gifted, special education, or twice-exceptional student.

A psychologist's report provides official documentation of the results of an assessment. In addition to stating test scores, a report should reflect the psychologist's interpretation of the child's behavior during the educational assessment in conjunction with information provided by the parent and teacher. All reports should serve as a means of informative communication. However, the length of the report may

be misleading with respect to information. For example, a 10-page document that goes into great detail with irrelevant background information and makes unfounded predictions concerning the child's future is unsuitable. Equally unsuitable, but at the other extreme with respect to length, is a two-sentence document that states, "Johnny earned an IQ score of 145 on the WISC. This score indicates that Johnny is gifted and should be placed in a program."

A good report starts with some brief demographic information about the student and then briefly explains the reason for referral. The initial reason for referral to the psychologist dictates the type of assessment (i.e., which tests are used). The specific tests used determine the information that is included in the report. During an assessment, the psychologist should spend an adequate amount of time getting to know the child and parents in order to develop an appreciation for the child's history as it pertains to the present situation. Because intelligence test scores have so often been used as the principle source of information for placement, classification, or both, they often are regarded as an end in themselves. It is our position that a useful assessment should provide diagnostic information that may result in a placement recommendation. The ultimate goal of an assessment should be to develop written recommendations that will guide the child's educational program.

During the assessment, the skilled psychologist will carefully observe how the child approaches different types of problems, attend to the level of language used by the child, look for indicators of impulsivity and stress, and interpret the child's general attitude in that type of setting. This information should be used anecdotally in discussing recommendations for the educational program.

The section of the report labeled "Recommendations" is actually the crux of the report and should serve as the foundation for any individualized educational plan being developed. The recommendations are action-oriented and suggest programming, as well as future assessments and evaluations of the student. This plan should include a description of the educational setting and materials that will be necessary for the student's success. It should always include a method for follow-up, which does not necessarily need to be conducted by the

psychologist. However, the psychologist should aid the teacher and parent in identifying the appropriate personnel for follow-up. Figure 3.1 shows an example of a psychoeducational report generated after the assessment of a mathematically talented child. In the report, the first test mentioned is EXPLORE, which was used as a talent search instrument. EXPLORE and other talent search tests such as the ACT Assessment and the SAT I are discussed extensively in Chapter 5.

Consumer Guidelines to Educational Assessment

1. *Know what question guides the assessment.* When considering a psychoeducational assessment, this is the first issue to address. The question that guides the assessment helps the professional choose the appropriate test and drives the recommendations. Parents and educators should work together to formulate the question and establish the most effective means for obtaining the information. We provide four brief examples of questions and encourage you to think about the kind of test that could provide the answer.
    - What is my child's level of mathematical talent and what is the appropriate level of instruction?
    - If my fourth grader skips into fifth-grade mathematics, will there be gaps in his or her mathematical background?
    - Will Algebra I be too hard (or too easy) for my sixth grader?
    - Does my child qualify for the gifted program?

2. *Know what tests are appropriate and useful for obtaining the needed information.* We don't expect parents to select and/or request specific tests for the student's assessment, nor is that even desirable because most parents don't have the professional background to do so. However, both parents and educators should be aware that a general ability test won't give enough specific information about a child's mathematical ability to

Student: Jason
Present Grade: Entering grade 6
Present School: Regal Parochial Middle School
Midsized Town, Any State
Assessment Dates: Summer before Jason entered sixth grade

Background and Reason for Referral

Jason has just completed grade 5 at Regal Parochial School. At present, the school offers two options for students who are talented in mathematics: (a) a grade 6/7 math class that is both compacted and accelerative; and (b) a sixth-grade prealgebra course. At the time of the referral, Jason did not qualify for either option. In view of the fact that Jason had completed several types of mathematical enrichment during his Extended Learning Program (e.g., working in bases other than base 10), his parents made the referral to obtain additional information about his aptitude in mathematics to assist them in their efforts to advocate for an appropriately challenging placement.

Records Reviewed

Iowa Tests of Basic Skills (ITBS)
Cognitive Abilities Test (CogAT)
EXPLORE

On the Iowa Tests of Basic Skills (ITBS), a group-administered grade-level achievement test, Jason earned the following percentile rankings (national norm group) when he was in fifth grade:

Concepts and Estimation	98th percentile
Problem Solving and Data Interpretation	98th percentile
Math Computation	83rd percentile

As a fourth grader, he earned scores at the 99th percentile in all three areas, when compared to the national normative group.

On the group-administered Cognitive Abilities Test (CogAT), Jason earned the following scores as a 5th grader:

Verbal	122
Quantitative	120
Nonverbal	124
Composite	124

Figure 3.1. Psychoeducational report.

Jason took the EXPLORE test, which is a test designed for eighth graders, as a fourth grader. His scores were as follows:

	English	Math	Reading	Science	Composite
Standard Score	13	17	12	14	14
Fourth Grade Percentile Compared to Academically Talented Fourth Graders	57	98	43	29	50
National Percentile Compared to Eighth Graders	50	75	42	30	50

Tests Administered

Wechsler Intelligence Scale for Children–Fourth Edition (WISC-IV)
Comprehensive Testing Program 4, Mathematics 1 & 2 (CTP 4)
Iowa Algebra Aptitude Test (IAAT)

Results From Tests Administered

Wechsler Intelligence Scale for Children–Fourth Edition (WISC-IV)

Composite Scores	Percentile Rank	Qualitative Descriptor
Verbal Composite	97th	Superior
Perceptual Reasoning	87th	High Average
Working Memory	75th	High Average
Processing Speed	7th	Borderline
Full Scale	82nd	High Average
General Ability Index	96th	Superior

Figure 3.1, Continued

Comprehensive Testing Program 4, Mathematics 1 & 2 (CTP 4)

Math Skill Area	Description	Number Correct/ Total Items	Percentage correct
Numbers and Number Relationships	Understands equivalent forms of fractions, decimals, and percents; uses numbers in these forms; understands multiple representations of numbers including expanded notation, exponential forms, and scientific notation; applies numbers using multiple representations; understands plotting fractions on a real number line.	9/12	75%
Number Systems and Number Theory	Understands place value with whole numbers and decimals; recognizes ordering of fractions and decimals; understands multiples, factors, primes, and composites; applies operations with fractions and decimals and number theory concepts in everyday mathematical situations.	10/13	77%
Geometry and Spatial Sense	Understands basic geometric language for describing shapes; understands basic properties of figures including applications in coordinate geometry; applies geometric properties and relationships, including congruence and similarity of figures, lines of symmetry, sum of angles in a triangle, triangle inequality, images under transformation, and 2- and 3-dimensional shape changes.	8/13	62%

Figure 3.1, Continued

Math Skill Area	Description	Number Correct/ Total Items	Percentage correct
Measurement	Understands basic systems of measurement and relationships among units of the same system; understands measures of figures including perimeter, circumference, surface area, volume, and angles; uses units appropriately; estimates measures using scale; understands the application of formulas in deriving measures; solves simple rate problems.	9/12	75%
Statistics	Reads and interprets various types of graphs including bar, single and double line, circle, pictorial, and tables; understands and computes average, median, mode, and range; understands Venn diagrams and sampling techniques.	8/10	80%
Probability	Determines probabilities and solves problems of chance using ratios and related probabilities; makes predictions based on mathematical probabilities; understands counting techniques including use of probability to determine amounts.	7/9	78%

Figure 3.1, Continued

Math Skill Area	Description	Number Correct/ Total Items	Percentage correct
Pre-Algebra	Represents patterns and mathematical situations involving tables, graphs, and equations; understands the use of variables, balancing equations, and completing mathematical sentences to find an unknown; applies variables, expressions, and equations to solve real-world problems; formulates equations from word sentences.	11/15	73%

Iowa Algebra Aptitude Test (IAAT)

Section	Raw Score	National Percentile Rank
Interpreting Mathematical Information	11	67th
Translating to Symbols	10	61st
Finding Relationships	13	85th
Using Symbols	12	87th
Total	46	77th

Summary and Recommendations

Jason has demonstrated evidence of high ability through tests as well as through performance in class. Notably, his scores on the fourth-grade ITBS were at the 99th percentile, indicating that there were not a sufficient number of difficult items to determine an appropriate level of challenge. In fifth grade, Jason's ITBS percentiles were at the 98th percentile, with the exception of Computation. The Computation score was *relatively* lower, but not significantly, and not at a level that would represent a concern.

Figure 3.1, Continued

Because of Jason's outstanding performance on grade-level achievement tests, he was invited to participate in the Elementary Student Talent Search. He took the EXPLORE test as a fourth grader. EXPLORE is a test developed by ACT for eighth graders, and thus Jason's performance was compared to eighth graders, as well as to other bright fourth graders who were Elementary Student Talent Search participants (i.e., they all earned scores on a grade-level test at, or above, the 95th percentile). Jason's composite score, which is an average of the four tests, was at the 50th percentile when compared to bright fourth graders and the normative eighth-grade group. The EXPLORE composite score provides information regarding a student's performance on all four tests: English, Mathematics, Reading, and Science Reasoning. Because the referral question referred specifically to placement and programming for mathematics, we focus our attention on his performance on this test.

Compared to the eighth-grade normative sample, Jason earned a score at the 75th percentile. That means that as a fourth grader, *without* exposure to the curriculum that the eighth graders would have studied, Jason surpassed 75% of the eighth-grade normative sample. His scale score of 17 surpassed 98% of bright fourth graders. This is remarkable and suggests that Jason will be underchallenged by a regular mathematics curriculum.

Jason was seen in a private clinic setting by a licensed psychologist; therefore, it was possible for an individualized intelligence test, the Wechsler Intelligence Scale for Children–Fourth Edition (WISC-IV) to be administered. These results were very informative and revealed noteworthy strengths as well as a significant concern. In brief, Jason's performance on the subtests of the Verbal Comprehension Index was at the Superior level. His performance on the nonverbal tasks measured by the Perceptual Reasoning Index was also very strong, and yielded an index score in the High Average to Superior Range. His working memory, which was measured by asking him to complete short-term memory tasks, was also in the High Average Range. However, his Processing Speed Index was extremely low, and yielded a score that was at the 7th percentile.

Because of the discrepancy in the four index scores, the composite score that would typically be reported, the Full Scale Score, is not as informative as the General Ability Index (GAI). The GAI uses subtests from the Verbal Comprehension Index and the Perceptual Reasoning Index, and Jason's GAI was at the 96th percentile. This would suggest that he has general ability,

Figure 3.1, Continued

which would support the prediction that he will be highly successful in an academic setting. However, the extremely low Processing Speed Index is a key component in understanding the apparent discrepancy between Jason's very high performance on the above-level tests (EXPLORE, IAAT, and CTP 4) and the teachers' perceptions that Jason has some learning difficulties and therefore does not qualify for either of the options for the mathematically talented students.

To determine whether Jason would have significant gaps in his mathematical knowledge if he were to skip the sixth-grade mathematics program, the CTP4 mathematics tests were administered. These tests measure mathematics knowledge, extrapolation of data, operations procedures, and deductive reasoning skills. Jason's performance was compared to students who had completed grade 6 and his overall score was at the 91st percentile, when compared to a national sample. This means that he scored as well as, or better than, 91% of students in the normative sample *who had completed sixth grade*. In other words, he demonstrated consistently strong performance across the various areas that were tested.

The IAAT is usually used with students who have completed prealgebra coursework, typically eighth and ninth graders. However, given Jason's very high score on EXPLORE, we determined that it would be appropriate to administer this test. Because of his very low Processing Speed Index on the WISC-IV, we allowed Jason extended time to complete the IAAT. He used this time extension on the first two sections, Interpreting Mathematical Information and Translating to Symbols, and scored at the 67th and 61st percentiles, respectively. His response rate increased on the final two components (Finding Relationships and Using Symbols) and he completed the tasks within the standard time limit, achieving scores at the 85th and 87th percentiles. Jason's overall score at the 77th percentile indicates that he has more than an adequate understanding of algebra topics and would most likely be appropriately challenged by either placement in prealgebra or algebra coursework.

To address Jason's complex learning needs, we recommend:

1. The assessment was conducted in a private clinic. Therefore, it will be important to share this information with Jason's teachers.

Figure 3.1, Continued

2. Placement in an advanced mathematics program (preferably within the school) is necessary in order to provide sufficient challenge that matches his very high level of mathematical aptitude. This type of placement will also require long-term planning to ensure that he has appropriately challenging mathematics throughout his school years. If Jason's needs for academic challenge cannot be met by the school, his parents may want to consider an arrangement with a mentor.

3. When available, involve Jason in academic enrichment activities, such as summer classes or weekend classes offered during the school year. Computer-based correspondence courses are available for talented students through Johns Hopkins University (http://www.cty.jhu.edu), Northwestern University (http://www.ctd.northwestern.edu), and the Educational Program for Gifted Youth (http://epgy.stanford.edu/epgy) that may be of interest to Jason.

4. Jason would most likely benefit from and enjoy participating in the Mathematical Olympiads for Elementary and Middle Schools (MOEMS) competitions. This would provide him with the opportunity to interact socially and academically with other mathematically talented students.

5. Given Jason's significantly low Processing Speed Index, it is recommended that when high-stakes and/or machine-scored tests are administered, that Jason be allowed two accommodations:
    a. untimed testing periods and
    b. the option to write in the test booklet rather than use the bubble forms.

6. Given the discrepancy between Jason's exceptional intellectual ability and his processing speed, we recommend reducing basic skill worksheets and instead offering assignments with fewer items at a more advanced level that incorporate these basic skills. These assignments could allow Jason to demonstrate his knowledge and skills on fewer items and reduce his frustration with lengthy written assignments.

7. Jason should participate in the middle-school level of the talent search. Because both the ACT and the SAT are offered by the majority of middle-school talent searches, it is recommended that he take the SAT when in seventh grade and the ACT when in eighth grade.

Figure 3.1, Continued

determine placement in a mathematics class. They also should be aware that a grade-level achievement test probably will not give enough information about what a student has learned, as well as what the student is ready to learn. Therefore, make decisions about the type of test(s) needed based upon the kind of information that is needed in order to make appropriate programming decisions.

3. *Confirm that the person conducting the assessment has appropriate training* (see Table 3.1). Some tests can be administered by a teacher who is familiar with the directions. Other tests require extensive training for both administration and interpretation; therefore, the person administering them usually has an advanced degree.
4. *Reaffirm the importance of effective communication among parents, educators, and the assessment expert about the results and recommendations.* "Communication is a two-way street" is a common saying. This adage is applicable when developing a program for mathematically gifted students. Sometimes the testing will be conducted entirely by the school. Other times, the testing may be conducted exclusively by an outside person. Most often, there is an in-between scenario. Ideally, there is mutual respect for the information being used in the decision-making process and "outside" professionals will provide results and tailored recommendations in a written form that includes the actual test scores and an interpretation within an educational context. We strongly recommend that, prior to testing from private professionals, parents and educators have communicated and have a plan for bringing the information together to discuss within the context of placement and/or programming.
5. *Results from testing should be completed and discussed in a timely fashion* (i.e., within one month after the assessment has been completed). Parents should be notified of any delays.
6. *Determine the format for the assessment.* In other words, are the tests individually or group-administered? Is the test electronically completed, does the student respond orally, or does he or she use paper and pencil to respond to test items?

Table 3.1
Summary of Tests

Test name (publisher)	Construct measured/ Group or individual	Purpose for administering	Age or grade level of student	Who can administer
Cognitive Abilities Test (CogAT-6) (Riverside Publishing)	Verbal aptitude, quantitative aptitude, nonverbal aptitude/Group	To measure cognitive ability; may be used for placement in gifted program	Grades K–12	Certified educator
Comprehensive Testing Program 4 (CTP 4) (Educational Records Bureau)	Achievement/Group Verbal and quantitative reasoning skills are assessed as well. Can also be administered above level.	Assessment of school material learned	Grades 1–10	Certified educator
Iowa Tests of Basic Skills (ITBS) (Riverside Publishing)	Achievement/Group When administered above grade level, it measures aptitude.	Assessment of school material learned	Grades K–8	Certified educator
Iowa Algebra Aptitude Test (5th ed., IAAT) (Riverside Publishing)	Specific readiness for learning algebra/Group or individual	Placement into prealgebra or algebra	Grades 7–8 (4–6 as above level)	Certified educator
Iowa Tests of Educational Development (Riverside Publishing)	Achievement/Group When administered above grade level, it measures aptitude..	Assessment of school material learned	Grades 9–11	Certified educator
KeyMath-3 Diagnostic Assessment (Pearson)	Math achievement and aptitude/ Individual	Diagnostic, primarily to determine presence of a disability; less useful for obtaining information about programming for math talented students	Ages 5–22	Certified educator

Table 3.1, Continued

Test name (publisher)	Construct measured/ Group or individual	Purpose for administering	Age or grade level of student	Who can administer
Measure of Academic Progress (MAP) (Northwest Evaluation Association)	Reading, mathematics, and language usage/Individually administered computer-adaptive test	Assessments in reading, mathematics, and language, which are aligned with state standards	Grades 2–10	Certified educator
Orleans-Hanna Algebra Prognosis Test (3rd ed.) (Pearson)	Specific readiness for learning algebra/Group or individual	Placement into prealgebra or algebra	Grades 7–11 (4–6 as above level)	Certified educator
Otis-Lennon School Ability Test (8th ed., OLSAT) (Pearson)	Verbal and nonverbal aptitude/Group	To measure cognitive ability; may be used for placement in gifted programs	Grades K–12	Certified educator
School and College Ability Test (SCAT) (Johns Hopkins Center for Talented Youth)	General measure of scholastic aptitude/Group	Primarily used as an above-level talent search measure	Grades 2–6	Certified educator
Sequential Tests of Educational Progress (STEP) (Johns Hopkins Center for Talented Youth)	Broad global achievement/Individual	Assessment of school material learned; often used above level	Grades K–12	Certified educator
Slosson Intelligence Test–Revised (3rd ed., SIT-R) (Slosson Educational Publications)	Emphasis on verbal ability/Individual	Appropriate for screening for gifted program; less useful for programming for math talented students	Ages 4–65	Certified educator or psychologist
Stanford Achievement Test (10th ed., Stanford 10) (Pearson)	General achievement of school subjects/Group	Assessment of school material learned	Grades K–12	Certified educator

Table 3.1, Continued

Test name (publisher)	Construct measured/ Group or individual	Purpose for administering	Age or grade level of student	Who can administer
Stanford-Binet (Revised Regularly) (Riverside)	General intelligence/Individual	Measure of general ability for placement and programming purposes; especially useful when diagnosing some disabilities	Ages 2–adult	Licensed or certified psychologist
Test of Early Mathematics Ability (3rd ed.) (Pro-Ed)	Achievement in mathematics/ Individual	Assessment of learning problems	Ages 3–8	Certified educator
Test of Mathematical Abilities (2nd ed., TOMA-2) (PRO-ED)	Achievement in mathematics/ Individual or group	Assessment of learning problems	Ages 8–18	Certified educator
Test of Mathematical Abilities for Gifted Students (TOMAGS) (PRO-ED)	Math reasoning and problem-solving ability/Individual or group	Screening for math reasoning	Grades K–6	Certified educator
Wechsler Individual Achievement Tests (3rd ed., WIAT-III) (Psychological Corporation)	Achievement in reading, mathematics, and language/Individual	Measure of general achievement often for placement and programming purposes; especially useful when diagnosing some disabilities	Grades pre-K–college	Licensed or certified psychologist

Table 3.1, Continued

Test name (publisher)	Construct measured/ Group or individual	Purpose for administering	Age or grade level of student	Who can administer
Wechsler Intelligence Scales (Psychological Corporation)	General intelligence/Individual	Measure of general ability for placement and programming purposes; especially useful when diagnosing some disabilities	Three versions of the test exist, one for each age level: Preschool/ Primary, School, and Adult (each is revised regularly)	Licensed or certified psychologist
Woodcock-Johnson III NU (Tests of Cognitive Abilities and Tests of Achievement) (Riverside)	Achievement in reading, mathematics, and language/Individual	Measure of general achievement	Ages 2–90+	Licensed or certified psychologist, or specially trained educator (computer scored)
Woodcock-Johnson III NU Tests of Cognitive Abilities (Riverside)	General cognitive ability/Individual	Measure of general ability for placement and programming purposes	Ages 2–90+	Licensed or certified psychologist, or specially trained educator (computer scored)

7. *Be aware of the cost of assessment.* The cost continuum corresponds to the school testing–private testing continuum. Sometimes, testing might be done through the school district at no cost to parents. At the other end of the continuum, parents might pay well more than $1,000, especially for a thorough assessment that includes an individualized intelligence test.

Conclusion

The main purpose of this chapter was to familiarize the reader with the notion of a psychoeducational assessment for gifted children. We want parents and educators to think of a psychoeducational assessment as a way to learn more about a child's cognitive characteristics (i.e., ability, aptitude, and achievement). The tests that are used in an assessment should be determined by the purpose for which the assessment is being conducted, and the information from tests should be meaningful within a context. A good psychoeducational assessment is the first step in advocating for an educational program. The next step is implementation of the program based upon the information obtained from the assessment. This is the model referred to as Diagnostic Testing→Prescriptive Instruction (DT→PI), which is the focus of the next chapter.

CHAPTER 4

The Diagnostic Testing→Prescriptive Instruction Model

KEY POINTS

- The Diagnostic Testing→Prescriptive Instruction (DT→PI) model is a five-step procedure designed to tailor instruction to a student's learning needs.
- Using tests developed for older students (above-level assessment) is an important component of the DT→PI process.
- Techniques for pretesting and posttesting are presented and the critical nature of this activity is discussed.
- The chapter presents examples of how to use the DT→PI process and offers suggestions for implementing it successfully.

Avoid trying to teach students what they already know.
—Stanley, 2001, p. 298

The purpose of this chapter is to explain the Diagnostic Testing→Prescriptive Instruction (DT→PI) model as developed by Julian Stanley for use with mathematically precocious students. This model is useful in working with exceptionally talented students to ensure that they study mathematics that is appropriately challenging, but that they do not skip important concepts or have "gaps" in their backgrounds. This chapter illustrates in a step-by-step manner the DT→PI model and includes examples of students who have used it.

The notion of diagnostic testing and prescriptive instruction has its roots in special education (Lerner, 1976). The approach is founded on a system of testing students "out of level." In special education, a test below the grade level of placement may be used. One of the first researchers in gifted education, Leta Hollingworth, who wrote the first gifted education textbook and is sometimes referred to as the "mother" of the gifted education field, recognized that this idea of out-of-level testing also could be useful for gifted students who could be assessed using tests designed for older grade levels (Hollingworth, 1942). (For more information about Hollingworth and her work, see the special issue of *Roeper Review*, Volume 12, No. 3, March 1990). Julian Stanley (1990, 2001) extended Hollingworth's ideas in the late 1960s and early 1970s and developed the DT→PI model to identify mathematically talented students' strengths and weaknesses and to pinpoint areas in which they needed work.

A key component of the DT→PI model is the systematic, objective approach to determining the most appropriate curriculum for mathematically talented students (Benbow, 1986; Lupkowski, Assouline, & Stanley, 1990; Stanley, 1978, 1979a, 1991, 2001; Stanley & Benbow, 1986). In the field of gifted education, Stanley's adaptation of the DT→PI model is especially useful for exceptionally mathematically talented students. Because it is an individualized approach, it can be adapted easily for each student, but it also can be used with a small group of students who demonstrate varying degrees of mathematical talent.

The DT→PI process begins by administering an above-level aptitude test to a talented student (see Chapter 3 for an extensive discussion of above-level aptitude testing). Students who do well on this initial screening measure may then participate in above-level achievement testing, where their strengths and weaknesses on particular topics are pinpointed. After this, students work with a mentor on the topics they did not understand. Once they have mastered a topic, they take a posttest and then reenter the process at the next higher level of achievement testing.

In reading through the description of the Diagnostic Testing→Prescriptive Instruction process, it may seem as though

students are taking many tests. Although this is the case in the initial phase of DT→PI, the purpose of the testing is to tailor instruction, and the percentage of time spent on testing is much smaller than the overall percentage of time spent on instruction.

Above-Level Assessment

Most regular classrooms use an on-grade-level curriculum that is designed to be implemented by a teacher with a general educational background who may not necessarily be a mathematics educator. The curriculum is not oriented to the individualization required by DT→PI. Typically, the gifted student in the regular classroom will have demonstrated mastery of the grade-level curriculum (often through home instruction, self-instruction, or both). In other words, gifted students know much, if not most, of the curriculum for their grade before they even walk into the classroom. When these students take a standardized test that was designed for their grade level, their responses to all or most of the items are correct, thus reaching what psychologists call the test's "ceiling." For these students, the grade-level test does not adequately measure their talents because answering all of the items correctly informs us that they performed well compared to other students in their grade. However, we don't know the extent of their talents due to the ceiling effect of the test. In the DT→PI model, the talented students simply take a test that was designed for older students, a strategy shown to be extremely effective with talented students (Benbow, 1992a; Benbow & Arjmand, 1990; Benbow & Lubinski, 1996; Benbow & Stanley, 1983; Keating, 1976; Stanley, Keating, & Fox, 1974).

As a case in point, when a student scores at the 95th percentile or above on a grade-appropriate mathematics test, the test may not have measured a student's full achievement in that area (see Chapter 3 for a detailed explanation of a percentile). This is an example of a student hitting the ceiling of a test. What is needed is a test that contains more advanced items. A more challenging test allows students to demonstrate what they know, what they don't know, and what they are ready to learn

next. Above-level tests spread out the scores of able students, helping us to differentiate talented students from exceptionally talented students. This information is extremely helpful for good educational planning (see Chapter 5 for more information about above-level testing).

The DT→PI model, designed with precocious junior high school students in mind, has been adapted for use with younger students (see Lupkowski & Assouline, 1992; Moore & Wood, 1988; Stanley, 1978). Each step of the model is detailed in Figure 4.1.

Steps in the DT→PI Model

Step 1: Determine Aptitude

The purpose of this step is to determine the mathematical aptitude of students who are performing well on grade-level mathematics. For example, elementary school students who have scored at the 95th percentile or above on a nationally standardized test such as the Iowa Tests of Basic Skills (ITBS) are eligible to participate in above-level testing. Students do not have to score at the 95th percentile on the overall test, but a score at the 95th percentile is strongly recommended on the mathematics subtests (e.g., Mathematics Concepts or Mathematics Problems and Applications) as a launching point for determining more precisely the student's aptitude in mathematics.

The 95th percentile cutoff is a guideline; it does not need to be adhered to rigidly. If a student scores slightly below the cutoff, but school personnel and parents have other evidence that the student has exceptional mathematics ability, then he or she may be eligible for above-level testing (see Ebmeier & Schmulbach, 1989; Lupkowski-Shoplik & Swiatek, 1999).

We strongly recommend the use of a standardized, nationally normed test in this phase of the DT→PI process. Extensive research has demonstrated that useful above-level measures of aptitude for students younger than seventh grade include the PLUS, School and College Ability Test (SCAT), EXPLORE, and Comprehensive Testing

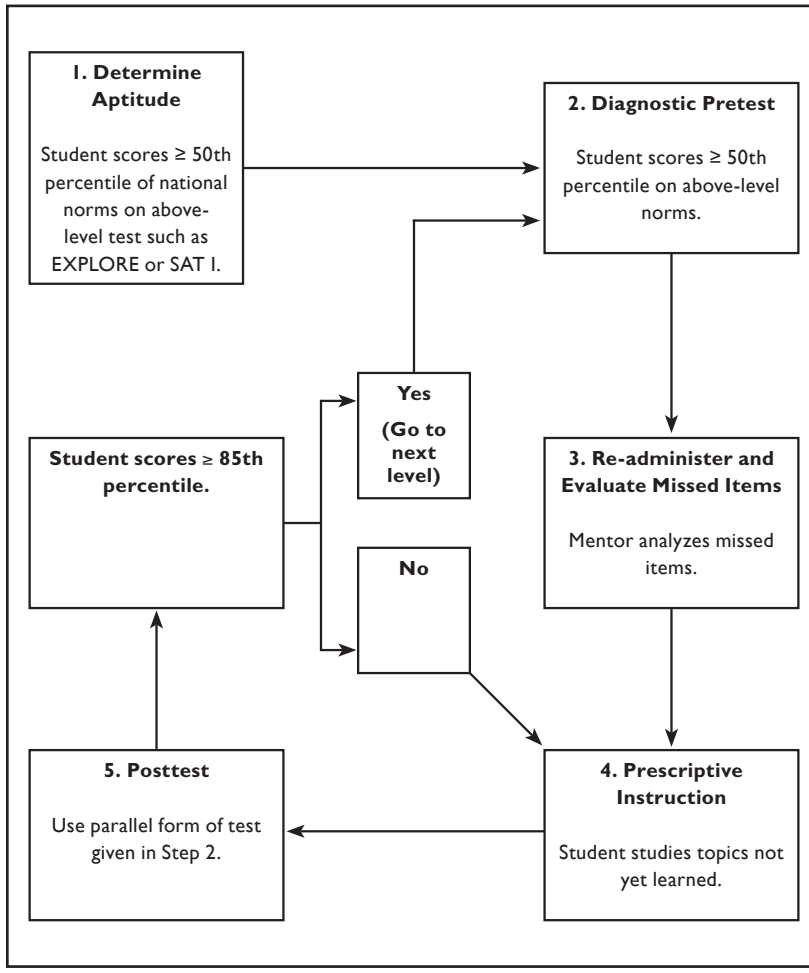

Figure 4.1. The DT→PI model. Model originally developed by Julian Stanley (Stanley, 1978). An earlier version of this figure is found in Lupkowski and Assouline (1992).

Program (CTP; Institute for the Academic Advancement of Youth, 1997; Lupkowski & Assouline, 1993; Lupkowski-Shoplik & Swiatek, 1999; Mills, Ablard, & Stumpf, 1993; Mills & Barnett, 1992). We also recommend that school personnel consider using a higher level of the quantitative sections of nationally standardized achievement tests they may already have available, such as the ITBS, Stanford Achievement

Test (SAT 10), or Metropolitan Achievement Test (MAT). If school personnel use one of these readily available tests, it is important to select one that is at least two grade levels higher than the student's current grade placement (see Table 4.1).

It is important to use above-level tests, not grade-level tests. Mathematically talented students may know advanced material that is not typically included on the grade-level test. The point of administering an above-level assessment is to determine what a student doesn't know so that instruction can be based upon filling in knowledge gaps. Standardized tests are especially effective for this purpose because they provide corresponding instructional objectives. Now the task becomes deciding how far above level to go. We recommend the following:

- *Kindergarten–grade 2.* Use a standardized test such as the ITBS or Stanford Achievement Test. However, select a level that is two grades above the child's current grade. For example, a first grader would take the third-grade form of the Stanford Achievement Test.
- *Grades 3–5.* Have the students participate in a talent search for elementary students (see Chapter 5 for extensive information about talent searches) and take a test such as EXPLORE, or select a level of a locally administered standardized test that is two or three grades above the child's current grade.
- *Grade 6.* Have the child participate in one of the elementary talent searches and take a test such as EXPLORE. If the student is also hitting the ceiling on EXPLORE, then have the student participate in one of the middle school talent searches.
- *Grade 7.* Have the child participate in one of the middle school talent searches and take the SAT I.
- *Grades 8–9.* Have the child participate in one of the middle school talent searches and take the ACT or the SAT I.

When Step 1 is completed, the scores will determine whether or not the student will go on to Step 2. Scores earned by talented students should be compared to scores for students in the above-level grade for which the test was designed. For example, scores earned by fifth graders who take EXPLORE are compared to scores earned by the

Table 4.1
Tests Recommended for Step 1, Aptitude Testing

Current grade level of talented student	Above-level test	Contact information
Grades 2–6	SCAT	CTY (Johns Hopkins University)
Grades 3–6	EXPLORE	Belin-Blank Center (The University of Iowa), C-MITES (Carnegie Mellon University), CTD (Northwestern University), TIP (Duke University)
Grades 1–6	Select the level of a standardized achievement test that is at least two grade levels above the student's current grade. Standardized achievement tests include: Iowa Tests of Basic Skills (ITBS), Stanford Achievement Test (SAT 10), or Metropolitan Achievement Test (MAT). Using an above-level achievement test serves as a measure of aptitude. (See Chapter 3 for more information.)	Local school district
Grades 7–9	SAT I, ACT	State, regional, or university-based talent searches listed above as well as in the Resources.

eighth-grade comparison group because EXPLORE was designed for eighth graders.

Standardized tests were developed with the 50th percentile as a key indicator of average performance for the grade level of the test. Therefore, when talented students earn scores at or above the 50th percentile on an above-level test, it is a good indication that their performance is comparable to average students at that grade level. With

that in mind, we recommend that talented students who earn scores at the 50th percentile on the test administered in Step 1 move on to Step 2 of the DT→PI. To illustrate:

> Helena, a third grader, earned a total mathematics score at the 98th percentile when compared to third-grade norms on the locally administered Metropolitan Achievement Test. Her teacher realized that this high percentile ranking did not provide sufficient information about where to begin instruction so that Helena was introduced to new concepts. The teacher then administered the fifth-grade level of the Math subtests of the Metropolitan Achievement Test. Helena's Math subtest scores were at the 65th percentile when compared to fifth graders. Therefore, she (and her teacher) were ready for Step 2 of the DT→PI.

In general, we do not recommend that talented students who score below the 50th percentile on the above-level aptitude test (Step 1) continue with the DT→PI process. Instead, we encourage curricular adjustments such as enrichment or problem-solving activities (see Chapters 6 and 7 for more information). Some students may have the opportunity to participate in planned enrichment activities in mathematics through their gifted program or through extracurricular activities such as Mathematical Olympiads for Elementary and Middle Schools (found in the Resources). However, as the reader may have noticed, when using above-level tests with very young students (e.g., third or fourth graders who take EXPLORE), the number of years between the student's grade level and the grade level for which the test was designed can be more than 2; in those instances, we recommend enrichment during the current academic year, but consideration of retesting during the following academic year.

Step 2: Diagnostic Pretest

In Step 1, Determine Aptitude, the goal was to discover mathematically talented students, those who demonstrated exceptional

mathematical ability and would benefit from an individualized, advanced program. Traditionally, this is accomplished by giving the students an above-level aptitude test, such as the talent search instruments of EXPLORE, PLUS, ACT, or SAT. Based upon this testing, a select group of students is identified for further testing and specialized programming. This testing is part of Step 2, the purpose of which is to measure talented students' mathematical achievement and determine where to begin instruction. In Step 2, we administer achievement tests, starting with the grade level that approximates the above grade level of the aptitude test used in Step 1.

In this step of the DT→PI model, we start by using an above-grade-level achievement test[1]. Why not use the grade-level end-of-course test for the student's current grade level as a beginning point? Although this might seem to be a good idea for "proving" to school personnel that the student has no content gaps, it doesn't really have much pedagogical advantage for programming. The goal is to determine what talented students know and do not know so that instruction can be tailored to their needs. Standardized achievement tests, as well as chapter tests and final exams provided by textbook publishers, are typically used in Step 2. If a standardized achievement test is used, it is helpful to have two parallel forms of the test because the second form of the test can be used in Step 5 (Posttest). Tests recommended for this phase of the DT→PI model are listed in Table 4.2.

The diagnostic pretest of Step 2 is administered under standardized conditions (e.g., the test proctor follows the instructions for administration as described in the test manual and observes time limits). One exception is made: Students are encouraged to answer all questions, but are told to indicate those items about which they are uncertain by writing a question mark to the left of the item number on the answer sheet. When the testing time has elapsed, the examiner takes the test materials from the student, scores the test, and determines the percentile rank of the student's score.

[1] When using DT→PI with primary-aged students, the test of choice in Step 1 may be an above-level achievement test. Therefore, with primary-aged students, it's possible that Step 1 and Step 2 of the DT→PI are actually combined.

Table 4.2
Tests Recommended for Step 2, Diagnostic Pretest

Current grade level of talented student	Above-level test	Contact information
Grades 2–8	• Comprehensive Testing Program 4 (CTP 4) • End-of-year tests	• Educational Records Bureau • Available through the textbook publisher
Grades 4–6	• Iowa Algebra Aptitude Test	• Riverside Publishing
Grades 4–6	• Orleans-Hanna Algebra Prognosis Test	• Harcourt Brace Educational Measurement

There are three possibilities for students in this step of the model, and only the second results in moving on to Step 3:

- They do very well on the diagnostic pretest (85th percentile or above): These students should take a more difficult version of the test.
- They do moderately well (earning scores between the 50th and 85th percentiles): These students should move on to Step 3, Re-administer and Evaluate Missed Items.
- They do not do well (the scores are below the 50th percentile): These students should exit from the model and receive enrichment instruction in specialized mathematics topics.

In the first of the above three possibilities, the highest scoring students, those who score at the 85th percentile or above when compared to the normative group on a standardized test, have demonstrated mastery even though they might not have perfect scores. Therefore, students earning scores at the 85th percentile or above should take the next level of the test.

In the second possibility, students who score between the 50th percentile and 85th percentile when compared to the appropriate normative group on the standardized test are ready to move on to Step 3, Re-Administer and Evaluate Missed Items (Stanley, 1979a). Those

students who earn scores between the 50th and 85th percentiles are likely to learn the material quickly.

The final possibility, when students earn a score below the 50th percentile on the diagnostic pretest, results in the students exiting from the DT→PI model. These students still need exposure to advanced topics in mathematics through enrichment instruction in specialized topics (e.g., tessellations, statistics and probability, Fibonacci sequences). Below are three examples representing the scenarios described above.

> Kim, a talented fifth grader, earned a score above the 50th percentile (when compared to eighth graders) on EXPLORE Mathematics (a test developed for eighth graders). Step 2 involved the administration of the Iowa Algebra Aptitude Test, a measure of readiness for algebra. On this test, he earned a score at the 99th percentile even though he had no formal instruction in algebra. Therefore, he was administered an algebra test to determine which algebraic concepts he needed to learn.

> In Step 1 of the DT→PI model, Helena, a gifted third grader, earned a score at the 65th percentile on the fifth-grade level of the Metropolitan Achievement Test. As part of Step 2 of the DT→PI model, Helena's teacher administered Level 5 of the Comprehensive Testing Program 4 (CTP 4) Mathematics Part 1 and Part 2 Tests (Level 5 was designed for fifth grades). She earned a score at the 57th percentile when compared to fifth graders, and therefore went on to Step 3.

> Shawn, a fourth grader, earned a score at the 52nd percentile on the Mathematics Total of the sixth-grade level of the Iowa Tests of Basic Skills. In Step 2, he was administered the sixth-grade level of the CTP 4 Mathematics Tests. On this level of test, his score was at the 35th percentile, indicating that his mathematics instruction could be provided outside of the DT→PI model. However, he still needed exposure to special enrichment topics in mathematics.

Step 3: Re-Administer and Evaluate Missed Items

The purpose of Steps 1 and 2 of the DT→PI model was to identify students ready to move on to Step 3. The purpose of Step 3 is to gain a more complete understanding of what topics the student does and does not know so that instruction can focus on new material.

The first activity of Step 3 is to return the test booklet to the student and have him or her rework missed or omitted items or items marked with a question mark. Unlike Step 2, when the student and instructor must adhere to published time limits, there are no time limits to Step 3. The examiner does not return the original answer sheet to the student or tell the student how the items were missed the first time. The student reworks the items on a separate piece of paper, showing all work.

At this point, the mentor becomes involved (although, in some cases, the examiner and mentor are the same person). The items are checked again, and the mentor evaluates the concepts not understood by the student. These concepts are related to predetermined goals and objectives provided by the test or textbook publishers. The mentor bases instruction upon this information.

> In Helena's case, she correctly answered (within the time limit) 38 items out of 84 on Level 5 of the CTP 4 Mathematics Test. She did not attempt 26 of the items because she ran out of time; 20 others she marked incorrectly. She put question marks next to four additional items (which had been answered correctly). The total number of items that she was asked to retry was 50. She was told to show all work and was given unlimited time. Based upon a careful analysis of the content of each item and Helena's responses to the 50 items, Helena's teacher determined that she was ready for instruction in numerous aspects of prealgebraic mathematics typically taught to fifth and sixth graders. These aspects included properties of geometric shapes, geometric formulas for volume and circumference, formulating equations from word sentences, charting information to find an unknown, algebraic notation, sequencing steps in a multistep

problem, making tree diagrams, solving problems of chance, and spatial problem solving.

The final activity of Step 3 is identifying who will best serve as the instructor for Step 4.

Step 4: Prescriptive Instruction

The purpose of the DT→PI process is to arrive at Step 4. The prescriptive instruction of Step 4 is based on a thorough analysis of the testing results obtained in Steps 1–3. Prescriptive instruction may or may not be provided by the classroom teacher, but it is critical that whoever does provide it has an excellent background in mathematics. This person is referred to as a mentor.

The mentor designs an instructional program based upon the diagnostic testing. The mentor carefully determines what principles the student does not yet understand and the areas in which the student needs further practice. The mentor also questions the student in order to gain more insight into what he or she does and does not understand.

In this important phase of the model, the mentor works with the student on the concepts—not the specific test items—he or she does not understand. Very little time is spent on the topics about which the student has demonstrated mastery. We recommend that the mentor design problems to help the student study a topic. Above-grade-level textbooks also are useful resources for practice problems.

The goal of prescriptive instruction is to master each topic before moving on to the next, not to work through every page of the textbook. Students do not repeat topics they have already learned. The DT→PI model also permits students to study concepts in greater depth than is typically possible in the regular classroom. They are given new material, which helps them to learn challenging mathematics at an appropriate pace. The goal for the student is to gain a rich, solid foundation in mathematics that will serve him or her well when studying more advanced mathematics.

The mentor scheduled 12 weekly 2-hour sessions in a study room at the local public library over a 3-month period. Helena was still placed with third graders for the mathematics period at school, but she was not required to do all of the work her classmates were doing. Helena worked on her mentor-assigned homework while the other students worked on the daily lesson. In the evenings, Helena's parents checked to see if her homework was completed (they were not expected to check for accuracy). The regular classroom teacher was supportive of Helena and the completion of the mentor-assigned homework, and she encouraged her to ask any time she had questions concerning the homework. If she completed her mentor-assigned homework before the other students completed their daily work, she was permitted to read, work on the computer, or to do other homework. Helena's teacher also made sure to include Helena in the classroom games and other group activities in mathematics.

The mentor developed the curriculum from a fifth-grade mathematics text that was part of the local district's curriculum. If Helena had questions about the homework, she was encouraged to contact the mentor via e-mail. Throughout the prescriptive instruction step of the DT→PI model, the mentor evaluated and documented Helena's progress. This information was helpful in future planning.

Step 5: Posttest

The final step in the DT→PI model is to posttest to determine if students have mastered the content. After completing the prescriptive instruction, students take a parallel form (equivalent in difficulty, but with different items) of the same test used for pretesting.

On the posttest, students who score above the 85th percentile for the appropriate above-level norms are considered to have mastered the material. Students who score below the 85th percentile require additional instruction and practice with the material. Even those who

score at the 85th percentile, but earn less than perfect scores on the test, require work on the topics they do not yet understand. After the student has demonstrated mastery and the mentor is satisfied that he or she has adequately filled in any knowledge gaps on that particular topic, the student reenters the process at Step 2, using achievement tests and materials for the next level or topic. Thus, the student studies the mathematics topics in a systematic, sequential fashion, demonstrating mastery before moving on.

> To measure Helena's mastery of the material, the mentor administered a parallel form of the CTP 4 Mathematics Test, Level 5. Compared to fifth graders, Helena earned a score at the 95th percentile. (Remember, prior to the mentor-paced instruction, Helena's score on Level 5 of CTP Mathematics was at the 57th percentile compared to fifth graders.) The mentor went back and reviewed with Helena the concepts measured by the items she had missed on the test. This reassured the mentor that Helena had mastered those concepts. Helena reentered the DT→PI process at Step 2 by taking the end-of-book test from the sixth-grade book.

Putting the DT→PI Model Into Practice

In the preceding sections, we described how to select students for a mentor-paced program. The remainder of the chapter focuses on how the model is implemented. Consideration needs to be given to who, when, where, and for how long the mentor and student(s) meet; how students are evaluated compared to the other students in their grade; and how issues of credit and placement are resolved.

The DT→PI process with elementary-age students has been used extensively in programs at Carnegie Mellon University, Johns Hopkins University, The University of Iowa, and the University of North Texas. Based on our experiences with these programs, we have listed our responses to issues encountered by parents, teachers, and other school

personnel. Also, we explain some of the details of developing the programs (see also Lupkowski, Assouline, & Vestal, 1992).

Forms included in this book, such as the one presented in Figure 4.2, were adapted from forms used in these university-based programs.

Who Participates in Mentor-Paced Programs?

The DT→PI model has been used successfully with mathematically talented students as young as first grade. Standardized testing does not begin until second grade in many schools, so the need for this type of program for young students is not documented. Because of the nature of standardized testing and the identification component for the gifted program in most schools, it is more common for the majority of students to start participating in the DT→PI model when in third grade or later.

Students must not only demonstrate a high level of aptitude and achievement in mathematics, but they also should be mature and highly motivated. Participation in this type of program is best when students are eager to study more mathematics. The program demands a great deal of time and effort on the student's part. It is critical for children who participate in such a demanding program to do it because they want to do it, not because it is something their parents or teachers choose for them to do.

How Important Is the 50th-Percentile Guideline?

Using the 50th percentile as a recommendation for including a student in a mentor-paced program means that the student has performed as well as the average student who has already completed that course. The talented student (who earned a score at the 50th percentile before being exposed to the curriculum) is a likely candidate for being able to complete the course quickly and well. This 50th-percentile guideline has been used extensively by university-based programs for talented youth because it successfully provides direction for programming (Bartkovich & Mezynski, 1981; Benbow & Lubinski, 1996; Stanley et al., 1974).

Teachers should use their professional judgment with a student who falls slightly below the recommended 50th percentile. A student who scored slightly below the 50th percentile could still continue on with the DT→PI process. This won't "hurt" the student, as it is a highly individualized process and the model will self-correct. In other words, a student who has difficulty with the material will be given more time by the mentor to master the material. For students who fall well below the 50th percentile, however, placing them in a mentor-paced program might not be the best use of resources. If a student's percentile ranking is around the 40th percentile, consider evaluating him or her at a later date (perhaps the following term) and look for other indicators that he or she may need a challenging mathematics curriculum (e.g., advanced cognitive reasoning skills and a high level of motivation).

What Is the Role of the Mentor?

Stanley (1979a) described the mentor's role as follows:

> For the "prescriptive instruction" one needs a skilled mentor. He or she should be intellectually able, fast-minded, and well-versed in mathematics considerably beyond the subjects to be learned by the "mentee(s)." This mentor must not function didactically as an instructor, predigesting the course material for the mentee. Instead, he or she must be a pacer, stimulator, clarifier, and extender. (p. 1)

Who makes the best mentors? It is not necessary for the mentor to be a trained mathematics teacher. In addition to high school math teachers, engineers, college professors, undergraduate math majors, and graduate students in mathematics have been successful mentors. It is critical that the mentor have a good understanding of both the mathematics the student is currently studying and the mathematics he or she will study in the near future. Although we have worked with numerous mentors who were not teachers, we have found that some schools will recognize a student's work with a mentor only if the mentor is a certified teacher.

We do not recommend that parents mentor their children, even if they have the appropriate background in mathematics. Parents may demand too much of their own child or may be unable to get enough cooperation from their child. They also may have too much ego or pride involved in the mentoring situation, or they simply may not have credibility with the school.

Sometimes, it is also suggested that a high school student might be a mentor. Our experience has been that high school students are busy with their own academic work and extracurricular activities and, therefore, would have difficulty managing the responsibilities of planning for mentoring sessions and meeting consistently on a weekly basis.

The mentor-to-student ratio is often 1:1. However, a skilled mentor can work successfully with up to five students (Moore & Wood, 1988). Students at approximately the same level of understanding of mathematics can be grouped and it is not necessary for them to be of the same age. The Center for Talented Youth (CTY) at Johns Hopkins University offers "Young Students Classes" based on the DT→PI model. Based on the students' performance on diagnostic tests, they are placed in small groups of two to five (Barnett, Albert, & Brody, 2005; Mills, 1992; Moore & Wood, 1988). The benefits of this approach include both appropriate placement within the mathematics curriculum and the opportunity to interact with intellectual peers.

How Are the Mentoring Sessions Conducted?

In a typical school, mathematics class usually takes place every school day for about 45 minutes. When using the DT→PI process, however, the learning is much more highly concentrated. Therefore, daily instruction is not necessary. We recommend that the mentor and student meet for a total of 2 hours per week. For students in fifth grade and older, it may be best to meet once a week for 2 hours. Because younger students may be more easily fatigued, we recommend shorter, more frequent sessions (twice a week for an hour each time). One school system that was extremely eager to have a student participate in the mentor-paced program allowed the mathematics mentor to go into the

classroom twice a week for an hour each time to work with the student. Between meetings, the mentor assigned the student homework. It is also helpful if the mentor is accessible to the student via telephone or e-mail to answer questions that arise between meetings.

During their meetings, the mentor works with the student on the principles (not the specific test items) the student didn't understand. Textbooks are used as resources, but mentors often devise their own material and problems. As this is a highly individualized process, assignments are tailored to a student's specific strengths and weaknesses. Students must demonstrate mastery of one topic before moving on to the next, but they are not required to work through every page of a text. This process is in sharp contrast to the spiral approach of most mathematics textbooks, where students are not expected to master a concept because they will be exposed to it the following year. If a textbook is used as a resource, students may not need to work through every page of the text because they will already have mastered much of the material (see Chapter 7).

What Happens Between Meetings?

Homework completed on a daily basis (4–5 days per week) is essential in a mentoring situation because the student does not meet with the mentor every day. However, the homework should not take more than 20 minutes to complete each day. Students with questions about the homework may contact the mentor by phone or by e-mail. It is critical for the mentee to take responsibility for his or her own learning; this means doing homework carefully in a timely manner. Students participating in the mentor-paced program should be self-motivated and self-disciplined. Parents remain actively involved by supervising the completion of homework.

> Homework is an essential aspect of any fast-paced mathematics program. The material is covered in class so quickly that it must be reinforced by extensive homework assignments. The amount of homework should not be so great that it places a burden on the student or becomes repetitive or busy work.

> Nevertheless, homework can help a gifted youth make the most of his or her abilities. Students . . . must develop good study habits in order to space out the homework over the interval between classes. The discipline of pacing themselves through the homework assignment also leads to better recall of the material. (Bartkovich & George, 1980, p. 21)

Although parents of younger students might need to supervise homework sessions closely to make sure the work is completed on time, we want to emphasize the importance of self-motivation. The student needs to be interested in participating in the DT→PI process, not just doing it to please his or her parents. The mentor (and the parents) should try to make it an interesting experience; ideally, the student will find participation in the DT→PI process more challenging and fun than regular math classes in school.

Parent, Student, Mentor, School: How Do They Get Along?

The answer to this question depends on how the DT→PI model is implemented (see the discussion about parent-school partnerships in Chapter 2). In some cases, the DT→PI model is implemented privately by parents who hire mentors to work with their children outside of school, with or without receiving school credit for the work completed by the student. On the other end of the spectrum is the situation in which the school develops its own mentor-paced program, systematically screens and serves students in this program, and has a plan for their placement in mathematics after the program is completed.

In our experience, many schools are eager to have their students participate in DT→PI programs, even if it is an external program that the parents have brought to the attention of school personnel. Educators understand that these types of programs supplement what the schools are able to provide. Cooperation is assured through communication: Letters explaining the program and reports of the student's progress, as well as meetings among parents, mentors, and school personnel facilitate communication and cooperation (see Figure 4.2, an adaptation of a form used in a university-based math mentor program).

Many school personnel demonstrate their support of mentor-paced programs through their cooperation. Sometimes, teachers are the first to recognize talent and encourage parents to contact program administrators. In other cases, parents investigate the program first. We always encourage parents to provide school personnel with information about their children's academic activities (e.g., test reports) and to discuss educational options for their child. Teachers find it helpful when parents share pertinent articles.

Who Should Administer the Achievement Tests That Assess the Students' Progress?

We recommend that an achievement test be administered by the mentor or in collaboration with school personnel to ensure that it is conducted according to the directions provided in the manual. Achievement test results are useful in determining appropriate credit for work the student has completed because that work is not a part of the regular curriculum (Lupkowski & Assouline, 1992).

Results of the testing should be presented in a written report that also includes a description of the child's behavior during the testing and recommendations for prescriptive instruction based upon the results. In addition to giving the report to the parents, the examiner should share it with the school principal, counselor, teachers, and other appropriate school personnel (Lupkowski & Assouline, 1992). More information about this type of report is found in Chapter 3.

Is It Better to Use Standardized Tests or Teacher-Made Tests in Step 2?

Standardized tests are usually accompanied by specific instructional objectives, which is a distinct advantage. This allows a mentor to link test results directly to prescriptive instruction, therefore making standardized tests very useful in the DT→PI process. Teacher-made tests have a great deal of face validity, however, and teachers often feel more comfortable with their own tests. Unfortunately, most teacher-made tests are not well designed (Popham, 2002), and, in some cases,

To: Educator and Administrator of Student's School

From: _____, Math Mentor Program Coordinator or Math Mentor

The purpose of this letter is to inform you that _____ (name of student) is eligible to participate in the Math Mentor Program. The goals of the program are discussed below. Your assistance is needed to help the student succeed in this program.

The Math Mentor Program was developed for select students who demonstrate exceptionally high aptitude and achievement in mathematics, as well as high motivation for studying mathematics. Mentors who work with these students have a strong background in mathematics. Mentors may meet with students up to 2 hours once a week in the evening or on weekends. Mentors may work individually with the student or with small groups of up to five students. Students are required to complete mentor-assigned homework before the next meeting.

The primary goal of the Math Mentor Program is to keep students challenged by providing advanced material. This means that the mentor will determine what areas of mathematics the student needs to study. This is accomplished through the Diagnostic Testing→Prescriptive Instruction approach, which is accelerative in nature. Students study advanced material at an appropriate level and pace for their exceptional aptitude in mathematics. This results in students studying material that is several years beyond their grade level.

To encourage students' successful participation in this selective program, educators and administrators are asked to do the following:

1. Excuse Math Mentor Program students from regular math class activities. Progress through the Math Mentor Program will be determined through an assessment procedure implemented by the mentor. Student progress will be assessed routinely by mentors via mentor-made tests and mentor-generated homework.
2. Allow students in the Math Mentor Program to complete mentor-assigned homework while students in the regular mathematics classroom are completing teacher-assigned work. This mentor-assigned work should be in lieu of regular math class work.
3. Encourage students in the Math Mentor Program to participate in enrichment activities including mathematics games, competitions, or work on the computer.

Figure 4.2. Math mentor program information form for schools. It is suggested that individuals implementing this program also attach articles, chapters, or summaries explaining the DT→PI process.

The mentor will:

- work with school personnel to evaluate the student's performance,
- recommend a grade for the student based upon work completed within the Math Mentor Program, and
- communicate with school personnel to address placement decisions for future study of mathematics.

Communication among the mentor, educator, and parents is essential for this program to be a success. Below are the names of the mentor and the Math Mentor Program coordinator and their phone numbers. Please contact us with any questions you may have.

Math Mentor:
Name: _____
Telephone number: _____
E-mail: _____

Math Mentor Program Coordinator:
Name: _____
Telephone number: _____
E-mail: _____

Please check the appropriate category and sign below:

_____ We will permit the student to work on mentor-assigned homework during the regular math class and facilitate the student's participation in the Math Mentor Program.
_____ We are not yet prepared to make this commitment. (Please note that the Math Mentor Program Coordinator will call you to answer your questions if you check this box.)

Teacher: _____ Date: _____
Principal: _____ Date: _____

Thank you for your interest and involvement in the student's progress.

Figure 4.2, Continued

they have been developed as a way of "getting back" at a mathematically talented student. In addition, information from these tests often is misused, and teachers may not apply a reasonable level of mastery to the situation. For all of these reasons, we prefer standardized tests, but we recognize that the best-case scenario would include both standardized tests and teacher-made tests.

Will Students Participating in the Mentor-Paced Program Have Less Opportunity to Interact With Others About the Material Being Studied?

The environment of a mentor-paced setting differs dramatically from the environment in a regular classroom. One of the most notable differences is that students in a mentor-paced program spend a great deal of time working individually. However, the student is not studying mathematics in isolation. He or she can easily exchange ideas with the mentor about the material being studied because of working in a small group or one-on-one. An additional benefit of working with a mentor is that the mentor can provide more accurate answers to questions than peers can. The mentor also has the knowledge and ability to extend the student's questions or to ask new questions of that student.

Sometimes, educators and parents are concerned that students in a mentor program will have fewer opportunities to interact with their peers and may even be isolated from them by participating in the mentor-paced process. Although they may not be working with their peers in mathematics class, students in a math mentor program may be in the regular classroom for their other subjects. Participation in mathematics clubs and contests (e.g., Mathematical Olympiads for Elementary and Middle Schools; see the Resources) will give students a chance to interact with other students in a mathematical and social context. In addition, we encourage them to participate in both unstructured play and more structured activities such as sports, music, and scouting programs. These varied activities will give them an abundance of opportunities to spend time with their age peers.

However, it is not necessary for students to spend every spare moment interacting with peers. This time should be balanced by

adequate time alone. This time alone, which many talented students prefer, allows them the opportunity to think about what they have learned and experienced. The time can be used to formulate new questions and to investigate them. Thus, what is sometimes perceived as isolation by adults actually has many positive aspects (Lupkowski & Assouline, 1992).

What Should Students Who Participate in the Mentor-Paced Program Do While Their Classmates Are in the Regular Math Class?

This important question should be addressed before a student begins a mentor-paced program. While other students are in the regular math class, the student might work on mentor-assigned homework, read, work on the computer, or go to the library. We recommend that students in a mentor-paced program be included in group activities such as classroom games. However, these students should not be required to sit through the regular mathematics class or complete the regular math assignments if they are already working with a mentor on more advanced mathematics.

For the students in a mentor-paced program, it is important to provide a quiet setting with adult supervision so they can work on mentor-assigned homework while other students are in the regular math class. These special arrangements should be structured so that students do not feel ostracized or punished. Parents, mentors, school personnel, and the students themselves should discuss this and ensure that all parties involved are fully aware of the reasons for the special arrangements (Lupkowski & Assouline, 1992).

Will Removing Talented Students From Their Classes Remove an Important Role Model for Average Students?

In light of the cooperative learning movement that was promoted in the 1990s and remains popular in many schools, this remains a common myth. This concern also is voiced in statements that question whether ability grouping is beneficial to students. Schunk (1987) has stated

that effective role models must be somewhat close in ability to those who would benefit from exposure to the models. Social comparison theory tells us that average students would not look upon exceptionally talented students as academic role models. In other words, average students view exceptionally talented students as too different from themselves.

There are other related issues. For example, large differences in ability within a classroom may promote arrogance on the part of the high-ability student. Conversely, if a teacher teaches a mixed-ability group at a high level and fast pace (appropriate for the extremely mathematically talented student in the classroom), an average student may experience unnecessary pressure from a set-up for failure. Furthermore, having mixed-ability grouping in mathematics with extreme variation in aptitude increases the teacher's management problems. On the other hand, teachers of students who participate in a mentor-paced program can concentrate on tailoring their teaching to the majority of their students and can be satisfied that their highly able students' needs are also being met (Lupkowski & Assouline, 1992).

How Are Student Evaluation Issues Determined?

For work that is completed, students should receive both a grade and credit. There are many different ways in which school personnel can address this issue. For example, a teacher might add special notes to a student's report card to indicate what grade-level mathematics the student studied that term. If the student works with a mentor outside of the school setting, the mentor might supply the student's grade to the regular classroom teacher to record on the report card. Sometimes, mentors give chapter tests to teachers to administer, and the grades on those chapter tests are used to determine the grade for that marking period. These careful records of a student's progress are essential; they are needed when the time comes to make placement decisions, especially if the student transitions from a mentor-paced program into the school-based curriculum (Lupkowski & Assouline, 1992).

How Do Educators Respond to the Mentor-Paced Student's Need for Acceleration?

By definition, mathematically talented students who participate in a mentor-paced program will be accelerated in mathematics. These students are participating in a program that is designed to move them through the curriculum at a challenging pace. Some educators will be very pleased by this acceleration, while others will be extremely defensive. Research over the last 50 years has shown that acceleration for mathematically talented students is an appropriate and useful option (e.g., Colangelo et al., 2004a; Southern & Jones, 1991; see also Chapter 6 for a detailed discussion of the merits of acceleration, along with supporting research).

Before having students begin participating in the DT→PI process, parents and school personnel should discuss the long-term plans for the students' mathematics education (Lupkowski & Assouline, 1992). To illustrate, if exceptionally mathematically talented students participate in a mentor-paced program for a period of time, they may be ready to take high school mathematics courses (e.g., algebra and geometry) several years before they are chronologically old enough to be high school students. If these students are to study mathematics in a regular mathematics class, special transportation arrangements may have to be made so they can move from one building to another during the school day. Scheduling math class will also be an issue. For example, it would be most efficient if math classes were offered at the same time so that accelerated students wouldn't have to miss class in another subject in order to take the right level of mathematics. Also, as students move through the grades, they may complete all of the mathematics courses a school district has available before they graduate from high school. Those students might need to take more advanced math courses through a local college or university or through a distance-learning program. Then, their study of mathematics will not be interrupted.

How Long Does a Mentor-Paced Program Last?

Mentor-paced programs can last for a few hours or for a few years. The DT→PI model is highly individualized, and the length of time in which the student participates in the program depends on his or her needs. We like to think of the mentor-paced program as a bridge that prepares students to take the next level of mathematics in their own schools. One precocious student who needed to use the DT→PI model for a short period of time was preparing to take Algebra II with older students. She spent the last several weeks of the summer meeting with a mentor to ensure that she had mastered the concepts taught in Algebra I, of which she had never taken in a formal course. In other cases, students might need the program for one semester or for a complete school year. An extreme case is that of an exceptionally mathematically talented student who studied high school geometry when he was 10 years old. That young man needed to participate in a mentor-paced program for several years in order to receive the right level of mathematics, while still being able to take other subjects with his age-mates.

Participation in a mentor-paced program is not an all-or-nothing activity. Students might participate in a program for a period of time, reenter the regular mathematics class (at a higher level) for a year or more, and then find that they are no longer being adequately challenged by the level of mathematics they are studying. These students might then return to a mentor-paced program for another period of time.

In order to make the best use of resources, we suggest that students complete a mentor-paced program and reenter a classroom setting (most likely with older students) as soon as is practical. There are many considerations involved in deciding when to reenter the classroom setting. One consideration is the student's level of precocity and another is the level of cooperation from the school system. The more precocious the student and the less cooperative the system, the longer a mentor-paced program needs to last.

For example, we recognize that the instruction prescribed for a third grader included in the DT→PI process will be more prolonged and more individualized than the instruction prescribed for a sixth grader.

We are not advocating having these young students rush into Algebra I. We want to be sure their formal reasoning skills are well developed so that they are indeed ready to study challenging mathematics. Students in this situation have the opportunity to participate in a great deal of challenging, enriching mathematics (see Chapter 7). For third graders and fourth graders in particular, the DT→PI process will probably require a longer term commitment and more resources.

Is There a Cost to the Program?

Yes. The mentors are not expected to volunteer their time. In fact, volunteering is not considered a good idea because neither party feels the same sense of investment if they do not make a financial commitment, as well as an intellectual commitment. If the mentoring occurs outside of school, parents are expected to pay the mentor an hourly fee that would be comparable to the fee parents usually pay tutors in that area. Parents may also need to pay for the cost of books and materials. Ideally, the local school district should pay for the program, especially if it is offered in school and mentors are school district employees. However, parents should not let the political realities, which may not match the ideal, interfere with implementation of the program.

Who Coordinates the Mentor-Paced Program?

Successful mentor program coordinators have been university staff members, gifted education teachers, or other members of the school staff familiar with standardized testing procedures and the school's mathematics curriculum. The person who coordinates the program identifies students who would benefit from the program, assesses their mathematical abilities and achievements via above-level tests, and matches them with appropriate mentors. The coordinator regularly consults with mentors to get updates on the students' progress and to help them secure appropriate materials for their students to use. The coordinator frequently administers posttests and keeps school personnel informed of the students' progress. Many times, the coordinator becomes the students' advocate, especially in the case of

a school district that resists having students participate in a mentor-paced program. In this situation, the coordinator needs to be aware of and present to school personnel the research demonstrating the success of these types of programs. One often-overlooked task of a program coordinator is program evaluation. Evaluation serves many purposes, including improving decision making for future students and adjusting the program to serve current students' needs (Borland, 2003; Callahan, 2009).

In the situation where only one student in a district participates in a mentor-paced program, the mentor takes on the coordinator's role. In this case, the mentor communicates directly with the parents, classroom teacher, and principal concerning the student's progress.

Mentor-paced programs such as the one described here are conducive to Clasen and Hanson's (1987) system of double mentoring. In double mentoring, two mentors attend to the student's needs. The first mentor, the classroom teacher, attends to the student's developmental needs, while the second mentor, the mathematics mentor, attends to the student's needs in mathematics. Thus, the two mentors work in tandem to foster both the student's intellectual and social-emotional development. Double mentoring is enhanced by a program coordinator.

What Makes the Mentor Stage of the DT→PI Process a Success?

Mentor-paced programs are most successful if:

- Mentor, school personnel, parents, and student cooperate.
- The mentor is involved in Steps 2 and 3, Diagnostic Pretest and Re-Administer and Evaluate Missed Items. These activities aid the mentor in developing a clear understanding of what the student is ready to learn.
- The mentor sets goals and outlines a linear progression of topics as part of developing a systematic plan for the student.
- The mentor has a good understanding of the scope and sequence of the K–12 mathematics curriculum.
- The curriculum is challenging and enjoyable (but not necessarily entertaining!).

- There is a coordinator.
- The student is excused from doing the classroom math work assigned to his or her age peers.
- The student receives credit for completing the mentor-assigned work.
- There is a clear plan for the student's future placement in mathematics.
- There is a clear plan for program evaluation (see Borland, 2003; Callahan, 2009; VanTassel-Baska, 2006).

What Types of Accommodations Might Be Established for Students Who Are Talented in Mathematics, But Would Not Benefit From a Mentor-Paced Approach?

We have found that the DT→PI model, as it is described in this chapter, is most effective for exceptionally talented youngsters who demonstrate significant motivation to move ahead in mathematics. For other students who may be mathematically talented, but not extremely so, it may be necessary to develop other accommodations. Various accommodations that might be appropriate for these students are discussed in Chapter 6, including ability grouping and enrichment within the regular classroom.

The Resources section provides an extensive list of materials that parents and teachers might use to enrich the mathematics curriculum. These include books, mathematics contests, computer programs, and other activities. Additional suggestions for enriching the mathematics curriculum for talented students are included in Chapter 7.

How Effective Is the DT→PI Model?

Most of the research with the DT→PI model has been conducted with students in seventh grade taking the SAT and subsequently completing high school mathematics courses. Their stories are highly successful. For example, students scoring 600 or above on SAT-Mathematics completed two high school mathematics courses in just 50 hours of in-class instructional time (Bartkovich & Mezynski, 1981).

Many students have mastered an entire year's worth of material with just 75 hours of instruction (Olszewski-Kubilius, 2004; Olszewski-Kubilius, Kulieke, Willis, & Krasney, 1989). Students participating in fast-paced classes do not suffer ill effects in their long-term retention of the subject matter (Benbow 1992b; Benbow, Perkins, & Stanley, 1983) and perform very well in the next course in the sequence offered in their schools. For example, most students completing Algebra I in a DT→PI program were placed in Algebra II or Geometry and earned A's in that course (Mills, Ablard, & Lynch, 1992). Olszewski-Kubilius (1998) noted that,

> Talent search summer programs have shown that some students can learn at a much faster rate and instruction can proceed at a much faster pace than heretofore believed without sacrificing the level of subject mastery or preparation for future courses, all with higher student satisfaction. (p. 111)

Conclusion

The goal of the DT→PI model is to match the level and pace of mathematics instruction to the academic needs of each student. Because mathematics is presented at an individualized pace in the DT→PI model, each student covers the material at an appropriate rate. Students are not required to study topics they already know well, which gives them more time to study new and challenging topics in greater depth than would be possible in the regular classroom. Students cannot move on to the next topic before attaining mastery, and, in this way, the model is self-correcting. The DT→PI model also avoids the pitfalls of the gaps that may occur if a student skips a grade. The student is consistently challenged by the pace at which the advanced material is presented.

Another benefit of the DT→PI model, in addition to ensuring that the students are consistently challenged in mathematics, is that these students avoid developing bad habits such as not writing out or checking their work when solving problems or doing complicated computations.

They have to develop good mathematical habits of mind because they are working on difficult topics at a rapid pace. Students also learn that testing is useful for reasons other than evaluation; in the DT→PI model, testing is diagnostic and is used to determine where instruction begins.

We would like to emphasize the importance of having parents, school personnel, and the student cooperate in making this process work. All individuals involved must consider the ramifications of participation in the DT→PI procedure. Important issues to consider before embarking on this procedure are earning school credit for advanced work completed, placing the student in advanced courses in the future, and how the mentoring experience will fit into the school's curriculum and grading procedure.

The DT→PI model allows students who are exceptionally able in mathematics to progress at an appropriately accelerated rate through a mathematics curriculum. These few students will move more rapidly through the school's curriculum than is typical. This can be an accelerative model, or what Elkind (1988) called "tailoring." According to Elkind,

> Promotion [in grade placement or subject matter] of intellectually gifted children is simply another way of attempting to match the curriculum to the child's abilities, not to accelerate those abilities. Accordingly, the promotion of intellectually gifted children in no way contradicts the accepted view of the limits of training on development, nor the negative effects of hurrying. Indeed, the positive effects of promoting intellectually gifted children provide additional evidence for the benefits of developmentally appropriate curricula. (p. 2)

The DT→PI approach provides a nice alternative to whole-grade or subject skipping. This is just one option for talented students, their families, and teachers to consider. Many more options for talented youths are discussed in Chapter 6.

CHAPTER 5

Talent Searches

KEY POINTS

- The goal of an academic talent search is to use above-level testing to discover students who are ready for advanced academic challenges that are not typically offered as part of the regular school curriculum.
- Academic talent searches have been in existence for almost 40 years. The first talent search was conducted in one community and served only a few hundred students in grades 7 and 8.
- The talent search model has grown to serve students in grades 2–9, is conducted throughout the United States and several other countries, and has expanded to include several types of above-level tests.
- Talent search participants, although much younger than the students for whom the tests were intended, typically earn higher scores on the tests than the students for whom the test was developed. Thus, initial concerns that students who participated in the talent search would be unnecessarily frustrated are unfounded.
- Advantages of participating in a talent search include educational diagnosis, tailoring recommendations to the abilities of the student, educational opportunities, educational information, and scholarships.
- The talent search model is more comprehensive than above-level testing. University-based talent searches also provide summer and academic-year programs for identified students, online programs, and teacher training, and they conduct research on the effectiveness of the model.

Since 1972, the impact of Julian Stanley's ideas has been tremendous. Literally millions of students' lives have been touched by his work. We can only imagine how the Talent Search model and the programs

the Talent Searches offer will influence the lives of future students. We owe this to the vision of Julian Stanley.

—Lupkowski-Shoplik et al., 2003, p. 214

In the late 1960s, Julian Stanley pioneered the talent search concept by offering a college entrance exam, the Scholastic Aptitude Test (now referred to as the SAT-I) to two bright eighth graders. These two boys had already performed very well in school, and they were searching for ways to be challenged (Stanley, 1996). Stanley used Leta Hollingworth's (1942) concept of above-level testing when he selected the SAT. He knew that bright youngsters who took grade-level standardized tests often answered correctly all or almost all of the items. Instead of developing his own test to measure academic talent, Stanley used a test that was designed for older students. He chose the SAT because it was an excellent test of reasoning ability with sufficiently difficult questions that were not overly dependent upon exposure to a specific curriculum (Stanley, 1991). In addition, the SAT was carefully developed and normed, was secure, and the scores were easily understood and interpreted by school personnel. The two young students performed very well on the SAT, and Stanley realized it had the potential to identify talented youngsters who could reason extremely well.

Stanley and two of his graduate students then administered the SAT to seventh graders in a large testing in January 1972, and the talent search was born. Since that time, under the auspices of university-based talent searches, hundreds of thousands of students have taken the SAT or the ACT, developed by ACT, Inc. These talent searches are now available in every state and in a few countries outside of the United States. Students who participate in these university-based talent searches are then eligible for many educational services, including summer programs, newsletters, parent meetings, and online programs (Assouline & Lupkowski-Shoplik, 1997; Cohn, 1991; Lee, Matthews, & Olszewski-Kubilius, 2008; Lupkowski-Shoplik et al., 2003; Olszewski-Kubilius, 2004; Stanley & Benbow, 1986).

Researchers and program planners realized that the same concept that worked so well for seventh-grade students could be adapted for

younger students. In 1981, Sanford J. Cohn pilot-tested a search for academically talented students younger than seventh grade at Arizona State University. As the first researcher to adapt the talent search process for younger students, Cohn (1991) used the Sequential Tests of Educational Progress (STEP) and the School and College Ability Test (SCAT) for students as young as age 7. In 1985, Cohn brought this concept to the Center for the Advancement of Academically Talented Youth (which later became the Center for Talented Youth [CTY]) at Johns Hopkins University, where its Young Students Program has grown steadily ever since.

Large-scale talent searches for students in sixth grade or younger took off in the 1990s (Lupkowski & Assouline, 1993; Mills & Barnett, 1992). The foundation for our work with extending the talent search concept to students in sixth grade and younger began when we were postdoctoral fellows with Julian Stanley's Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins University in the late 1980s. At SMPY, we frequently received calls from parents of mathematically talented fourth and fifth graders who were looking for more challenges. At the time, there were no large, systematic talent searches for students younger than seventh grade to refer the families to, so we realized there was a need for work in this area. Building on Stanley's talent search concept, we began by testing a few students individually. As we were initiating the Elementary Student Talent Search, CTY at Johns Hopkins University began using the Upper Level of the Secondary School Admission Test (SSAT-U) in a widespread talent search (Mills & Barnett, 1992). CTY worked with the Educational Testing Service (ETS) to develop the PLUS test as an above-level test for fifth and sixth graders in its Young Students Talent Search (Institute for the Academic Advancement of Youth, 1999). CTY now offers programs to elementary-age students on an annual basis.

For our above-level test, we experimented with the Lower Level of the Secondary School Admission Test (SSAT-L), which was designed for students in grades 5–7. We administered this test to third, fourth, and fifth graders, and we found that the test effectively discriminated among mathematically talented students (Lupkowski & Assouline,

1993) and was useful for identifying students for special programs (Lupkowski et al., 1992).

We were pleased with the results of our "talent searching" with third through fifth graders and broadened our use of the SSAT-L in several large talent searches in Texas, Iowa, and Pennsylvania. After using the SSAT-L for a few years, we decided to try another test that had just come on the scene—the EXPLORE, developed over a 3-year period by ACT, Inc. The EXPLORE met the same requirements that the SSAT-L had previously met: It was a carefully developed, standardized, nationally normed test of mathematical and verbal reasoning. The EXPLORE offered additional advantages:

- EXPLORE contained a measure of Science Reasoning, which was not explicitly measured on the SSAT-L.
- ACT was willing to work with us to establish local test sites that would be convenient for our students.
- EXPLORE was less expensive to administer, resulting in lower costs for student participants.

The first Elementary Student Talent Search using the EXPLORE test was conducted by Susan Assouline at The University of Iowa and Ann Lupkowski-Shoplik at Carnegie Mellon University in January 1993. A total of 4,089 Pennsylvania and Iowa students took the EXPLORE at that time (a detailed presentation of the Iowa results is found in Colangelo, Assouline, & Lu, 1994). We have used the EXPLORE each year since then. The Center for Talent Development at Northwestern University and the Talent Identification Program at Duke University began using EXPLORE in 1994, and the Elementary Student Talent Search has continued to grow. Today, approximately 15,000 students per year take EXPLORE as part of a university-based Elementary Student Talent Search (personal communication, J. Fletcher, December 1, 2009). Students in all 50 states in the U.S., as well as in parts of Canada and Australia, have access to above-level testing and subsequent advanced programming in a variety of subjects through one of the university-based talent searches (see http://www.hoagiesgifted.org/talent_search.htm for links to many of these programs). Students may choose from the EXPLORE test or the SCAT test. (See Chapter 3

for more information about SCAT, or contact the Center for Talented Youth at Johns Hopkins University at ctyinfo@jhu.edu). The remainder of this chapter will focus on the EXPLORE test.

How Are Students Selected for the Talent Search?

Students first take a standardized, nationally normed achievement test such as the Iowa Tests of Basic Skills (ITBS), Stanford Achievement Test, or Metropolitan Achievement Test (MAT), usually conducted as part of the school's regular testing program (Colangelo et al., 1994). Criteria vary somewhat from program to program, but the university-based talent searches usually use the 95th or 97th percentile cutoff. Research has shown that the 95th percentile is an appropriate cutoff for talented third through sixth graders taking the EXPLORE test (Lupkowski-Shoplik & Swiatek, 1999). Students scoring at or above this cutoff on the Composite, Math Total, Language Total, Vocabulary, Reading, or Science subtests are recommended for further testing. These students have already demonstrated that they can do very well on tests designed for their age group. In fact, they have reached or are near the ceiling of the test. They have answered all or almost all of the items correctly, and the test does not contain enough difficult items to measure their abilities accurately. Therefore, taking a more difficult test that was designed for older students is appropriate.

Elementary Student Talent Search participants pay fees ranging from $60 to $80 to cover the costs of test administration, mailings, educational information, and newsletters and other program information. Students on the free or reduced-cost lunch program at school may receive a fee waiver. To measure their abilities more accurately, students take an above-level test. In this case, fourth- through sixth-grade students take EXPLORE (some talent searches allow very capable third graders to participate) as an above-level test because it contains more difficult items (and thus has a higher ceiling) and allows students to demonstrate the full range of their abilities. Above-level tests spread out the scores of able students, helping us to

differentiate talented students from exceptionally talented students. This information is useful in educational planning.

EXPLORE was developed by ACT and first used as a test for eighth graders in 1992. It is a multiple-choice instrument that includes the following tests: English, Mathematics, Reading, and Science Reasoning. The tests are designed to measure eighth-grade students' "curriculum-related knowledge and the complex cognitive skills important for future education and careers" (ACT, 2007, p. 5). Each of the four multiple-choice EXPLORE tests takes 30 minutes, and scale scores are reported on a range of 1–25. The Composite is an average of the four tests. Usually, when students take the EXPLORE, they take all four tests.

The EXPLORE–English test contains 40 items and is comprised of two subtests: (1) Usage/Mechanics, which measures a student's understanding of standard written English (e.g., punctuation, grammar); and (2) Rhetorical Skills, which measures a student's understanding of the use of strategy, organization, and style in writing.

EXPLORE–Mathematics is a 30-item test that measures mathematical reasoning, rather than the ability to do involved computations or memorize formulas. The Mathematics test emphasizes the ability to solve practical quantitative problems and covers four areas: knowledge and skills, direct application, understanding concepts, and integrating conceptual understanding. Topics include prealgebra, elementary algebra, geometry, and statistics/probability. Students have calculators available when taking the test.

The 30 items on the EXPLORE–Reading test measure reading comprehension by focusing on skills needed when studying written materials from different subject areas. Students must derive meaning from three prose passages; determine implicit meanings; and draw conclusions, comparisons, and generalizations.

The EXPLORE–Science Reasoning test contains 28 items presenting sets of scientific information in one of three different formats: data representation (graphs, tables, and other schematic forms), research summaries (descriptions of several related experiments), or conflicting viewpoints (descriptions of several related hypotheses inconsistent with

Table 5.1
Average EXPLORE Scores for 2008–2009

Grade	English	Math	Reading	Science	Composite
Third *n* = 921	12.4	11.6	12.6	14.5	12.9
Fourth *n* = 2,609	14.4	13.9	14.2	16	14.7
Fifth *n* = 6,186	16.8	16.2	16.1	17.7	16.8
Sixth *n* = 5,083	18.2	17.3	17.3	18.9	18.2
Talent Search Total *N* = 14,799	16.6	16.1	15.9	17.6	16.7
Eighth (National Normative Group*) *N* = 4,879	14.9	15.7	14.6	16.4	15.5

* The norms were based upon a sample of eighth graders from Spring 2006.

each other). The Science Reasoning test measures how well students understand scientific information and draw conclusions from it.

All four tests of EXPLORE are scored on a scale ranging from 1–25 and were normed on a national sample of eighth graders. Average scores of Elementary Student Talent Search participants from the 2008–2009 academic year, as well as the eighth-grade normative sample, are reported in Table 5.1.

How Well Do Students Perform on the Above-Level Test?

Research findings on the EXPLORE test have been consistent each year. One of the most important findings is illustrated in Figure 5.1. Section A of Figure 5.1 shows the normal curve that results from all students in a particular grade taking a grade-level achievement test.

Scores from students in the top 5% of their age group—students who were at or near the ceiling of the grade-level test—were spread out when they took the EXPLORE as an above-level test (Colangelo et al., 1994; Lupkowski-Shoplik & Swiatek, 1999). This is demonstrated in Section B. Section B shows that most Elementary Student Talent Search participants earned scores in the middle of the EXPLORE range, while a small number earned low scores and a small number of students earned high scores. Through our research, we have concluded that the EXPLORE effectively discriminates among students at high ability levels, thus helping us to make more appropriate educational recommendations for them.

In looking at the EXPLORE test results for Elementary Student Talent Search participants similar to those reported in Table 5.1, Colangelo et al. (1994) and Assouline and Lupkowski-Shoplik (1997) noted the following:

- The scores earned by students at each grade level consistently increased (i.e., fourth graders scored higher than third graders, fifth graders scored higher than fourth graders).
- Not many third graders were encouraged to take EXPLORE because it is so far above grade level. However, those who did participate performed relatively well on this challenging test, which was designed for students with 5 more years of educational experience.
- Fourth graders performed close to the average score for eighth graders.
- Fifth and sixth graders outperformed the average eighth graders on all four tests of EXPLORE.
- Table 5.2 reports frequencies and percentile rankings for third- through sixth-grade students participating in the Elementary Student Talent Search during the 2008–2009 academic year in the Midwest, Southeast, portions of the Western U.S., and Pennsylvania.

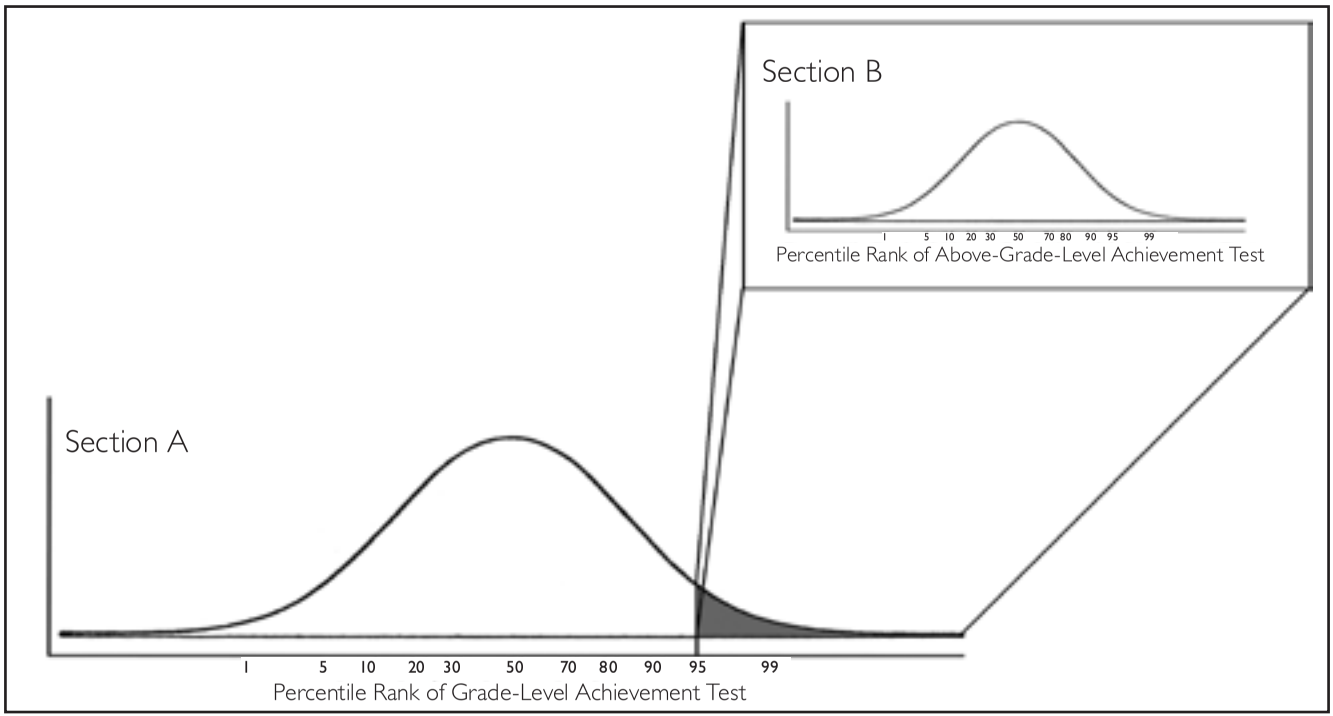

Figure 5.1. Above-level test score distribution of students scoring high on a grade-level test. Adapted from "Talent searches: A model for the discovery and development of academic talent," by S. G. Assouline & A. Lupkowski-Shoplik, in N. Colangelo and G. A. Davis (Eds.), *Handbook of gifted education* (2nd ed., pp. 171), 1997, Boston, MA: Allyn & Bacon. Reproduced with permission of Pearson Education, Inc.

Table 5.2

Frequencies and Percentile Rankings on the EXPLORE Mathematics Test for Elementary Student Talent Search Participants, Grades 3–6, 2008–2009

Score	Grade 3 Freq.	Percentile	Grade 4 Freq.	Percentile	Grade 5 Freq.	Percentile	Grade 6 Freq.	Percentile
25	0	100	3	100	96	99	196	98
24	0	100	6	100	81	98	165	94
23	0	100	0	100	0	97	0	93
22	2	100	6	99	124	96	234	90
21	0	100	20	98	165	94	297	85
20	7	99	31	97	201	91	368	78
19	3	99	40	93	286	87	370	71
18	10	98	111	87	749	78	926	58
17	33	96	195	81	918	64	844	40
16	24	92	138	67	494	52	385	27
15	109	85	536	48	1447	36	772	16
14	98	73	398	36	702	18	233	6
13	71	63	222	24	217	11	72	3
12	127	52	351	15	318	6	55	1
11	82	40	112	10	72	3	16	1
10	80	30	129	6	70	2	11	0
9	68	22	66	4	30	1	5	0
8	52	15	43	3	16	0	2	0
7	0	12	0	2	0	0	0	0
6	40	10	39	1	14	0	1	0
5	36	5	22	0	2	0	1	0
4	18	2	7	0	0	0	1	0
3	7	1	4	0	0	0	0	0
2	0	0	1	0	0	0	0	0
1	3	0	0	0	0	0	0	0
Number	870		2,480		6,002		4,954	
Talent search average (50th percentile) score	11.6		13.9		16.2		17.3	
National norm average scores	14.4		14.4		14.4		14.4	

How Can the Scores Be Used by Parents and School Personnel?

Results of above-level testing are much more useful than grade-level testing for exceptionally talented students. On grade-level tests, students "bump" against the ceiling of the test and do not have an opportunity to show their abilities. To illustrate this, consider the example of two students: Hector and Leslie. Both students took the California Achievement Test (CAT) in fourth grade. They both performed exceptionally well on the Mathematics sections of the test (see Table 5.3). Their CAT grade-level scores were identical, which might cause educators to believe that they would need similar curricular adjustments in mathematics. When they took the EXPLORE test, however, Leslie's EXPLORE–Math scale score was 24 (out of a possible 25, placing her at the 98th percentile when compared to the eighth-grade norm group), while Hector scored 12 (placing him at the 23rd percentile when compared to the eighth-grade norm group).

Although these two students appeared to have identical profiles on the grade-level standardized test (the CAT), they clearly have very different abilities and needs in mathematics, as demonstrated by their scores on the EXPLORE. Unfortunately, the two students are currently placed in the regular classroom for mathematics. Although they are in the gifted program at their respective schools, the programs are enrichment-based; therefore, they do not receive any differentiation for mathematics. Both students need additional challenges in mathematics, including participating in contests and competitions, being grouped with other talented students for mathematics instruction, and perhaps compacting a course sequence by taking 2 years of mathematics in 1 year. Leslie's needs in mathematics are even more pronounced. She would benefit from all of the options previously suggested, plus individually paced instruction during the school year or summer, as well as course skipping or grade skipping, if all academic areas are advanced.

Table 5.3
Grade-Level Test Scores and Above-Level Test Scores of Two Students

Test	Hector	Leslie
Grade-level test taken in fourth grade: California Achievement Test, Mathematics Computation	99th percentile	99th percentile
California Achievement Test, Mathematics Concepts and Applications	99th percentile	99th percentile
California Achievement Test, Math Total	99th percentile	99th percentile
Above-level test taken in fifth grade: EXPLORE-Mathematics (percentile compared to eighth graders)	Scale score = 12 (23rd percentile)	Scale score = 24 (98th percentile)

College Readiness Standards for the EXPLORE Mathematics Test

Parents and school personnel can use information developed by ACT to help them interpret students' EXPLORE scores and make appropriate recommendations for them. ACT has developed the College Readiness Standards (ACT, 2009), which are statements describing what students who score in various score ranges are likely to know and to be able to do. Complete College Readiness Standards are listed on ACT's website (http://www.act.org). The College Readiness Standards report information allows parents and teachers to compare the performance of their students with the performance of students representing the entire nation. The standards are cumulative, meaning that students scoring in a higher range are likely to demonstrate knowledge and skills in the lower score ranges.

The College Readiness Standards are provided for each of the four academic areas measured in EXPLORE—English, mathematics, reading, and science reasoning—and are provided for four score ranges (13–15, 16–19, 20–23, and 24–25) along the EXPLORE score scale. Students who score between 1 and 12 are most likely

beginning to develop the skills and knowledge described in the 13–15 score range. Information included below is detailed in *Connecting the College Readiness Standards to the Classroom for Mathematics Teachers* (ACT, 2009). In addition to providing detailed information about the standards students have met, ACT (2009) also provides "Ideas for Progress," which are useful for curriculum planning. Much more detail about the College Readiness Standards can be found at http://www.act.org/standard/guides/explore/pdf/EXPLORE_Math.pdf and http://www.act.org/standard.

Score Range 13–15

Students can perform one-operation computation with whole numbers and decimals. They can identify the location of a point with a positive coordinate on the number line. They can recognize equivalent fractions and fractions in lowest terms and perform a single computation using information from a table or chart.

Score Range 16–19

Students can solve some routine two-step arithmetic problems, and they can calculate the average of a list of numbers. Students can read tables and graphs and perform computations on data from tables and graphs. In addition, they can identify a digit's place value and recognize one-digit factors of a number. Students scoring in this range can substitute whole numbers for unknown quantities to evaluate expressions, and they exhibit some knowledge of the angles associated with parallel lines. They are able to compute the perimeter of polygons when all side lengths are given.

Score Range 20–23

Students can solve routine two- or three-step arithmetic problems that involve concepts such as rate and proportion, percentage off, and computing with a given average. They can determine the probability of a certain event, and they can calculate the missing data value, given

the average and all data values but one. They demonstrate knowledge of number concepts that include rounding, the ordering of decimals, absolute value, primes, and greatest common factor. They can add and subtract simple algebraic expressions, locate points in the coordinate plane, and find the measure of an angle using properties of parallel lines. They can compute the area and perimeter of triangles and rectangles in simple problems.

Score Range 24–25

Students can solve multistep arithmetic problems that involve planning or converting units of measure. They can manipulate data from tables or graphs, order fractions, work with scientific notation, work with squares and square roots, and identify solutions to simple quadratic equations. Students can compute the area of triangles and rectangles when one or more additional simple steps are required. They can compute the area and circumference of circles after identifying necessary information.

Programmatic Guidelines for Students Who Have Taken the Above-Level EXPLORE Test

After determining what students know and are able to do in mathematics, the next step is to devise an appropriate program for them. We used the 2008–2009 EXPLORE data collected on more than 14,000 students who participated in the Elementary Student Talent Search to help us relate scores on the EXPLORE assessment to various program options. The program options we considered were based on almost 20 years of experience in creating programs for the range of talents demonstrated by the gifted students who take EXPLORE.

First, we divided EXPLORE scores into three ranges: Range A, Range B, and Range C (listed in Table 5.4). We deliberately made the ranges overlap (e.g., for fourth graders, Mathematics Range A contains scores from 1 to 9, while Mathematics Range B contains scores from 9 to 12). This is done to emphasize that the skills associated with these

Table 5.4
EXPLORE–Mathematics Scores Ranges A, B, and C

Range	Grade 3	Grade 4	Grade 5	Grade 6
A	1–9	1–12	1–14	1–16
B	9–12	12–15	14–17	16–19
C	12–25	15–25	17–25	19–25

Note. Each of the ranges in a grade overlap to emphasize that the skills associated with these scores are on a continuum.

scores are on a continuum. The score guidelines in Table 5.4 were not designed to be strict cutoffs. Rather, they were developed to give parents and educators an idea of how to interpret EXPLORE scores and help students to be challenged appropriately in school.

In Range A, each of the cells begins with 1 (e.g., 1–9), and they constitute the bottom range of the EXPLORE scale. Very few students earned a score lower than 7.

Although Range B, the middle range, is rather narrow (only four points on the EXPLORE scale), this band represents the largest group of students taking the EXPLORE test. The students in this group have performed in approximately the top half of their age group when compared to other gifted students in the Elementary Student Talent Search.

Range C represents the highest scoring group of students in the Elementary Student Talent Search. Students in this group have scored well above other gifted students in their age group (approximately the top quarter of their age group when compared to other gifted students taking EXPLORE). This is the most talented group of students with the greatest need for a differentiated mathematics program. The reader will note that, for Range C, the third-grade scores appear to cover quite a broad range; however, most of the scores clustered around 12–18, with very few students earning scores above 18.

It is very important to note that students earning scores within Range C may still have different needs. For example, in the 2008–2009 sample, there were 870 third graders tested. Of those, 2 students earned a score of 22, which means they have very different needs in

mathematics compared to the 127 third graders who earned a score of 12, even though we have placed all of them in Range C. Similarly, a third grader who earns a score of 25 may have very different needs than a fifth or sixth grader who earns that score.

Figure 5.2 illustrates the Pyramid of Educational Options for Elementary Student Talent Search participants (Assouline & Lupkowski-Shoplik, 1997). We related the options depicted on the pyramid to score ranges A, B, and C. Again, we would like to emphasize that the scores we recommend for certain options are meant to be guidelines. These are not hard and fast cutoff scores. The options on the pyramid are in ascending order, with enrichment-based options at the bottom and accelerative options at the top. All students participating in the Elementary Student Talent Search would benefit from the options listed at the bottom of the pyramid, while the more accelerative options would be recommended for those students earning higher scores on the EXPLORE test.

Acceleration and enrichment as programming options are thoroughly discussed in Chapter 6. Our goal in developing Ranges A, B, and C and relating scores to the options listed on the Pyramid of Educational Options was to give educators and families guidance in selecting appropriate options for their talented students. It is important to keep in mind that the score is not the only piece of information to use when selecting program options for a student. In addition to consideration of the locally available resources, one must also bear in mind the age and maturity level of the student, as well as the student's grade and individual learning style.

So, for Hector and Leslie, the two students we mentioned earlier, we can use the pyramid for guidance in selecting programmatic options. Hector's scaled score of 12 places him in Range A. He would benefit from all of the options listed at the base of the pyramid. Leslie's score places her in Range C, and she would benefit from all of the options listed in the pyramid. She clearly needs acceleration in mathematics beyond the regular curriculum.

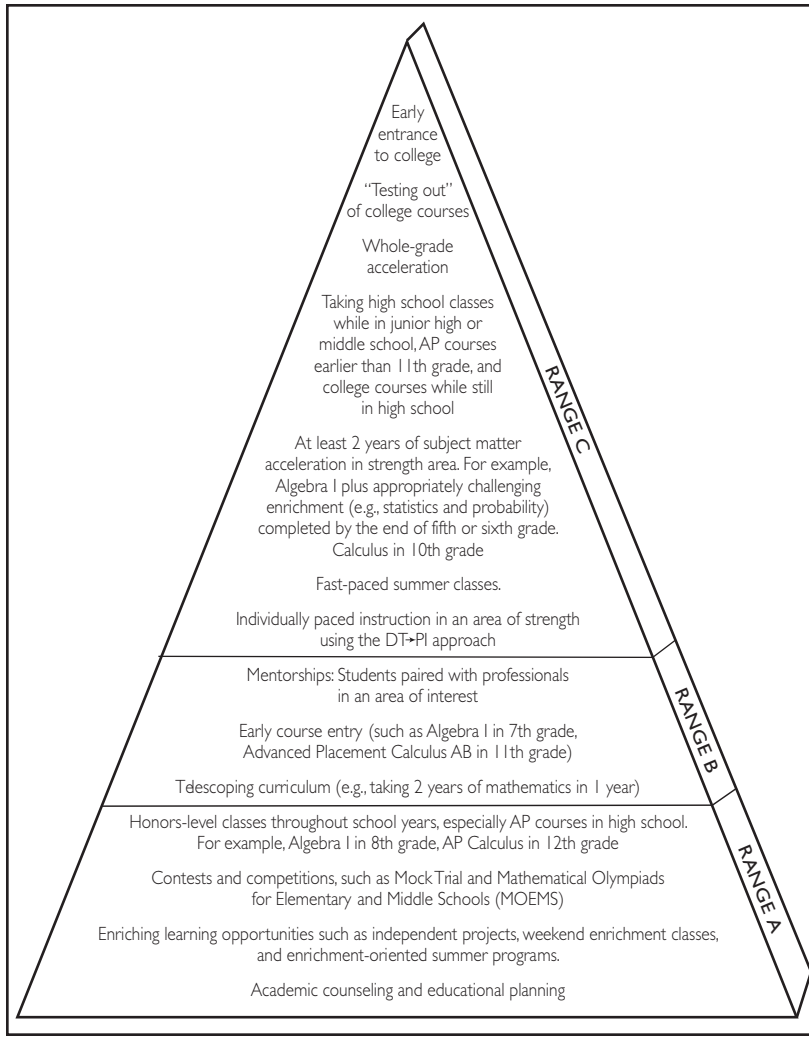

Figure 5.2. Pyramid of educational options. Students who participate in above-level testing while in grade 3 and who earn a Mathematics score of 10 or lower should consider retaking the EXPLORE test in grades 4, 5, or both. Because students experience so much cognitive growth during those 2 years, recommendations regarding advanced coursework from testing subsequent to the testing in grade 3 are more valid. Adapted from "Talent Searches: A Model for the Discovery and Development of Academic Talent," by S. G. Assouline and A. Lupkowski-Shoplik, in N. Colangelo and G. A. Davis (Eds.), *Handbook of gifted education* (2nd ed., pp. 170–179), 1997, Boston, MA: Allyn & Bacon. Reproduced with permission of Pearson Education, Inc.

What Happens After Students Participate in a Talent Search?

After participating in a talent search, students are given much educational information. They receive their test results and an interpretation of their test scores. Talent search personnel also offer them advice about appropriate educational options. Talent search participants become eligible for special programs, including summer classes, by-mail tutorials, and weekend workshops. Finally, the elementary student talent searches provide resource booklets and newsletters containing lists of educational information, including program opportunities, books to read, and resources for parents and teachers (Lupkowski-Shoplik et al., 2003).

Summer and Weekend Programs Provided by the Talent Searches

Summer programs offer students the opportunity to study advanced topics in depth with intellectual peers. The regional talent searches, including those offered at Carnegie Mellon University, Duke University, The University of Denver, The University of Iowa, Johns Hopkins University, and Northwestern University offer summer programs for talent search participants. These summer programs may be residential or nonresidential and may last from one to several weeks. Some programs are accelerative in nature, while others offer excellent academic enrichment courses. For example, Challenges for Elementary School Students (ChESS) is provided for participants in the Belin-Blank Exceptional Student Talent Search, offered by The University of Iowa. These classes are offered at various sites in Iowa. Classes for elementary students in this nonresidential program are typically 3 hours per day and last 1–2 weeks. Most of the classes are connected with the major subjects (English, math, and science), but may include computers and foreign languages. Another program, Carnegie Mellon Institute for Talented Elementary and Secondary Students (C-MITES) offers summer programs at locations throughout

Pennsylvania. These summer classes are similar to the ChESS classes, and courses include Robotics, Mathematical Problem Solving, Math Olympiads, Mathematics Through the Ages, Computer Math Mania, Probability and Statistics, and Informal Geometry. Numerous other summer programs for talented students are available. Although families must pay tuition for most of these programs, need-based financial aid is available.

Many of the regional talent searches offer students courses on weekends during the school year. These courses, frequently offered on college campuses, give students a chance to interact with instructors who are experts in their field and to join other like-minded students for a brief academic experience. For example, the C-MITES weekend workshops and the Weekend INstitute for Gifted Students (WINGS) program offered by the Belin-Blank Center offer 3-hour classes in robotics, lasers, bugs, architecture, and chemistry. They also offer workshops for parents. More information about the university-based talent search programs can be found in Lee et al. (2008).

Advantages of Participating in a Talent Search

Obviously, we are advocates of the talent search approach to identifying mathematically talented students. The benefits of the approach, as described by Rotigel and Lupkowski-Shoplik (1999) and Olszewski-Kubilius (2007), include the following.

Educational Diagnosis

Above-level tests, such as EXPLORE, measure talented students' abilities more accurately than grade-level achievement tests do. They spread out the scores of academically talented students, resulting in a bell-shaped distribution (Lupkowski-Shoplik & Swiatek, 1999). Hector and Leslie's test results, described earlier, illustrate this point clearly. Only after we have an accurate measure of students' abilities are we able to offer specific recommendations for their educational programs.

Educational Recommendations Tailored to the Abilities of the Student

Researchers who have used the SAT with hundreds of thousands of seventh graders have developed guidelines for educational recommendations for students scoring at particular levels (Cohn, 1991). These recommendations represent a continuum of modifications that best match a child's demonstrated ability and achievement, ranging from enrichment options to accelerative options (Cohn, 1991; Olszewski-Kubilius, 1998). The goal is to match the level and pace of the curriculum to the child's needs by finding the "optimal match" (Robinson & Robinson, 1982). After testing thousands of third through sixth graders, we developed similar guidelines for the students taking the EXPLORE as depicted previously in Table 5.4 and Figure 5.2.

Educational Opportunities Provided by University-Based Talent Searches

University-based talent searches offer summer programs, weekend programs, correspondence courses, and online programs (Lee et al., 2008; Olszewski-Kubilius, 1998). These enrichment and accelerative programs allow students to study topics that might not be offered at their local schools (Lupkowski-Shoplik & Kuhnel, 1995; Piskurich & Lupkowski-Shoplik, 1998). Residential summer programs provide the opportunity for students to make gains in emotional and social development, as well as to take advanced classes (VanTassel-Baska, 1996). Through these programs, students meet and study with other talented youngsters. When students participate in these programs, it may be the first time they are truly being challenged in class and are truly engaged in learning. Exceptional students may require intensive instruction beyond what their schools are able to provide; courses provided by university-based talent searches help fill that void (Olszewski-Kubilius, 2007; K. B. Rogers, 2007).

A True Peer Group

Participating in educational programs offered by university-based talent searches gives students the chance to interact with intellectual peers who are similar in age. Because most academically talented students are placed in heterogeneous classrooms most of the time in their local schools, they have few opportunities to be around others like themselves. Placement in these special classes provides general social support for the students, as well as a greater degree of intellectual stimulation and challenge. Powerful academic effects and moderate affective effects are found when gifted students are grouped with like-ability peers (K. B. Rogers, 2007).

Appropriate Educational Information

By participating in a talent search, students obtain a more accurate measure of their academic abilities and, therefore, learn more about their abilities. When they participate in specially designed programs to explore topics of interest, they are better informed about their own achievements and abilities when the time comes to select a college or a career (Brody, 1998). Talent searches provide printed material and extensive websites on educational opportunities and encourage students to maximize their abilities. Students enter college better prepared and ready for even more challenges (Brody, 1998). The invitation to participate in a talent search also does much to enhance the individual student's academic self-concept (Swiatek & Lupkowski-Shoplik, 2000a). This is especially true for students who qualify for the talent searches, but have not qualified for their local school's gifted and talented program.

Honors, Awards, and Scholarships

Students may earn a variety of awards due to their excellent performance on a talent search measure (Cohn, 1991). Demonstration of advanced reasoning may result in special opportunities at some of

the university-based programs. For example, students may receive merit scholarships to attend university courses.

Opportunities for Educators

Several talent search centers also provide specialized training for the educators who provide accelerative and enrichment programming for talent search participants. An equally important consideration is the contribution that this training has for these educators who often transfer their new skills to the K–12 school setting.

The Success of the Talent Search Model

> The evidence overwhelmingly supports the use of talent search to identify highly able reasoners who are eager to develop their abilities. Higher scoring participants tend to earn more honors and more advanced standing, enter college earlier, and complete college with greater success than those who score less well. . . . An important fact to remember is that the accomplishments of thousands of these academically able young people are dramatically large. They are accomplishments unheard of prior to the start of the talent searches. In research studies documenting the value of the SAT as a predictor of academic success the findings are not merely statistically significant. The programs are profound in their capacity to enable highly able and eager students to proceed educationally as fast and in as great a depth and breadth as they wish. (Cohn, 1991, pp. 169–170)

Due in large part to the success of the talent searches, there has been an exponential increase in the opportunities available to gifted students, especially with regard to opportunities for residential academic programs on college campuses. The foundation of the talent search model, above-level testing, is a simple, yet powerful, method

of identifying talented students. Since the first talent search with seventh and eighth graders in 1972, Julian Stanley and his colleagues at numerous universities have identified millions of students who have benefited from the early identification and special programming options the talent searches offer (Lee et al., 2008; Lupkowski-Shoplik et al., 2003).

The DT→PI model, advocated by the talent searches, allows identified students to move ahead in their areas of strength at an appropriate level and pace. It effectively challenges academically talented students and has demonstrated long-term positive outcomes (Stanley, 1996; Swiatek & Benbow, 1991b).

> The talent search concept has deeply affected our ideas about appropriate programming for gifted and talented learners. Educational interventions need to be talent specific and responsive to individual needs. . . . The true beauty of aptitude testing, such as that done through the talent search, is that it provides administrators with important diagnostic information for curriculum planning. (VanTassel-Baska, 1996, p. 239)

As described by VanTassel-Baska (1996), the talent searches, with their strong research base and practical approach to educating talented youth, highlight the following concepts:

- Programs and curricula should be matched to the abilities of a student.
- The more academically talented the student, the greater the need for intensification and extension of services.
- Acceleration is a necessary aspect of gifted programming.
- Accelerated, advanced, and challenging work on an ongoing basis is a powerful incentive to achievement at later levels (e.g., Swiatek & Benbow, 1991a).
- The talent search concept has had an important impact on curriculum and instruction. For example, we have seen more interest in classical subjects such as Latin, Greek, philosophy, and rhetoric and greater interest in rigorous treatment of standard courses.

- Higher education institutions have become involved in educating academically talented precollege students. Many colleges and universities offer courses for precollege students, have developed an ongoing research agenda concerning talented youth, and have developed graduate programs to provide for the education of teachers and other professionals in gifted education (Olszewski-Kubilius & Lee, 2005).
- The staff at talent searches model a mentoring approach. They often work individually with students and their families to help the students realize their talents by helping them select appropriate educational options.
- Talent searches have provided a research and developmental base for content and other forms of acceleration (Brody & Mills, 2005).

Research Findings on the Elementary Student Talent Search

Although the talent searches for elementary school students are relatively new when compared to the talent searches for middle school students, they have begun to establish a sound research base. The research that has already been conducted answers a variety of questions, from "Who should be tested?" to "How well do the students do?" Some of the research questions are addressed below.

How Do We Know Which Level of a Test Is Appropriate for Young Academically Able Students?

An important consideration with any test is whether or not the items are appropriately difficult. The test must meet the "Goldilocks Standard"—not too hard, not too easy, but just right! A talent search instrument must have a sufficient number of difficult items to be challenging for the extremely able students, but also must not be unnecessarily frustrating for the less-exceptional students. For academically talented students, above-level tests are generally more

appropriate than grade-level tests, and EXPLORE is a highly effective talent search instrument for third through sixth graders (Colangelo et al., 1994; Lupkowski-Shoplik & Swiatek, 1999). EXPLORE has been used in the Elementary Student Talent Search with more than 150,000 students. A review of the distribution of the scores earned by bright elementary students reveals a bell curve. In other words, EXPLORE has enough easy items and enough difficult items, to allow us to distinguish among an already bright group of students (ACT, 2007).

Additionally, EXPLORE has enough difficult items for the older, very bright students (i.e., talent search sixth graders). A study completed by ACT confirmed that the EXPLORE–Mathematics test was an appropriately challenging above-level instrument for bright elementary students, neither too easy nor too difficult. Of note, however, is the fact that, on many items, talent search fifth and sixth graders performed as well as, if not better than, the normative sample of eighth graders (ACT, 1996; Colangelo et al., 1994).

Do Students Who Take EXPLORE Come From an Appropriately Selected Sample?

Students are invited to participate in the elementary student talent searches based upon scores they earned on grade-level tests. We recommend they earn a score at the 95th percentile or above on at least one section of a grade-level standardized test before taking EXPLORE. For students meeting this initial criterion, EXPLORE has been found to be appropriately challenging, without being a frustrating experience (Assouline, Colangelo, McNabb, Lupkowski-Shoplik, & Sayler, 1993; Lupkowski-Shoplik & Swiatek, 1999). Specifically, Lupkowski-Shoplik and Swiatek (1999) found that less than 10% of the third-grade Elementary Student Talent Search participants meeting the initial 95th percentile equivalent earned very low scores ("chance" scores) on the EXPLORE–Mathematics test. For fourth through sixth graders, less than 2% of the students earned those very low scores. Therefore, we concluded that the 95th percentile criterion is appropriate.

We do tend to be cautious, however, when we are helping families of third graders make the decision of whether or not to take the EXPLORE test. We remind them that EXPLORE was designed for eighth graders and may be quite difficult even for a bright third grader who has had little standardized testing experience. Although less than 10% of the third graders taking the test earn very low scores, we are still concerned that the third graders have a much higher potential for being frustrated by taking the EXPLORE. Therefore, we suggest to families that they think carefully about their child. Children who don't handle frustration well or tend to become very upset if they don't know all of the answers are not good candidates for taking EXPLORE in third grade; we recommend that they wait until fourth grade to take the test.

Are There Differences in the Performances of Boys and Girls?

Researchers with the SMPY at Johns Hopkins University have reported consistent gender differences in mathematics in gifted junior high school students: Boys score significantly higher than girls on the SAT–Mathematics, and there are more boys than girls who score at the highest levels of the test (Benbow, 1988; Benbow & Stanley, 1980). Researchers with the Elementary Student Talent Search are interested in investigating the question of gender differences in the population of younger mathematically talented students. Because the talent search with elementary students is still relatively young, researchers have just begun to investigate gender differences in this population.

Gender differences favoring boys in the elementary student talent searches have been found, but the magnitude of these differences has not been consistent. Colangelo et al. (1994) and Assouline and Doellinger (2001) found that boys scored significantly higher on EXPLORE–Mathematics than girls did. These differences were statistically significant, although effect sizes were in the negligible range, indicating that the differences were not of practical significance. Swiatek, Lupkowski-Shoplik, and O'Donoghue (2000) found the same type of results in their sample: Boys scored an average of 13.22 on

EXPLORE–Mathematics, while girls scored 12.62. This difference, although statistically significant, was not meaningful.[1]

In looking at the highest scoring students (those earning a score of 22 or higher on EXPLORE–Mathematics), however, the researchers found twice as many boys as girls earned such high scores. Therefore, they concluded that important gender differences are evident in mathematically gifted students, even as early as elementary school. This research coincides with our anecdotal evidence that many more boys than girls are exceptionally mathematically talented. The gender differences found in large research samples help us to realize how important it is to provide special programming for mathematically talented girls in elementary school.

Are There Gender Differences in Attitude About Mathematics?

In their research sample of Elementary Student Talent Search participants, Swiatek et al. (2000) found a pattern similar to that found in older gifted students: Boys demonstrated more positive attitudes than girls toward science and computers, while girls showed more positive attitudes than boys toward English, writing, and reading. Findings reported by Colangelo et al. (1994) were similar, although there was no significant difference found between boys and girls in attitude toward science. We need to note, too, that the difference between boys and girls in liking mathematics was "negligible" in the Swiatek et al. study, although Colangelo et al. found that a significantly greater number of boys than girls indicated that they loved mathematics. Statistical analyses in the Colangelo et al. study revealed that more boys than girls responded "I love it" to the question, "How do you feel about mathematics?"

These two studies of gifted elementary students may indicate a trend that becomes more apparent as students grow older. Taken together, these observations are consistent with the suggestion of Benbow (1988)

[1] The effect size was calculated to be .18. Effect size is a statistical indication of the importance of the magnitude of the difference between two means (averages). Cohen (1988) arbitrarily classifies effect sizes below .20 as "negligible."

and Benbow and Stanley (1982) that the important issue may not be that academically talented girls like math and science less than boys do, but that they like verbal subjects more than boys do.

What Is Known About How the Talent Search Students Perform in the Areas Measured by EXPLORE–Mathematics?

Assouline and Doellinger (2001) conducted a study examining the individual items of the EXPLORE–Mathematics test. This test covers four areas of math: prealgebra, algebra, geometry, and statistics/probability. They found that the talent search sixth graders exceeded the performance of the normative sample of eighth graders on 23 of the 30 items, while fifth graders outperformed the normative sample on 8 of the 30 items. In general, their performance was very strong in prealgebra, geometry, and statistics/probability. The "weakest" area of performance by the talent search students was in algebra (which most of the students had not taken, of course). This means that academically able students in sixth grade and younger perform as well as, or better than, average eighth graders on a test of mathematical achievement. The evidence is clear that these students need something beyond that which the typical sixth-grade curriculum provides: They are indeed ready for more challenging material, such as that offered in Prealgebra and Algebra I courses.

Further analysis of the data revealed that talented elementary students performed well in the content areas of statistics and probability, geometry, and prealgebra. Such a strong performance suggests that, in general, these content areas should be introduced earlier than they are, and the treatment of these topics should be at a higher level of challenge for these talented youngsters.

What Can Above-Level Testing Tell Us About the Curriculum?

The developers of the EXPLORE test have been able to tie score ranges for each test to specific skills and objectives. Therefore, used in combination with the College Readiness Standards described earlier in this chapter, the results from EXPLORE provide an appropriate

starting point when determining the programs and curricula for talented students. Chapter 7 presents a thorough discussion about curricula and materials for mathematically talented students.

CHAPTER 6

Programming

KEY POINTS

- There are many educational options for mathematically talented students, ranging from enrichment within the regular classroom to radical acceleration. No one option is "right" for all students.
- The "optimal match" means obtaining objective information about the student's abilities and achievements and then matching those abilities and achievements to the appropriate level and pace of the mathematics curriculum.
- Acceleration is appropriate and necessary for many mathematically talented students. Acceleration as a successful programming option has been strongly researched for several decades.
- Several exemplary programs offer illustrations of the variety of approaches for programs for talented youth.

If I tell you what to do on Monday, who will tell you what to do on Tuesday?

In the previous chapters, we have discussed both philosophical and practical issues concerning assessment and planning. In Chapter 5, we presented evidence concerning the range of mathematical talent, with an emphasis on the importance of using objective information from testing as a way to better understand the student's need for additional challenge. This chapter focuses on specific programs that can be used to meet the academic needs of mathematically talented students.

Mathematically talented students have many different educational options from which to choose. Rather than giving educators and parents

a list of clever activities to do with students in a random fashion, we present a smorgasbord of choices available to students and suggest how to match these choices to the students' needs in a systematic manner. Our goal is to help educators determine not only what to do on Monday morning, but also on every day of the week throughout a student's elementary, middle, and high school years. For the students, our goal is consistent exposure to new learning.

Finding the Optimal Match

On a daily basis, we try to help parents who want to know how to challenge their children in school. Inquiries usually sound like "My child is gifted, but she doesn't seem to be challenged by the gifted program. What can I do to challenge her?" or "My son is very good in math, but he hasn't been identified as gifted, and he's bored with the regular math curriculum. What can we do?" The first step in helping these families and the school personnel who work with their children is to obtain objective information about the student's abilities and achievements. The next step is to match the ability and achievements of the student to the appropriate level and pace of the elementary and middle school mathematics curriculum. Robinson and Robinson (1982) called this finding the "optimal match" (see Figure 6.1). An optimal match between the curriculum and the student's abilities occurs when a student feels sufficiently challenged and engaged by the material. The student is "stretched," but not overly frustrated. Both the pace and the level of instruction need to be matched to the student's abilities and prior achievements. In this chapter, we discus how this is accomplished.

Step 1: Assess academic abilities and achievements.

Before selecting a program or curriculum for a student, we first need to obtain specific information about his or her academic abilities and achievements. A variety of measures can be used for this purpose. Some of these measures—such as teacher-developed tests, student products, and grade-level standardized tests from the school's regular testing program—are already available and widely used. However, these

Step 1: Assess academic abilities and achievements.

Step 2: Choose the general grade level of curriculum for further assessment of specific knowledge.

Step 3: Offer curriculum-based assessment using chapter tests and pretests.

Step 4: Place the student in the appropriate level of the curriculum for instruction.

Step 5: Reassess—Is the student being challenged but not overly frustrated?

Figure 6.1. Finding the optimal match.

measures may not give a complete picture of the student's level of ability or achievement because they are not as reliable or valid as standardized measures (see Chapter 3).

Other measures also may be administered. Above-level testing (such as EXPLORE testing for third through sixth graders) usually provides a more accurate indicator of talented students' abilities than grade-level standardized tests because the former raise the ceiling typically imposed by the latter. In other words, the information from above-level tests helps us determine the level of challenge that is appropriate for gifted students. For example, Joshua, a fifth grader, took the EXPLORE test, which was developed for eighth graders. He scored 13 out of a possible 25 on the Mathematics section of the test, placing him at the 41st percentile compared to eighth graders in the norm group. This indicates that he performed as well as or better than 41% of eighth graders taking the test that year. This performance has specific implications regarding his need for challenge throughout his academic years, indicating that he needs to be placed in a math class that is at a more challenging level and is taught at a more rapid pace than the typical fifth-grade math class. He will continue to need advanced programming throughout his middle school and high school years.

Steps 2 and 3: Choose the general grade level of curriculum for further assessment of specific knowledge. Then, offer curriculum-based assessment using chapter tests and pretests.

It is not enough to test a student and then say that he or she is exceptionally talented in math, science, or reading. As explained in Chapter 3, testing is a necessary (but not sufficient) first step to prescribing an appropriate educational program developed to match the curriculum with the student's abilities. After determining the student's mathematical aptitude (often by using above-level testing), school personnel can choose the general grade level of curriculum at which to begin further assessment of a student's specific knowledge. This is called a *curriculum-based assessment*. Curriculum-based assessment tests are developed from the actual curriculum used in the school or district and allow for the most valid programming by a school for a student. For example, chapter tests and pretests (from the appropriate grade level) can be used to measure mastery of specific content and topics. This step in the process is important because it helps determine what the student already knows and what the student has yet to learn.

Joshua's EXPLORE test results were obtained during the middle of his fifth-grade year. After his results came back (and because his mathematics score was at the 41st percentile when compared to eighth graders), it was appropriate to consider advanced programming in mathematics. His teacher administered the end-of-book test for fifth-grade mathematics. He correctly answered 97% of the items, indicating that he had mastered a good deal of the fifth-grade mathematics curriculum. He was also tested using the sixth-grade mathematics chapter tests for the first 6 months of the school year (Chapters 1–8). Joshua's scores ranged from 92 to 100 on each of the tests.

Step 4: Place the student in the appropriate level of the curriculum for instruction.

When studying the curriculum, the student might be placed with age peers or with older students to study mathematics. The student might study mathematics in a classroom setting or individually with a teacher or mentor. More information about these various options is offered later in this chapter.

Joshua had performed so well on the sixth-grade chapter tests that he was placed in the sixth-grade mathematics class in the middle of his fifth-grade year. His teacher worked with him closely to ensure that

there were no gaps. He completed the sixth-grade curriculum with an A average. Joshua also enjoyed participating in enrichment mathematics courses offered on weekends at a local university.

Step 5: Reassess—Is the student being challenged, but not overly frustrated?

When matching the curriculum to the student's abilities, it is important to discern his or her optimal learning rate and precise academic strengths. Students should feel that they are making steady progress in mastering a subject, not moving too quickly, and not doing unnecessary review work. The goal is to match student ability with academic challenges (Center for Talented Youth, 1993; Noble & Childers, 2008). Good programs for gifted students (and for all students) have one primary goal: matching the abilities of the students to appropriately challenging coursework and programs. If the material is too difficult, students will become frustrated and will not learn; if it is too easy, they will become bored and "turned off."

In Joshua's case, the teacher asked him periodically if he understood the material and if he was having any difficulty. Joshua reported he was enjoying the challenges presented by the sixth-grade class. The following year, as a sixth grader, Joshua was placed with seventh graders in prealgebra. Again, he performed very well and responded positively to the challenge.

Enrichment and Acceleration

Enrichment

The debate about acceleration versus enrichment is decades old and very complex. In brief, enrichment proponents are concerned that accelerating students means that they will learn information in a rapid, superficial manner, while advocates of acceleration are frustrated by the repetitive tasks students are asked to do even if they have already demonstrated mastery of a topic. George, Cohn, and Stanley (1979) presented an excellent treatment of the issues.

Stanley (1979b) described four forms of enrichment and several issues associated with enrichment, all of which remain relevant today. Despite the fact that Stanley's descriptions were generated in the 1970s, there have been no new developments. The four forms of enrichment are:

- *Busywork*: More of the same, but in greater quantity than required of most students in the class.
- *Irrelevant academic enrichment*: A special subject or activity meant to challenge the academic experiences of the student; however, the subject has very little to do with the talent area (e.g., the mathematically talented student spends enrichment time working on a class newspaper).
- *Cultural enrichment*: Although not relevant to the specific talent area, the activity has cultural merit (e.g., attending a theatrical or musical performance).
- *Relevant academic enrichment:* There is exposure to special topics in the specific talent area, such as a unit on tessellations, which may or may not be related to the topics being covered in the class.

Chapter 7 presents many resources for enrichment topics that can be used effectively by the regular classroom teacher and others. The remainder of this chapter presents programming ideas for systematically exposing students to the mathematics curriculum and moving them through it at a pace commensurate with their aptitude.

Acceleration

When thinking about acceleration, we often focus on whole grade-skipping. However, there are many different types of acceleration, and each one is appropriate for different situations. Benbow (1986) and others (Rogers, 2004; Southern & Jones, 2004) listed many ways for students to accelerate their educational program. The options range from entering kindergarten early or skipping a grade, to participating in an individually paced program (see Table 6.1).

Table 6.1
Examples of Accelerative Options

- Enter kindergarten early
- Skip a grade
- Participate in fast-paced classes
- Condense 4 years of school into 3
- Receive credit by examination
- Participate in subject-matter acceleration
- Participate in individually paced programs

When encountering a decision to accelerate a student, educators are often very concerned about the social implications of such a move. Will placing a student with older students for math class have a negative effect on the young student, either socially or academically? To respond to this question, it is important to look at the many research studies investigating the short- and long-term effects of acceleration in mathematics. Extensive reviews of the literature have demonstrated that the evidence for acceleration is generally positive for students in prekindergarten through graduate school (e.g., Benbow & Stanley, 1996; Daurio, 1979; Janos & Robinson, 1985; Kulik & Kulik, 1984, 1987; Passow, 1996; Reis et al., 1993; Sosniak, 1997; Southern & Jones, 1991). Brody and Benbow (1987) reviewed data from a follow-up study of SMPY students and found no harmful effects of acceleration and a number of positive effects. Students who were more accelerated in school earned more awards, attended more selective colleges, and were more likely to plan to earn doctorates or other advanced degrees than the less-accelerated students. They found no differences in the accelerated and nonaccelerated groups in terms of social and emotional adjustment. Kulik and Kulik's (1984) meta-analytic review of 26 studies on the effects of acceleration on gifted students revealed that gifted students' achievement was significantly higher in accelerated classes than nonaccelerated classes. Brody, Lupkowski, and Stanley (1988) found that college students who had previously been accelerated were very happy overall that they had done so. They were pleased to be

placed in a more challenging environment, and they reported that any inconveniences they suffered (e.g., not being able to drive at the same time as their older classmates) were temporary and relatively minor.

"Literally hundreds of studies have shown that advancing talented students in their studies so that they are working with curricula designed for older students (i.e., acceleration) is effective in enhancing achievement" (Lubinski & Benbow, 2006, p. 334). For those concerned with the long-term effects of acceleration, a number of follow-up studies have been conducted. For example, 10 and 20 years after mathematically talented 13-year-olds participated in fast-paced mathematics classes, their achievements were compared to equally able students. One finding was striking: The students who had participated in accelerated math classes in junior high school were *twice* as likely to be in math-science career tracks years later. Lubinski and Benbow (2006) concluded "[B]eing challenged by intellectually rigorous math-science educational opportunities that are responsive to one's learning needs increases the likelihood of being in a STEM career 20 years later" (pp. 334–335). In summarizing findings of his meta-analysis, Kulik (2004) repeats the point: "The overall message from these studies is therefore unequivocal: Acceleration contributes greatly to the academic achievement of bright students" (p. 15).

Much of the recent research on acceleration has been summarized in the two volumes of *A Nation Deceived* (Colangelo et al., 2004a, 2004b). Parents and educators advocating for mathematically talented students will find these volumes gather together the current research on acceleration in a very readable, user-friendly format. Another important resource for parents and educators is the Institute for Research and Policy on Acceleration (IRPA) at The University of Iowa. IRPA was founded in 2006 in an effort to help educators, parents, students, administrators, and policymakers learn more about why and how academic acceleration works for meeting the needs of highly able students. See the website for more information (http://www.accelerationinstitute.org).

The major issue is that acceleration as an educational strategy for challenging gifted children is clearly supported by research, but it has received only minimal support from educators in the field. Starting in 1972, the Study of Mathematically Precocious Youth (SMPY) at Johns

Hopkins University began offering fast-paced mathematics classes to academically talented students in grades 7 and 8, with the goal of allowing students to move through mathematics at a pace matched to their abilities. Thus, many accelerated math programs were initiated by university-based programs for precollege students. The outcomes of these programs have been impressive. For example, junior high students who successfully completed SMPY's first series of fast-paced precalculus classes achieved much more in high school and college than matched-ability students who elected not to participate. They were also much more accelerated in their education than nonparticipants, and they reported that they were satisfied with their acceleration (Stanley & Benbow, 1986).

Examples of accelerative math programs. Mills, Ablard, and Gustin (1994) described an accelerative mathematics program for third through sixth graders that used a curriculum developed specifically for exceptionally mathematically talented students. In this program, developed by the Center for Talented Youth (CTY) at Johns Hopkins University, students attended 3-hour classes on weekends during the school year and worked on required homework an additional 2–3 hours per week. Students worked individually, in small groups, and as a whole class. They worked through the mathematics curriculum in a systematic, linear fashion by studying a topic thoroughly, demonstrating mastery, and then moving on. The pace of the class was matched to the students' abilities. Students were placed in the class based on ability, and there were no restrictions on age or grade. For example, a class could have been composed of third, fourth, fifth, and sixth graders all completing the same course within 7 months.

Careful study of students' progress revealed that participants in this program showed remarkable gains. Their progress was evaluated by giving them above-level assessments (see Chapters 3 and 5). Their scores were compared to those of older students in the comparison group for that test. At each grade level, students improved their performance significantly. Third graders showed the greatest increase (moving from the 35th to the 81st percentile when compared to the sixth-grade norm group). In each grade level, at least 87% of the students exceeded

normative gains. In other words, almost 9 out of 10 students achieved at higher levels than would have been expected.

When considering participation in a fast-paced, accelerative program, many people ask if students can retain the knowledge learned for any length of time. To address this important issue, the CTY students took a posttest 5 months after completion of the course, and scores remained relatively the same (Mills et al., 1994).

Another accelerative math program is the University of Minnesota Talented Youth Mathematics Program (UMTYMP), which has been provided for students in grades 6–12 for more than 30 years. Highly accelerated, in-depth mathematics courses are offered for exceptionally talented youth. Eligible students are recommended by their schools and take the UMTYMP Algebra Qualifying Examination. Nearly 900 students test for this program each spring.

Classes are offered in Minneapolis/St. Paul and in Rochester, MN. Students begin by studying Algebra I and may continue through college calculus, differential equations, and linear algebra. The UMTYMP courses are equivalent to one full year of high school credit. Successful students may also receive credit from the University of Minnesota for the calculus course. Nearly 400 students were enrolled in 26 different classes during the 2008–2009 school year; these classes included both high school and college level courses. Students take a 2-hour class one or two days per week and are assigned 8 to 10 hours of homework each week. In one 2-hour session held after school, students cover the equivalent of 3 weeks of regular high school mathematics content. For more information, visit http://www.itcep.umn.edu/umtymp/index.php. The student handbook is available at http://www.itcep.umn.edu/umtymp/WebHandbook09.pdf.

The Academically Talented Youth Program (ATYP) represents another example of content acceleration and fast-paced instruction designed for academically talented students. McCarthy (1998) noted the nearly ideal application of the talent search model, upon which ATYP was established, to communities in Michigan. Mathematically talented middle school students in the Kalamazoo, MI, area were provided with 180 hours of instruction scheduled over 2 school years. This permitted the completion of 4 years of high school mathematics

(i.e., 2 years of algebra, plane/solid geometry, and precalculus prior to 10th grade). The point of ATYP was not to rush the students through the mathematics curriculum, but rather to have a process that "matches students to the curriculum and instructional pace appropriate to their need" (McCarthy, 1998, p. 122).

McCarthy (1998) described in detail the identification, programming, and evaluation of ATYP, which has been in existence since 1982. Best of all is the fact that the model has been replicated in other communities in Michigan. Although each community has added its local flavor to the model, the integrity of the talent search assessment process and the match of the assessment to the program are integral components of the program's success. Since its inception, this program has provided accelerated classes for more than 3,000 students from 60 Michigan schools.

Comments concerning acceleration. In our experience, many school officials are categorically opposed to students accelerating in school (Assouline et al., 2009; Colangelo et al., 2004a; Southern & Jones, 1991). This opposition is often rooted in district policy or seems to stem from either one bad experience with acceleration or a major concern with students' social development. School officials and parents are concerned that, if a student is placed with others who are one or more years older, the accelerated student will be at some type of disadvantage, develop poor social skills, be "behind" the other students, or stand out compared to other students.

In response to these concerns, Assouline and colleagues (2009) developed the *Iowa Acceleration Scale*. This scale was designed to help educators and parents systematically consider all aspects of a student's development while discussing the decision to accelerate or not. It includes a discussion of the issues relevant to acceleration, and it guides educators systematically through questions about academic ability, school factors, family considerations, motivation, attitude toward learning, academic self-concept, developmental factors (age, physical size, and motor coordination), emotional development, relationships with peers, attitude toward acceleration, and parental support.

Accelerated students may have fewer opportunities to interact with same-age peers. However, we do not feel that this is a sufficient

reason to hold students back. Students can have many opportunities to interact with age peers through informal play with neighborhood children, participation in church groups, extracurricular activities at school, and so forth.

Lupkowski and Assouline (1992) listed five important points concerning acceleration:

- Healthy social development means learning to get along with people of all ages and skills.
- There are many opportunities outside of school to interact with age peers.
- Having a large number of same-age friends isn't as critical to a child's healthy social development as having a few close friends with whom to share ideas.
- An indicator of social readiness for acceleration is a preference for playing with more mature children.
- Finally, long- and short-term planning is essential. Students, parents, and teachers need to remember that there may be a few years during adolescence when minor inconveniences become major ones. (p. 28)

Lupkowski and Assouline (1992) also noted:

> Acceleration accomplishes much that is useful. When implemented properly, it guarantees that an appropriate match between the talented student and curriculum has been found. Acceleration provides the opportunity for challenges that children might not otherwise experience.
>
> If a student is not permitted to accelerate, he or she may be locked into the traditional pace and sequence of study and denied the opportunity to study a more advanced subject later. For example, a student who waits until ninth grade to study algebra with the rest of his or her classmates may not be able to schedule more advanced mathematics courses such as calculus or differential equations, or advanced science courses requiring high-level mathematics. Although one might counter that these subjects can be studied in college, we disagree with that

> argument. . . . [W]aiting to study a subject eliminates the chance to take new, more challenging courses. The mathematically talented student who craves higher level mathematics and related subjects would be wise to pursue those interests. (p. 30)

Linking Acceleration and Enrichment

In the face of so much evidence regarding the needs of academically gifted students, why has there been so little progress in educational programming for these students? What has changed in the decades since George et al. (1979) published their book?

We believe that much of the lack of educational progress is rooted in the acceleration versus enrichment debate. Politically, there is considerable concern about acceleration, and enrichment, no matter how irrelevant, seems to be the approach that is most widely preferred and used. One notable treatment of the acceleration versus enrichment issue merits mention. Schiever and Maker (2003) carefully explored the various forms of enrichment and acceleration and concluded, as did Stanley (1979b), that a combination of acceleration and enrichment makes the most sense. By definition, relevant enrichment in mathematics must have some acceleration, and appropriate acceleration will have an enrichment component.

What Are the Program Options for Mathematically Talented Students?

We have already made a strong case for allowing exceptionally talented students to accelerate in math, and we have described several exemplary programs that utilize acceleration. There are many more educational options for mathematically talented youth. We recognize that students have a range of abilities. Our focus is to try to help them match their academic strengths to the appropriate curricular option. Because not all students are equally able in all academic areas, they need different programs and different approaches for various subjects.

We recommend looking at each student individually and fitting the various programs the school offers to their needs.

We view the needs of mathematically talented students as being on a continuum. Students whose talents are somewhat above average need differentiation within the regular classroom, while students who are quite a bit above average need subject matter acceleration or curriculum compacting. Extraordinarily talented students need radical acceleration and individually paced programs. The Pyramid of Educational Options described in Chapter 5 shows the many educational choices available to mathematically talented students and relates those options to scores earned on the EXPLORE test.

Appropriate Options Within the Regular Classroom

Bright students who love math and do well in class might be able to receive the challenges they need within a regular classroom. Perhaps they are the first ones done with their seatwork, and their teachers are always looking for ways to keep them challenged. Possible options within the regular classroom are listed in Table 6.2 and are described below.

Breadth/depth approach. The goal of this approach is to refocus the objectives of an assignment from quantity to quality—in other words, to avoid giving students "more of the same." For example, if the class is working on computation skills and the assignment is to complete the even-numbered problems on a page, the gifted student is assigned all of the problems. On the other hand, an example of breadth/depth (i.e., quality) is to assign the gifted students the challenge of completing computations in bases other than base 10 (Pratscher, Jones, & Lamb, 1982).

The breadth/depth model is relatively simple for the regular classroom teacher to deliver because it does not require the development of a totally separate program for the gifted. Instead, the teacher matches in-depth activities with each level of the existing curriculum (Lupkowski & Assouline, 1992). However, there are two potential pitfalls of this option.

Table 6.2
Instructional Options Within the Regular Classroom

Appropriate Options

- Breadth/depth approach: The same curriculum, but greater depth.
- Enrichment topics: Extend or enrich the regular curriculum.
- Math-related independent study projects: Investigate a math topic.
- Curriculum compacting: Eliminate some curriculum to allow more time for other activities.
- Telescoping: Spend less time in a course of study (complete 3 years of high school in 2 years).
- Subject-matter acceleration in mathematics: Move up a grade for mathematics.
- Ability grouping: Groups of advanced students study math together. Students might be grouped within the regular classroom, or an entire group of math talented students might study an honors-level math course together.

Less-Appropriate Options

- Tutor other children: Help others who are having difficulty in math. This is not a good substitute for learning new material.
- Isolated, self-paced instruction: Student works ahead in the textbook at his or her own pace. This may result in feelings of isolation; student probably won't learn the material well.

First, the student might see the additional activities as punishment for being talented in math. For example, if Sonja is required to do the same set of practice problems as the rest of the students and is also expected to complete another activity or problem set, she will probably resent the additional work. Sonja is more likely to remain engaged in the assignment if the teacher allows her to demonstrate mastery of the skill by permitting her to do fewer practice problems (i.e., one half of the amount assigned to the rest of the class). As soon as mastery is demonstrated, she should move on to the curriculum-related extension assignment.

The second pitfall is potentially more serious. It is highly probable that the teacher is not prepared to provide substantial horizontal or vertical enrichment that presents a broader range of mathematics concepts to these students or presents challenges to them that are typically reserved for older students. For example, elementary school teachers are not typically required to have a substantial background in mathematics in order to receive a teaching certificate. This is why it is so important to continue to upgrade teacher preparation programs and to offer additional training to experienced teachers. In the report, *Doing What Matters Most: Investing in Quality Teaching* (which was prepared by Linda Darling-Hammond [1997], executive director of the National Commission on Teaching and America's Future), the following point is made:

> No other intervention can make the difference that a knowledgeable, skillful teacher can make in the learning process. At the same time, nothing can fully compensate for weak teaching that, despite good intentions, can result from a teacher's lack of opportunity to acquire the knowledge and skill needed to help students master the curriculum. (p. 8)

Enrichment topics in the regular classroom. The variety and availability of enrichment activities has increased substantially in the past 20 years. The delivery of these activities is often through centers or stations that are set up by the teachers, but selected by the students. For example, most of the students in Mrs. King's fifth-grade regular classroom are expected to complete centers on metrics, estimation, and multiplication. In addition, there are centers on tessellations and the Fibonacci sequence available to students who have the time, interest, and motivation to work on them. Examples of appropriate enrichment topics for these centers include probability and statistics, mental arithmetic, spatial visualization, algebra, geometry, and discrete mathematics (Wheatley, 1988). In Chapter 7, we will discuss additional enrichment topics, as well as games, competitions, computer programs, and other activities for bright students in the regular classroom.

The disadvantages of this approach are the same as those mentioned previously: The enrichment topics might be seen as punishment for completing the other activities early, and the teacher may not be sufficiently comfortable with the activities. The resolution of these disadvantages is complex. At a minimum, to avoid the perception that the enrichment centers are a punishment, the students could be allowed to substitute a higher level, more challenging activity for one they have already mastered (Winebrenner, 2001).

Math-related independent study projects. The underlying assumption of this option is that gifted students will be given the same assignment as their classmates, but will be able to complete the assignment more quickly than the others. Therefore, they will have time to investigate a mathematical topic on their own with the teacher's guidance and perhaps the help of a community mentor.

Examples of independent study projects range from applied to theoretical. An example of an applied project is conducting a survey and reporting the results as part of an effort to resolve a community issue. An example of a theoretical project would be working with a university-level mathematician on an area of interest such as fractals. Working on math-related independent study projects is recommended as a supplement to the regular curriculum, but it is not meant to be a substitute for curriculum compacting and proper pacing (see the next section).

Curriculum compacting. The goal of curriculum compacting is to determine which parts of the curriculum can be eliminated from a student's course of study and to use the time saved for other activities. A student's mastery of the subject is determined by pretesting. Curriculum compacting is designed to give the student additional time to study enrichment topics in the same subject, complete independent study projects, move more quickly through the curriculum, or pursue other worthwhile academic activities (see Winebrenner, 2001, for an extensive treatment of compacting).

Curriculum compacting is best accomplished by pretesting appropriately. We use the word *appropriate* because many educators use the pretests that are part of the textbook, but do not use the results to program for the student. For these teachers, the pretest is

another activity, and the results have little impact on programming. Few teachers have training in the application of pretesting results. Because pretesting is the preliminary activity in compacting, teachers are often derailed from successfully implementing compacting at this stage (Chapter 4 offers more information about appropriately applying pretesting). This would be in contrast to the teacher who uses the time saved to have the student participate in an activity not related to mathematics, such as watching a movie.

Once the teacher determines what a student knows and doesn't know, the next step to compacting is documenting content still to be mastered and choosing appropriate replacement activities for the topics that have already been mastered. These may be enrichment-oriented activities that are related to the topics already mastered, accelerative activities that are more advanced, or unrelated activities. In a sequential subject such as mathematics, it may be difficult for regular classroom teachers to select appropriate enrichment activities that are not accelerative in nature. Also, teachers may not feel confident in choosing appropriate accelerative activities. Therefore, unfortunately, the replacement activities chosen are often unrelated to mathematics. They may keep the student occupied, but they do not move the student systematically through the curriculum or broaden the student's understanding of a topic. Chapter 7 offers some suggestions for appropriate activities.

Telescoping. In telescoping, students spend less time than normal in a course of study. For example, they may complete 3 years of high school in 2 years, or they may complete a 1-year mathematics course in a semester (Southern & Jones, 1991). Michigan's ATYP program, described earlier in this chapter, is a good example of telescoping. According to McCarthy (1998), the typical student in the ATYP program learns 2 years of high school math content in one-fourth the instructional time usually allotted in high school classrooms.

Subject-matter acceleration. Moving up a grade just for the mathematics class can be a good option for students who are exceptionally talented in only one content area, and this is a relatively easy way for schools to make accommodations for mathematically talented students. The major advantage to this approach is that the

regular classroom teacher no longer has to search for additional enrichment materials because the talented student has been removed from the class during math period. The student is more likely to be grouped with intellectual peers, because he or she is cognitively ready for the more advanced material. Another advantage is that students will be given credit for work completed, which is not always possible when acceleration occurs within the regular classroom. One disadvantage of this acceleration is that the pace of the class a grade higher might still be too slow for these quick learners, and there may be very little content that is substantially new (Flanders, 1987; Plucker & McIntire, 1996). Table 6.3 presents the advantages and disadvantages of subject-matter acceleration in mathematics.

Ability grouping. When students of similar ability are grouped together, it is known as homogeneous grouping. This can occur when an entire classroom is composed of students of similar mathematical abilities or when a classroom of students with a wide range of abilities is divided into several groups based on ability. When implemented in the regular classroom, this grouping arrangement requires careful planning by the teacher. It can be an excellent way to meet an individual student's needs because the pace of the curriculum is matched to the pace of a small group of learners, rather than the whole class. Thus, talented students are given challenging activities, but are not forced to wait for everybody else to catch up.

Unfortunately for mathematically gifted students, homogeneous grouping became an unpopular option at about the same time that mixed-ability grouping became popular and was "sold" to administrators as a way to solve their scheduling problems induced by the inclusion and middle school movements. Also contributing to the wholesale application of mixed-ability grouping was the inaccurate association of the term *grouping* with tracking. The research by Oakes (1985), which described the deleterious effects of tracking on lower ability students, was interpreted by well-meaning educators as a signal to dispense with any type of grouping. The effects of the convergence of the anti-tracking movement with the inclusion, middle school, and back-to-basic-text movements contributed negatively to the education of mathematically talented students.

Table 6.3
Advantages and Disadvantages of Subject-Matter Acceleration in Mathematics

Advantages

- Regular classroom teacher does not have to search for materials for the advanced student, because that student is removed during math class.
- Student is more likely to be grouped with intellectual peers.
- Student is given credit for work completed.
- Student is appropriately challenged and therefore remains interested in mathematics.
- Academic success and social development have been empirically demonstrated to be overwhelmingly positive.

Disadvantages

- The pace may still be too slow for the student.
- If only one year of acceleration is employed, there may be relatively little new content for the student.
- The student might "run out" of mathematics before graduating from high school if long-term planning is not implemented.
- Student may not receive credit for high school math done before enrolling in high school.

Gifted educators have been steadily advocating for a return to ability grouping, especially for mathematically talented students (Rogers, 2002). In their advocacy efforts, gifted educators cite research demonstrating that ability grouping is one of the preferred options for academically talented students. For example, Kulik and Kulik (1997) reported that, when academically talented students were grouped for accelerated and enriched classes, they demonstrated academic gains. Grouping middle and lower aptitude learners, however, has little or no effect on their achievement test scores. Therefore, grouping students does not have the harmful effects that some fear, and it has large positive effects for academically talented students (Kulik & Kulik, 1997). When gifted students are grouped for enrichment or accelerative classes, they can interact with other students who have similar interests and abilities,

they are given assignments at an appropriate level of difficulty, and they are allowed to work at a pace matched to their abilities.

With homogeneous groups, teachers can tailor instruction to a group of students, which is more effective and easier for the teacher than trying to teach a large group of diverse learners at once.

> [W]hen gifted students are in class with other students of similar ability, curriculum can be designed that goes well beyond the regular content in both depth and breadth. The syllabus for average learners can be compacted and the topics elaborated, but most importantly, the thinking can be on a higher plane with topics unified and synthesized. No other grouping model offers such potential. (Wheatley, 1988, p. 253)

Ability grouping may occur every day or just a few times a week. One district we work with groups math talented students by ability only twice a month, when they have "Math Challenge Day." The students do fun enrichment activities on those days. In this example, students are grouped by ability, but their math progression is not systematic nor is it accelerative.

Other schools might have more systematic pull-out programs in math. For example, students are removed from the regular classroom twice a week during the math period so they can be grouped with similarly talented students for instruction. Students might study challenging topics that may or may not be related to the regular math program. For example, a talented fifth grader takes the regular fifth-grade mathematics class but is pulled out weekly to work with talented peers on unrelated math topics. This student is responsible for all assignments in the regular classroom and must make up any work or tests that are missed. This type of pull-out program serves three purposes:

- it recognizes students' talents,
- it groups students with other talented peers, and
- it gives students more challenging mathematics to study.

A major drawback to this program is that instruction in the regular classroom still isn't differentiated. Because students are required to complete the regular assignments and tests, it can lead to frustration and a feeling of being "punished" with extra work for being "smart" in math.

Less-Appropriate Options for Mathematically Talented Students

Tutor other children. It seems like a good use of resources to ask gifted students to tutor others in the class who have difficulty in math. After all, we all know that you learn something very well if you have to teach it to someone else. So, why isn't this a good idea?

First, this would be asking students to do the teacher's job, for which a teacher has been trained for a number of years. Second, these students already know the material with which their classmates are struggling; they don't need additional time to master it. For example, they do not need more practice with two-digit addition because they mastered that concept long ago. Simply put, it is not a good use of talented students' time to tutor others instead of learning something new. Therefore, this option is not recommended as a means of challenging mathematically talented students.

Isolated, self-paced instruction. Having a student work ahead in the textbook at his or her own pace while other students receive the regular mathematics instruction from the teacher is one of the easiest ways to fill a student's time, and it is a frequently chosen option. The student is permitted to approach the teacher with questions, but spends most of the time learning the material on his or her own. Although the student may study mathematics at a faster pace, he or she might experience feelings of isolation and probably will not learn the new material well or to any great depth. This option is also not recommended.

Options Outside of the Regular Classroom

Although many mathematically talented students may be well served by their regular classroom teachers, they may also benefit from

Table 6.4
Instructional Options Outside of the Regular Classroom

- Mathematics competitions and clubs
- Summer programs for gifted students
- Weekend programs for gifted students
- Individually paced programs: The DT→PI model
- Magnet schools
- Distance-learning courses
- Work with a community mentor or teacher on a math-related project

participating in options outside of the regular classroom. These options include competitions, summer programs, and weekend programs, all of which have the advantage of grouping talented students together, as well as individually paced programs, which have the advantage of tailoring the curriculum to each student's abilities and achievements (see Table 6.4).

Mathematics competitions and clubs. Competitions offer students the opportunity to practice higher level math skills than they might use in the regular classroom. They also give students the chance to interact with other mathematically talented students and adults who enjoy mathematics. Some competitions do not require students to travel away from their home schools in order to participate. This is an option that is relatively easy for even a small school to implement because students can participate on their own time or they can use their enrichment time within the regular classroom to prepare for the competition. Preparation for competitions often takes place in school mathematics clubs. Some competitions may require a faculty sponsor, so a student might not be able to participate on his or her own. Through these competitions, students meet others with similar interests and abilities (Dauber, 1988). Usiskin (2000) has advocated for the role of mathematics clubs and school teams in promoting a culture that makes it acceptable for students to express and develop their interest in mathematics. Recommended competitions include Mathematical

Olympiads for Elementary and Middle Schools (MOEMS; http://www.moems.org), MATHCOUNTS (http://www.mathcounts.org), the American Mathematics Competitions (AMC 8 and AMC 10; http://www.unl.edu/amc), and the American Regions Mathematics League (http://www.arml.com). More information about these competitions can be found in the Resources. The goal of these competitions is to provide fun and creative problems to students to challenge them mathematically.

Summer programs for gifted students. Residential and nonresidential summer programs offer an excellent option for students who need additional enrichment, acceleration, or both in mathematics over and above what their school provides. Summer classes for academically talented students in elementary, middle, and high school are offered by many universities, including Carnegie Mellon University, Johns Hopkins University, Northwestern University, and The University of Iowa. The Resources section contains more information about summer programs for mathematically talented students, as well as several books listing additional opportunities for talented students. Several exemplary summer programs are described below.

The C-MITES Summer Program, offered by Carnegie Mellon Institute for Talented Elementary Students (C-MITES), is a 2-week commuter program offered at more than 30 sites throughout Pennsylvania. Students are chosen on the basis of above-level test scores (such as EXPLORE), teacher recommendations, grades in school, and academic activities. During the program, students study one topic in depth. Courses include Probability and Statistics, Informal Geometry, Amusement Park Physics, Explorations in Science, Programming Using Alice, and Robotics. Instructors are experts in the subject they teach, and most are certified teachers.

At The University of Iowa, the Belin-Blank Center for Gifted Education and Talent Development offers commuter programs for elementary students and residential programs for secondary students. The purpose of the Belin-Blank Center programs is similar to that of the other talent search centers: to bring together bright students with similar interests so that they can enjoy intensely focused study in the company of their academic peers. The Belin-Blank Center courses are

open to any talent search participant and include math problem solving, computer applications, science investigations, literature and writing, and language and culture. The summer program faculty members are all experts in their area of instruction. Some members of the faculty are certified teachers, while others are university professors or clinicians.

The Center for Talented Youth at Johns Hopkins University also provides rigorous summer programs for gifted students. Most of the courses are available through the residential program, offered at several sites in the U.S. and abroad (although many students have taken advantages of the distance-learning program as well). Commuter options are available for younger students, and students in fifth grade and older may participate in the residential programs. Courses are offered in mathematics, science, and humanities. Students study one topic for 3 weeks, and they may be eligible for school credit upon successful completion of the course.

The advantages of participating in a summer program designed specifically for gifted students include:

- Participants study a topic in depth for an extended period of time.
- Academically talented students are grouped together, giving them the opportunity to work together at a faster pace and in greater depth than they would in a typical class in school. They also enjoy interacting with other like-minded students.
- In an accelerative summer class, students may complete one or more years of mathematics and receive credit for one or more school courses. Therefore, they may be placed in a higher level, more challenging mathematics class when they return to their regular school in the fall.
- Research studies have demonstrated the effectiveness of participating in fast-paced, accelerative summer programs. For example, students completing a summer mathematics course at Johns Hopkins University took more advanced courses in high school and took more Advanced Placement exams at an earlier age than students who had scored similarly on above-level tests, but did not participate in summer programs (Barnett & Durden, 1993). Students participating in fast-paced

mathematics classes were also more likely to attend a selective undergraduate institution and to enter college earlier (Swiatek & Benbow, 1991b). Olszewski-Kubilius (2007) provided an excellent summary of the advantages of these programs.

A drawback of these programs is they do not necessarily have a direct impact on the mathematics program in school. For a student who studies advanced topics in a summer program, for example, there is no guarantee that the mathematics in his or her regular school program will be appropriately modified. However, parents and students still view these opportunities as extremely valuable, because they help the student to encounter a challenging educational experience (perhaps for the first time), enjoy studying rigorous mathematics, and meet peers and adults who also love math.

Weekend programs for gifted students. Weekend programs may be offered by local schools or universities, and they can include a series of sessions offered on successive Saturdays or 1-day courses on a Saturday or Sunday. Many are enrichment courses, but some universities offer courses that provide high school or college credit for younger students.

Students who do not live near a center offering programs designed for mathematically talented students can check with local museums, science centers, and universities concerning enrichment classes they offer. Mathematically talented students might take courses designed for older students in order to be challenged. The director or coordinator of the program can provide guidance in selecting challenging courses. Listed below are several examples of weekend programs designed specifically for academically talented youth.

C-MITES offers Weekend Workshops throughout the school year on the Carnegie Mellon University campus in Pittsburgh and Ardmore, PA. These 3-hour experiences are offered to academically talented students in kindergarten through ninth grade. Weekend Workshops include Algebra Aces, Discover Division, Ice Cream Science, Mini Math Olympiad, Robot Race Cars, and Write Your Own Web Page. Instructors are selected for the program on the basis of their expertise in the subject and their experience in working with gifted youth.

Students may attend one or more of the 3-hour workshops a semester. Scholarships are available based on financial need. More information about these workshops can be found at http://www.cmu.edu/cmites.

The Saturday Enrichment Program, offered by the Center for Talent Development at Northwestern University, provides courses for students in preschool through ninth grade. These courses are available on the Evanston, IL, campus to students who meet the following criteria: (a) a score at the 95th percentile or above on a nationally normed, standardized achievement test in the area that matches the class for which the student is applying, and (b) a recommendation from a current teacher or gifted program coordinator. Students may attend one course for 8 weeks. Courses are offered in science, math, literature, drama, and other areas. Courses include Math Is a Blast, Problem Solving, Discovering Geometric Concepts, and Real-World Math Problems.

The Weekend Institute for Gifted Students (WINGS), offered through The University of Iowa's Belin-Blank Center, offers Saturday enrichment classes for students in grades 3–8. The classes are 3 hours in length. The range of course topics is wide, as indicated by the titles, which include HyperStudio; Fossil-Mania; It's Force, of Course; Dentistry and Forensic Science; Animal Medicine; Math Olympics; Brain Teasers and Head Scratchers; and Making Plays.

The Belin-Blank Center also offers a series of Saturday courses through the Challenge Saturday Program. Coursework in science, math, or writing is provided over a series of five consecutive Saturdays. The curriculum mirrors the programming that is offered during the summer program, and thus students study in one specific area for an intensive time period (3 hours) throughout the semester.

Individually paced programs. The DT→PI model is useful for students who are extremely talented in mathematics and who cannot typically receive adequate challenges in the regular classroom. Using this model helps us to know if a student would benefit from program options that are more accelerative in nature or more enrichment-oriented. Briefly, the DT→PI model includes five steps: aptitude testing, achievement pretesting, re-administering and evaluating missed items, prescribing instruction, and posttesting. The DT→PI model is discussed

in depth in Chapter 4. It is an excellent option for students who can't be challenged by the regular school curriculum taught at the usual pace.

Magnet schools. Many large communities have magnet schools, which bring gifted students together to learn as a group. These schools are able to offer special advanced coursework in specific content for their students. Bringing gifted students together in this way has several advantages: It is more efficient in terms of teacher time and district expense to have students grouped together in one building, and students benefit both academically and socially from being grouped with other talented peers. Grouping gifted students with their peers for instruction makes it easier for teachers to tailor instruction (Rogers, 2002).

One example of a secondary-level magnet school is Iowa's Des Moines Central Academy, created by the Des Moines Public School District in 1985. During its first year, 52 academically talented eighth graders from Des Moines Public School District's middle schools attended the magnet secondary school for part of their school day. The purpose of this "experimental" school was to prepare students for Advanced Placement (AP) courses and tests by the end of their sophomore year. As of 1985, not a single student in Des Moines had taken an AP test (although coursework had been offered). Three years later, in 1988, the first sophomore class took a total of 41 exams, and 85% of the class scored 3 or better on the tests. The participation of Des Moines Central Academy students in the AP program skyrocketed (up 400%) from 1988 to 1998, and in 2008, each student took an average of six AP exams.

Distance-learning courses. Advances in technology over the last 25 years have fueled an explosion of distance-learning opportunities for talented youth. These distance-learning courses permit students to work at a level and pace commensurate with their abilities because they are computer-based. Most students work independently; however, the work of some students is monitored by a local teacher-mentor. School personnel might be concerned that a student would experience feelings of isolation if he or she is always studying mathematics alone. Although students taking computer-based courses may have few interactions with other students studying the same material, they have the benefit of taking a course that might not be available in their home school due

to low enrollments or lack of teacher expertise (Wilson, Litle, Coleman, & Gallagher, 1998). Distance-learning classes may be an excellent solution for the student who has "run out of math" to study at school. There is no need to transport the student to another building for math or to hire a special math teacher.

Students who are successful in distance-learning classes are highly motivated and work well independently. They must have the capacity to persevere when things get difficult. With distance-learning courses, schools can provide younger students with advanced classes without needing to deal with the social issues that might arise when placing a younger student in a class for older students (Miller, Adams, & Olszewski-Kubilius, 2007). However, the cost of distance-learning courses can be prohibitive; they can cost as much as $1,000 per semester. Parents are frequently asked to pay these costs.

Since 1985, the Education Program for Gifted Youth (EPGY) at Stanford University has offered computer-based courses in mathematics, physics, expository writing, and computer programming to high-ability students (see http://epgy.stanford.edu/overview/index.html). More than 50,000 students ranging in age from 5 to 18 have taken courses since the program began (see the Resources for more information). Courses in mathematics include algebra, precalculus, and calculus. Students qualify for this program based on math aptitude testing, and the courses they take are designed to replace the math classes taught at school. EPGY courses can be established at a school or on an individual basis. Instructors interact with students via e-mail, telephone, or "virtual classroom" sessions.

EPGY courses are designed specifically for gifted students. Students must show that they fall within the top 15% of students in mathematical ability on the basis of standardized testing. Courses are individualized so that fast learners can move quickly through the mathematics sequence with little repetition, while slower learners will receive additional instruction. EPGY mathematics courses use symbolic computation environments in which students construct proofs of their answers. Such exercises ensure that students know both how to solve problems and why their solutions are correct. Mathematics courses offered include K–2 Mathematics, 3–4 Mathematics, 5–6 Mathematics,

Prealgebra Mathematics, Beginning Algebra, Intermediate Algebra, Precalculus, Honors Geometry, Calculus A–C, Linear Algebra, Differential Equations, Multivariable Integral Calculus, and other university math courses.

Because the EPGY courses are computer-based, students can take advanced courses without leaving their schools. They also eliminate the problem of being placed in a class with much older students. In EPGY, students progress through courses at their own pace, making it possible for them to move on to university-level courses in several subjects before leaving high school.

The University of Nebraska–Lincoln (UNL) Independent Study High School (ISHS) has provided distance-learning opportunities since 1929. As a high school, ISHS is fully accredited by the Nebraska Department of Education and the North Central Association Commission on Schools. Although originally developed as a way to offer subjects to rural Nebraska students that were comparable to those offered in urban areas, the student body has grown throughout the decades to include students from around the world. Students may enroll in ISHS independently, or a student's school may enroll him or her.

All students are required to have a local supervisor who is responsible for maintaining test security, establishing testing conditions, proctoring tests, and submitting completed tests to ISHS. ISHS recognizes that many people, including parents, supervisors, and others (even siblings), play a supportive role in terms of helping students complete their assignments.

ISHS currently offers courses in all major curricular areas. Although course offerings may include basic courses for students with special needs, they also include advanced courses for accelerated students. For mathematically talented students, ISHS offers a full spectrum of classes, from the prealgebra and pregeometry levels, to calculus.

Although the high school community of ISHS is virtual, it is clearly a high school and includes traditional services such as individual academic advising and a newsletter, the UNL's High School World.

Distance-learning programs are also provided by the Center for Talent Development at Northwestern University, the Center for Talented Youth at Johns Hopkins University, and others.

Work with a community mentor or teacher on a math-related project. Students who are unable to take an advanced mathematics course or work in a systematic fashion with a teacher on advanced math may still find an opportunity for additional challenges by completing a math-related independent project. This project may be completed with the supervision of the regular classroom teacher, gifted coordinator, or a community mentor.

One example of a structured independent study program is the Pennsylvania Junior Academy of Science. Students in grades 7–12 who are interested in mathematics can complete a project involving algebra, calculus, geometry, statistics, topology, operations research, or number theory. Then, they have the opportunity to present their project to panels of judges in an annual competition at the local and state levels. For more information, visit http://www.pjas.net and http://www.pjas-region2.org/index.asp. Students looking for interesting project ideas might consult the following website: http://www.sciencebuddies.org.

Exemplary Mathematics Programs

Some school districts have made a conscious effort to identify their mathematically talented students and provide a carefully planned program for them. Three exemplary programs for mathematically talented elementary students are described below. Two programs are accelerative in nature. In other words, they move students forward in the curriculum at a faster pace. The other program described here, C^2Math, differentiates instruction according to students' abilities, but its orientation is to enrich the curriculum, rather than to accelerate.

Southern Valley School District's Math Mentor Program. One suburban school district offers a unique program for its mathematically talented students. Students are systematically identified for mathematics enrichment and accelerative programs using a two-step testing process. Starting in the spring of fourth and fifth grade, students' testing records are reviewed. Those who have scored at the 97th percentile or above (local norms) on the Mathematics Total of the Metropolitan Achievement Test as part of the regular district standardized achievement testing

are invited to participate in Level 2 Mathematics Screening, which includes administration of the Quantitative Reasoning, Mathematics Part I, and Mathematics Part II sections of the Comprehensive Testing Program 4 assessment (Levels 6 and 7 of this test, which were normed on sixth and seventh graders).

The top-scoring 2%–3% of fifth- and sixth-grade students in the Level 2 screening are invited to participate in a mentor-paced mathematics program. During the fifth-grade year, these accelerated students meet with their mentor alone or in small groups twice a week during the school day. These 45-minute sessions occur in the students' home schools, and the mentor travels from one school to another each day. The sixth graders participate in three 45-minute sessions per week.

Participation in the school district's mentor-paced program results in the students being accelerated at least 1 year in mathematics. These students complete fifth- and sixth-grade mathematics in the same year, and they take Integrated Math I (a seventh-grade course) in sixth grade.

In the mentor program, students study mathematics intended to replace the regular mathematics requirements. Their report cards and permanent records reflect their participation in this special program. The district chose to offer this accelerative program in response to needs they had seen in their students. As a way of being conservative, they decided to offer the option to accelerate in mathematics by only 1 year. Students do not typically advance 2 or more years in this program.

Those students who are invited to take the Level 2 screening test, but do not meet the criteria for participation in the Math Mentor Program, are automatically considered for enrichment in the regular classroom. They participate in planned enrichment activities on an as-needed basis. Students are also pretested on each chapter. If they demonstrate mastery level learning (defined as 90% correct), they receive enrichment. The enrichment activities are frequently developed by the elementary math mentor, and they are delivered by the regular classroom teacher.

After the Math Mentor Program was in place for a few years, the district hired a secondary math mentor to continue working with talented students as they progress through the secondary grades. When students have completed all math courses offered on the school district

campus, they may take classes at a nearby university or take Internet Calculus using computers in the school district.

The Houston County Schools.[1] Personnel at the Houston County Schools in central Georgia realized they needed to develop a comprehensive plan for identifying and providing services to their mathematically gifted students (Culpepper et al., 2000). The director of gifted education, gifted education teachers, mathematics coordinator, math facilitators, and the middle school and high school Mathematics Vertical Team leaders worked together to plan and implement a new program for all students in the school district called the C²Math (Curriculum Compacting in Mathematics) program. These individuals were interested in making sure mathematics was appropriately challenging vertically; that is, from one grade level to the next, kindergarten through 12th grade.

Personnel recognized that students in their schools learned mathematics at different rates, had different backgrounds in mathematics, and were ready for different material at different times. They also recognized that many textbooks available today offer a weak treatment of mathematics to students in the elementary and middle school grades. They decided to develop their own comprehensive mathematics program using the compacting model of differentiated instruction.

The goals of the C²Math program were to improve mathematics instruction for all students in the district, offer gifted services to every student in his or her home school on a daily basis, provide talented mathematics students with instruction by gifted-certified teachers, and provide these services to mathematically talented students who might not have qualified for the district's gifted program under state criteria.

C²Math is an assessment-driven program. This means that students are tested to determine what they know and curricula are tailored to their needs, rather than using a set curriculum that is delivered to all students, regardless of ability. First, school personnel developed pretests based on the Georgia state curriculum standards. Students take these pretests before beginning each unit. Based on the results of the

[1] We would like to thank R. Culpepper, S. Gilreath, and S. Powell for their helpful comments on an earlier draft of this section.

pretest, students are assigned to one of three flexible groups: Concept Development, Skills, or Extension. Scores are ranked in ascending order and students are grouped for instruction accordingly. Students in the Concept Development group have limited prior content knowledge and require the most basic instruction on the concept. Students in the Skills group have some knowledge of the topic. They have splintered skills on content knowledge and need instruction to fill in the missing pieces. Students in the Extension group have correctly answered most pretest items, thus demonstrating that they have a good knowledge of the topic and that they are ready for extended content. These students then receive mathematics instruction at the enrichment level.

The Concept Development, Skills, and Extension groups are flexible. Students are placed in different groups during the year depending on their prior knowledge of each unit's topic. For example, fourth-grade students studied six units during the year. At the beginning of the year, they were tested on the first topic, number sense, and then they were placed in one of the three groups for the first 6 weeks of the program so they could study that topic at the right level and pace for them. After those 6 weeks, students were pretested on the second topic, multiplication. Students were then assigned groups for mathematics based on their multiplication pretest results. Many students switched groups. In fact, during the year, 90% of the fourth graders changed groups at least once (Culpepper et al., 2000).

C^2Math is not textbook dependent. Carefully tied to the Georgia state curriculum, it is a collaboration among the gifted education staff, the mathematics staff, the classroom teachers, and the teacher leaders (called the C^2 Facilitators). For the students in the Extension group, the planning team made sure that the mathematics was taught by a teacher who had earned the Georgia certification in gifted education. When the Extension group finishes working on the topic-related activities the planning team developed, they often have time for additional enrichment in mathematics. For example, the fourth graders completed the number sense topic in 4 weeks, instead of the 6 weeks allocated. In the 2 weeks they had left before beginning the next topic, the teacher provided enrichment on prealgebra topics.

After the Extension group of fourth graders completed work in the data/probability unit, they explored graphs of linear functions using graphing calculators. In every unit, extension teachers integrate more problem-solving and higher order thinking skills than in the Skills or Concept Development groups. Because this district's goal did not include acceleration but focused on enrichment, the teachers wanted to integrate as much challenge into their units as possible without accelerating the students. Instead of having the students move on to a new concept or new topic, they had them look at the same topic in a different way.

Houston County's middle and high school component of the "seamless curriculum" was developed using the Math Vertical Team approach developed by the College Entrance Examination Board (2001). The goal of this program is to align the curriculum vertically and to prepare more students to take advantage of the most challenging math courses in high school and college.

Vertical teaming eliminates repetition of previously learned material and provides students with meaningful instruction. Like C^2Math personnel, these middle and high school math teachers found that the mathematics curriculum they were planning was richer and more challenging for all students. For example, they introduced graphing calculators in sixth grade and used them extensively as the students learned about functions, rates of change, and accumulation. Using graphing calculators allowed students to see the visual connections among abstract concepts. Teachers found that they were teaching mathematics concepts in greater depth than ever before.

The members of the planning group have worked hard to overcome challenges. For example, it was an adjustment for the C^2Math teachers to be restricted to teaching one unit during a prescribed period of time and to teach all of the agreed-upon units in the same order. However, this restriction has allowed the program as a whole to be much more flexible and responsive to the abilities and needs of the students. One additional important limitation of this enrichment-based program as it is currently implemented is readily recognized by members of the planning team: It does not include a specific plan for acceleration for

mathematically talented students. As the team noted, this is the next step in their program planning.

Comprehensive Math Program: A 12-Step Plan for Change[2]. Dr. Wendy Leader and her colleagues at Mesa County Valley School District, in Grand Junction, CO, implemented a comprehensive program designed to address several issues concerning their students who had been identified as gifted in mathematics when they were in elementary school. First, there was the issue of inconsistent placement from the elementary schools into middle school math classes. This likely happened because of a strong tradition of local (i.e., building) control, which meant that some elementary programs were more progressive than others; also, many of the elementary teachers, although generally excellent teachers, were not perceived to have a strong background in mathematics. Second, there was ambiguity among the educators regarding best practice for enrichment and acceleration. Also, there was the disconcerting fact that many parents "shopped" for a middle school that would place their children as quickly as possible into Algebra I. Finally, there was concern that students who had been identified as gifted in mathematics based on advanced achievement in elementary school were failing to maintain advanced math scores as they moved through district secondary schools.

With support from the central office administration, specifically the Executive Director of Middle Schools, Deb Bailey, Dr. Leader worked with math specialists from the district. One in particular, Carla Haas, was already working with middle school teachers to implement a new curriculum. The timing of these activities seemed especially prescient because the high schools, with direction from Ann Peterson, the high school math content specialist, were also in the process of adopting new curriculum. The team researched extensively the literature on mathematically talented students and used that knowledge, in combination with their own expertise and advice from colleagues to draft a 12-point plan:

- Students are placed in middle school math classes based on a set of criteria. Teacher input on student motivation may be

[2] This section was contributed by Dr. Wendy Leader, Coordinator of Gifted Programming at Mesa County Valley School District.

considered; however, placement is not based only on elementary teacher recommendation or parent request.

- Each middle school offers a regular grade-level course and an honors math course at each grade level.
- Heterogeneous grouping occurs in the regular grade-level course with scaffolding and special education coteaching for struggling learners.
- Honors course placements are based on multiple criteria, including achievement data, presence of an Advanced Learning Plan (ALP; required for all identified gifted students in that school district) in math, evidence of high achievement in math, and recommendations from previous teachers.
- Throughout the academic year, "switch" points are put into place so that students may move between classes if placement is inappropriate. During the first month of school, all math teachers offer similar content so that switching of placements occurs smoothly.
- An end-of-sixth-grade math exam is developed in order to do above-grade-level testing of advanced fifth graders.
- An end-of-course exam for Algebra I is developed to be used in middle schools and high schools to ensure consistency in rigor.
- All seventh graders who are at or above grade level take the Iowa Algebra Aptitude Test to determine algebra readiness at the end of the third quarter of seventh-grade math. Those students whose scores indicate readiness for algebra will receive prealgebra and eighth-grade content instruction during the second half of the seventh-grade math class if they are not already doing so.
- Middle schools consider offering math during common blocks of time so that students may move easily to other grade-level courses if necessary. This also allows for cross-teaming, grouping, and regrouping.
- Any math class containing a cluster of students on Individualized Education Programs (IEPs) or 504 Plans for math is not focused solely on remedial elementary content, although it may spiral through those concepts and procedures.

- Students are not placed into an inappropriate level of math (one not matching instructional needs) simply because of schedule or because class size cannot accommodate them.
- Schools support concept-based instruction and differentiation in their Professional Learning Communities (PLCs).

Options for advanced learners. Leader and Haas recognized that advanced math students needed additional options, including a regular grade-level math class, honors grade-level math, or subject-skipping into honors math at the next grade level. Compared to regular math classes, the honors math classes were designed to be faster paced; focused on in-depth, real-world problem solving; and included complex applications of math concepts and processes. They established three criteria for placement:

- score of 95th percentile or higher on math portion of Northwest Evaluation Association's Measure of Academic Progress (MAP) test, which was already being given in the district;
- math score in the upper two thirds of the advanced range on the state standards-based test; or
- above-level end-of-course test (i.e., end-of-sixth-grade test for fifth graders) with a score of 70% or above, or a score yet to be decided on the Iowa Algebra Aptitude Test (IAAT).

Students meeting two of the three criteria would be considered for subject-skipping if they, their previous math teachers, and their parents agreed that the desire and motivation to do advanced math was present. Students meeting one of the criteria would be considered for honors placement in grade-level math, provided they demonstrated sufficient interest and motivation. Students meeting none of the above criteria, or lacking the desire to be advanced in math, would be placed in a grade-level math class with differentiation and extensions offered.

Program articulation between elementary and high school. Elementary schools adopted the Investigations in Number, Data, and Space mathematics curriculum (see http://investigations.terc.edu/index.cfm). Elementary Math Content Specialist, Julie Schmalz, taught classes to increase the capacity of elementary teachers to deliver high-

quality, concept-based math instruction. Because the new concept-based curriculum represents a change in the way the majority of elementary students learn math, few gifted elementary students are facing boredom. However, as the conceptual method of learning math becomes easier for elementary students, leaders in the district expect that the issue of acceleration for advanced elementary math students will require a long-term fix.

At the middle and high school, educators were implementing conceptually integrated curricula. The Connected Math Project 2 (CMP2; http://connectedmath.msu.edu) was adopted for middle schools, and Core Plus Mathematics (http://www.wmich.edu/cpmp) was being phased in at high schools. (Students already in the Algebra I-Geometry-Algebra II-Precalculus sequence were allowed to complete it, because *Core Plus* is an integrated math program and gaps could be created in students' learning if they switched to a different curriculum midsequence.) Middle school counselors placed students into math classes based upon data from the previous year. Parents realized that their children's achievement data trumped their own opinions, and those of their children's elementary teachers in some cases, in course placement.

Through a partnership with the local college, middle school teachers took advantage of the opportunity to become highly qualified in mathematics. Elementary gifted education teachers who teach advanced math in elementary schools were expected to avail themselves of the same professional development opportunities afforded to middle school math teachers, so that they could implement sixth-grade CMP2 curriculum with those students who need it. The Math Content Specialists created pacing guides and established monthly grade-level meetings for teachers to attend after school so they could learn from each other and from their math content specialist. They also created a share point available in-district that contains a wide variety of informational and planning documents and links.

Because CMP2 features far less repetition and spiraling than do traditional curricula, the criteria for acceleration in middle school math were revised. For subject-skipping, students are required to obtain a score of 85% on the off-level end-of-year exam. The IAAT is still given

midway through seventh-grade math as a formative assessment, even though the courses that follow are no longer limited to algebra content. Any students who are not placed into honors math in sixth grade still have a chance to move into the honors sequence in seventh grade by meeting criteria based on achievement data for their grade level.

The middle school honors sequence telescopes 4 years of math instruction into the 3 years of middle school. Thus, students who are assigned to honors sixth-grade math, or who are placed into the honors sequence in seventh grade, can complete the first course of high school math, with optional high school credit, by the end of eighth grade. This has been a relief to students and parents who felt that, because they did not meet the criteria to subject-skip, they somehow weren't recognized as advanced in math anymore. State test results showed an increased percentage of middle school students who are gifted in math performing at the advanced level in math, with approximately 76% at the advanced level compared to 50% the previous year.

The district's central administration committed to providing busing to the nearest high school for middle school students who needed more advanced math classes than were offered at their middle schools. Bill Larsen, Executive Director of High Schools, pledged to work with all high schools to make first-hour math classes available to middle school students, so that those students could begin the school day at the high school and then be transported back to their middle schools for the rest of the day.

Incorporating Response to Intervention. Deb Bailey, Executive Director of Middle Schools, introduced a standardized set of Response to Intervention (RtI) protocols for all middle school students in reading and math. Struggling students are characterized as falling into Tiers 1, 2, or 3, based on their escalating needs for intervention, while gifted students are characterized as falling into Tier 3 (i.e., needing a high level of intervention). Standard interventions for gifted middle school students are expected to include cluster grouping into content classes, acceleration in math (honors math classes) as warranted by data, whole-grade acceleration when appropriate, and gifted and talented classes taught by qualified gifted education teachers who are at least half-time at each middle school. Interventions for gifted students are not

considered extra or outside of the norm in Mesa County Valley School District middle schools, but are an integral part of the RtI process, as recommended by the Colorado Department of Education.

By initiating change through a broad-based coalition of administrators and teachers representing the full range of learners, the district is confident that it can assure quality instruction to meet the needs of its advanced elementary students in years to come.

Challenges in Developing Special Programs

When a school or school district devises a special program to recognize the needs of its exceptionally talented students, there may still be some difficulties. For example, if students are pulled out of the regular classroom and grouped to study mathematics for part of their school day, scheduling might be a challenge. If the school does not schedule math at the same time for all grades, students will be pulled out of mathematics class at different times. This can make life very challenging for the teacher or mentor, as well, as he or she might end up traveling between several buildings several times a day, so that much time is wasted going from one site to another. For example, we know one teacher whose Tuesday schedule dictated that she should be in each building no longer than one period; in effect, she had to be in seven different buildings in a seven-period day.

When students participate in a special program that pulls them out of the regular mathematics class, the math mentors need to be very sensitive to the needs and feelings of the regular classroom teachers. They need to be aware that some teachers might resent the fact that someone else provides the curriculum for their mathematically talented students, which might imply that the regular classroom teacher cannot provide it appropriately him- or herself. In some cases, the mentors are also expected to provide the regular classroom teacher with activities for the students' enrichment, and again this must be handled with sensitivity.

Issues in Planning Programs for Mathematically Talented Students

To this point, we have discussed many of the different program options for mathematically talented children. Each of these program types has advantages and disadvantages. As school personnel design programs and parents make decisions about appropriate options for their talented children, they need to think about many issues, which are discussed below and are summarized in Table 6.5.

A "One-Size-Fits-All" Program Doesn't Fit All

Mathematically talented students have varying abilities in mathematics. Some students identified as mathematically talented may be somewhat advanced compared to other students in their grade, while others may be exceptionally advanced. To challenge these students appropriately, we encourage educators to match the program and the curriculum to the students' abilities. Some talented students might be ready to skip ahead two grades in mathematics and work with a mentor; others might need enrichment within the regular classroom. What is important is that the curriculum be matched to the abilities of the student by adjusting the pace and the depth at which the material is presented.

Students May Be Gifted in Math, But Not in Other Subjects

Many mathematically gifted students with whom we work are exceptionally mathematically talented, yet they are not equally talented in other academic areas. For this reason, they might not be placed in their school district's gifted program or have opportunities to advance their mathematical skills. It is important not to deny mathematically talented students opportunities because they have not been labeled "gifted." For example, one school district did not permit a mathematically precocious student the opportunity to participate in a Mathematical Olympiads for Elementary and Middle Schools competition because that student had not been identified as gifted.

Table 6.5
Issues in Planning Programs for Mathematically Talented Students

- A "one-size-fits-all" program doesn't fit all.
- Students may be gifted in math, but not in other subjects.
- The gifted program might not address the needs of the mathematically talented students.
- "Acceleration vs. enrichment" is a false dichotomy.
- Acceleration doesn't necessarily produce gaps.
- Students extremely talented in mathematics may make computation mistakes.
- Special programs need to be integrated into districtwide objectives so they can survive changes in personnel.

The Gifted Program Might Not Address the Needs of the Mathematically Talented Student

Unfortunately, in some cases, even though a mathematically precocious student might be identified for a special program for gifted students, the program itself might be inadequate to meet his or her needs. In many school districts, the gifted program is a pull-out program, meaning that students are pulled out of the regular classroom for a specified period each week to work on challenging assignments. In many cases, the gifted program is enrichment-oriented. Time might be devoted to field trips, writing plays, or learning music history. Although all of these may be worthwhile pursuits, they do not address the everyday needs of the mathematically talented student to learn new, challenging mathematics. "The gifted program will meet mathematically talented students' needs only if the students are permitted to move ahead in the mathematics curriculum at an appropriate pace and depth, not if they are given random enrichment activities" (Lupkowski-Shoplik, 1996, p. 7).

"Acceleration vs. Enrichment" Is a False Dichotomy

Enrichment and acceleration are not mutually exclusive concepts. Good enrichment in mathematics is accelerative, while good acceleration practices include adding enriching activities and concepts. Allowing students to move along at an appropriately rapid pace and encouraging them to study mathematics in great depth are both needed for the curriculum to be correctly matched to the students' abilities.

Acceleration Doesn't Necessarily Produce Gaps

One major concern that is often mentioned when considering accelerating students is that there will be gaps in their backgrounds if they skip ahead in mathematics. An effective way to alleviate these concerns is to test students to determine what they do and do not know (see Chapters 3 and 4). What concepts do they still need to master before being ready to advance? A teacher or mentor can work with the students on the concepts they have not mastered. Within a fairly short period of time, the accelerated students will have filled in the gaps and will be ready for the new challenges of the more advanced class.

Students Extremely Talented in Mathematics May Make Computation Mistakes

Students who have advanced understanding of mathematical concepts may not demonstrate the same high level when it comes to computation tasks (Lupkowski-Shoplik et al., 1994). For example, a student who understands the concept of multiplying two 2-digit numbers might make an addition error while solving a problem. In a study of 1,667 students, Rotigel (2000) found that mathematically talented youth consistently scored higher on measures of mathematical concepts than on measures of mathematical computation. She listed several explanations, including:

- Talented students might perform better on conceptual tasks than on computation tasks because conceptual tasks seem to be easier for them to solve using their reasoning abilities,

while solving computation problems may be more dependent on direct instruction (Lupkowski-Shoplik et al., 1994).

- These mistakes may be a result of carelessness if talented students have become bored and have developed the bad habit of doing mathematical calculations too quickly, without writing down their thought process.
- Because mathematically able students may have impressed others with their logical reasoning abilities, they may have been able to avoid the memorization and drilling needed to be more accurate when working computation problems.

Special Programs Need to Be Integrated Into Districtwide Objectives So They Can Survive Changes in Personnel

Frequently, special programs for mathematically talented students are established because of the commitment and hard work on the part of one or two individuals. These programs might thrive as long as the key individual is still in the same position. If that key person leaves, however, is there a guarantee that the program will still continue? We have developed a "To Do" list to help institutionalize the program so that it becomes a part of the school and not dependent upon one or two individuals:

- *Create ownership for the constituents (parents, students, principals, gifted coordinators, and regular classroom teachers).* People feel the program is "theirs" if they are involved from the beginning. They begin by taking a role in the planning phase of the program. Through a group approach, they come to an agreement on how to provide services for the mathematically talented students they have identified. They are involved in the design of the program and in decision-making as the program begins. In the case of a program using a mentor approach, for example, regular classroom teachers and mathematics specialists can be included so they have input into making the program work smoothly.
- *Document what has happened.* Gather a copy of all of the letters sent to parents. Keep careful records of test results and

placement decisions. Track students' progress through the program. Make a handbook for the new people who will be joining the team. This handbook might include an annual calendar that indicates what needs to happen and when. For example, when should assessment occur? When are placement decisions made? The handbook should also include a statement of purpose, an explanation of how students are identified for the program, summaries of program evaluations, and so forth.

- *Devise a written plan.* Develop a written schedule of goals for the program for the next several years. Develop a plan for growth of the program. Provide information that addresses the question, "What happens to the students after the program ends?"
- *Gather data and conduct evaluations that demonstrate how the program works.* Concrete data, such as test scores showing the students in the program have made greater gains in achievement test scores than students of equal ability who are not in the program, are important.
- *Listen for the success stories.* Students who have completed the program can return to tell teachers, school board members, and others how the program had an impact on their education. Written testimonials about what a difference this special program made can be powerful. Copies of newspaper articles and other public relations materials are also useful. Keep these in the program handbook.
- *Make sure there is articulation within the curriculum.* People who will be involved at the high school level need to be aware of this program so they can begin planning for these students when they get to higher grade levels.

Long-Term Planning

The following is a list of eight points for students, families, and teachers of mathematically talented youth to consider as they plan a student's educational program.

"Elementary students who are extremely talented in mathematics need time to develop the necessary cognitive structures that characterize mathematical maturity" (Lupkowski & Assouline, 1992, p. 129).

"How do I know when my child is ready to study algebra?" asks the parent of a mathematically talented second grader. Although we are strong advocates of acceleration for mathematically talented youths, we also realize that few students are ready to study algebra at this young age. The goal for elementary students is "mathematical maturity" before moving on to algebra, geometry, and so forth. This maturity is characterized by a thorough background in general mathematics (arithmetic), the structure of the number system, arithmetical problem solving, and Piagetian formal operational thinking (Piaget & Inhelder, 1969).

"In order to start at the right point and ensure that students develop strong mathematical underpinnings, it is extremely important to assess carefully what skills and content have been mastered" (Lupkowski & Assouline, 1992, p. 130).

This point follows directly from the point listed above. For the parent of the precocious second grader, the answer is not to rush the student into algebra. Instead, we recommend carefully assessing his or her background and mathematical abilities and moving ahead at a pace appropriate for the student (see Chapter 4). This might result in the student accelerating one or more years in mathematics. At the same time, the precocious second grader might demonstrate mastery of skills that have not been formally taught to him or her; instruction should begin at the point where the student has mastered the content and is ready for new material.

"Mathematically brilliant youths should study mathematics at their appropriate level of mental functioning and at a steady rate" (Lupkowski & Assouline, 1992, p. 130).

Students do not benefit from racing ahead in mathematics only to find themselves so far ahead of the mathematics delivery systems at their schools that they go for months (or even years) at a time without

studying new topics. Students do not need to study mathematics intensively for hours every day. Instead, the goal is to offer students opportunities to study mathematics at a steady rate throughout their schooling. This might mean weekly meetings for 2 hours with a mentor, rather than five 45-minute periods of mathematics every week. Students might need to participate in afterschool or summer programs to be challenged adequately. Other students who are in a challenging mathematics program during the school year won't need an accelerative summer program. They would benefit from other pursuits.

"Mathematically talented students need to balance their accelerated study of mathematics with the study of other academic subjects and participation in extracurricular activities" (Lupkowski & Assouline, 1992, p. 131).

Parents and teachers need to see to it that students do not become too focused on their study of mathematics. Accelerated students have the luxury of time; if they have moved ahead of their age-mates by one or more years, they can take the time to do an in-depth math or science fair project. They have the time to explore nonacademic areas and study academic subjects in depth. Although our goal is not to guarantee that mathematically talented students are "well-rounded," students do benefit from exposure to different subjects, trying out different hobbies, playing with friends, having time alone, and having unstructured time. Many enrichment programs on diverse topics are available at local museums, summer camps, and so forth.

"Teachers, mentors, clubs, and competitions can enrich an accelerated mathematics curriculum for talented youths" (Lupkowski & Assouline, 1992, p. 131).

Just as students benefit from studying mathematics in a formal setting, they can also benefit from learning mathematics in an informal setting. Teachers and mentors can give students interesting problems to try, and students can participate in math clubs or competitions (see the Resources). Thus, students are moving ahead in mathematics, but they are also taking advantage of the range of mathematics enrichment opportunities that are available to them.

"Moving ahead extremely fast in the mathematics sequence is likely to catapult the elementary student beyond the offerings of the school system before high school graduation" (Lupkowski & Assouline, 1992, p. 132).

It is extremely important to consider carefully the impact a grade skip or subject-matter acceleration will have on a student's educational program. The problem is not that the student will "run out" of mathematics to study. It is simply that the school district might not have any more formal mathematics courses for the student to take. There is always more mathematics for students to study (ask any mathematics professor who is conducting mathematical research and trying to invent new mathematics daily). The question is, how does the student get to the class? There might be problems with transportation, timing of courses, not taking mathematics with age-mates, and so forth. All of these things are important to think about. Plan for the eventuality that the student will move beyond the course offerings of the school system. He or she might take more advanced courses by attending college part-time, working one-on-one with a math teacher, or taking a distance-learning course.

"Academic programs offer varied opportunities for able students to forge ahead in mathematics" (Lupkowski & Assouline, 1992, p. 132).

School is not the only place where students have the opportunity to study mathematics. They may participate in challenging mathematics opportunities through local universities or through a distance-learning program. Today, there are hundreds of interesting opportunities for motivated students, ranging from accelerative classes, to competitions, to science projects, to mentorships. These talented students should be made ready by age 12 or 13 to study algebra and subsequent courses quickly and well. The use of the DT→PI model ensures that students do not skip important concepts. A variety of challenging programs are listed in the Resources, and academic talent searches are discussed in Chapter 5.

"Before moving on to college coursework, students should take the most challenging coursework available at the high school level" (Lupkowski & Assouline, 1992, p. 133).

It does not make sense to rush through high school and move on to college just to be faced with needing to take remedial courses. Taking the most challenging courses available in high school ensures that the student is well prepared for college. Advanced Placement (AP) courses provide college-level work to students before they graduate from high school. Students may earn credit for one or many courses, leading to early graduation from college or the opportunity to take more advanced or varied courses in the undergraduate curriculum. Students may also benefit from online courses, such as those offered by Stanford University's EPGY, which was mentioned earlier in this chapter.

Evaluating Programs for Mathematically Talented Youth

Belcastro (1987) outlined a set of principles that gifted programs for mathematically talented elementary students should meet and proposed a series of excellent questions that program personnel should ask in reviewing their program:

- *Is the program for mathematically talented elementary students a part of the regular curriculum?* It is recommended that delivery of services occur through the subject area (mathematics), not through a general gifted program where mathematics would be only a part of the activities. This program should be guided by a teacher assigned to the regular elementary curriculum who also has a mathematics specialty. The program offered by the Southern Valley School District, mentioned earlier in this chapter, is a good example of a program that addresses this principle.
- *Is there a rigorous identification procedure?* Students selected for advanced programming in mathematics should be identified by assessing their mathematics abilities (e.g., through a mathematics achievement test).

- *Is the program in effect every day?* Students in the regular classroom study mathematics every day, and mathematically talented students should have that same opportunity. The advantages of having the program in effect every day include constant interaction with and attachment to a teacher of the mathematically gifted, the enthusiasm for mathematics that can be achieved only through continuous time, and the joy of moving ahead rapidly in mathematics or delving deeply into it.
- *Does the program provide placement and interaction with peers who are mathematically talented?* Research shows that ability grouping is an extremely effective method for educating gifted students. The advantage of ability grouping for mathematics is that students study mathematics with others who have similar abilities, feelings, interests, problems, and goals.
- *Is faster pacing of the mathematics group facilitated?* Mathematically talented students need less time than others to learn information, and they don't require constant repetition. The pace of the program should be matched to the student's learning rate.
- *Are students challenged at their own level using advanced strategies?* Belcastro (1987) responded to his question,

> The mathematics curriculum designed for mathematically gifted elementary students should be different from the mathematics curriculum offered other students. It should pay greater attention to advanced skills and techniques and advanced conceptualizations that by definition cannot be easily understood by students of similar age but of average or below-average ability in mathematics. Further, the teachers of the mathematically gifted must present mathematical content on higher levels of abstraction and in more complex form than is done for students average in mathematics. The program for the mathematically gifted should challenge them at their own levels in mathematics. (pp. 8–9)

- *Are teachers selected who are trained in the education of the mathematically gifted?* Teachers in a program designed for mathematically gifted students should be excellent teachers who have a content specialization in high-level mathematics and therefore can answer students' high-level questions. These teachers will also be familiar with the scope and sequence of the K–12 mathematics curriculum. It is imperative that they understand the unique characteristics of mathematically talented youth and that they be aware of challenging opportunities where these students can interact with their mathematically talented peers.

  We would like to add another question to Belcastro's list: *Was a needs assessment conducted when the program was designed?* It is important to discern why a program is being developed, for whom it is developed, what will happen, and who the key players will be because they all have an important role in the planning process. This creates a sense of ownership and helps ensure that the program will exist even if one or two key players leave.

Conclusions

After mathematically talented students are identified, it is important to recognize the kinds of programs that will be beneficial to them. A wide range of programming options is available to mathematically talented students, from enrichment within the regular classroom, to fast-paced, individualized instruction. We have tried to emphasize in this chapter and throughout this book that the important idea is achieving the "optimal match," that is, matching the right level and pace of curriculum to the abilities of the talented student.

Learning occurs only when there is an appropriate match between the child's ability and the circumstances he or she encounters (Hunt, 1961). We recommend carefully assessing each student's level of general ability, specific aptitudes, and achievement and matching the curriculum to them. For some students, enrichment programs provide enough stimulation. Other students need a program that is radically accelerated.

CHAPTER 7

Curricula and Materials

KEY POINTS

- Mathematically talented students need differentiated curricula to be appropriately challenged.
- When the core curriculum is used with mathematically talented students, enrichment activities that are provided should be within the context of continuous and systematic progression through the curriculum.
- Mathematically talented elementary and middle school students need learning opportunities with a variety of topics, including problem solving, geometry, algebra, arithmetic, number systems, probability and statistics, spatial visualization, and ratio and proportion.
- Teachers of mathematically talented students will need to go beyond the textbook to differentiate the curriculum by using manipulatives, math games, computer programs, and out-of-class opportunities.
- The National Council of Teachers of Mathematics' (NCTM, 2000) *Principles and Standards for School Mathematics* does not specifically address the needs of mathematically talented students; however, the *Principles and Standards* may be adapted to accommodate the need for advanced material.

My child, then in second grade, had tested at the eighth-grade level on [a standardized math] test. I tried advocating for him with his math teacher, who considered this for a moment and thoughtfully replied, "I can't teach your child. I have to teach the curriculum."

—Zelda, quoted on The Ridiculous Things I Heard Today (http://www.hoagiesgifted.org/ridiculous_things.htm)

When we began to write this chapter on curricula, our desks were covered with stacks of math enrichment books, algebra books, and elementary math curricular materials. How should we guide teachers and parents in selecting the appropriate curriculum? We quickly realized that we couldn't single out one curriculum or book to recommend for all mathematically talented students or we would be like the second-grade teacher in the quote above. The point we had to make is that the everyday curriculum offered in most schools is not challenging enough for mathematically talented students, and our objective with this chapter is to provide guidance on how to supplement and differentiate it for the individual student.

While considering the multiple approaches to modify the core curriculum for gifted students, we found ourselves returning to Dana Johnson's (1994) succinct, yet classic statement,

> Appropriate curriculum and instruction for mathematically gifted learners cannot be delivered only through a supplemental program or enrichment materials. The core curriculum for those students must be modified so that the daily exposure to mathematics is closely aligned with their abilities. (p. 233)

Johnson is absolutely right in stating that mathematically talented students need something different from what is offered in the typical elementary classroom. Not only do they need a modified program, but they also need a predetermined scope and sequence that allows them to progress systematically through a challenging mathematics curriculum. This predetermined scope and sequence will most likely be developed according to the resources available within a district, because this is not a prepackaged curriculum that you can go out and buy. However, we recognize that few schools have developed a predetermined scope and sequence for their mathematically talented youth. They are not able to offer a truly differentiated curriculum for mathematically talented students. Therefore, the goal for this chapter is to offer the classroom teacher the tools needed to *differentiate* the "regular" curriculum for highly able students.

Acceleration and Enrichment

Nearly a decade after Johnson's (1994) statement about the ineffectiveness for high-ability learners of curriculum that was only supplemental or enriched, she addressed ways to adapt the mathematics curriculum for these learners within the NCTM (2000) *Principles and Standards for Mathematics*. A key point related to our discussion is that "A mathematics curriculum has to be accelerated before it can be effectively enriched" (Johnson, 2003, p. 189).

The National Mathematics Advisory Panel (2008) report, *Foundations for Success* reviewed more than 16,000 (!) research studies and related documents, all of which led to a conclusion similar to Johnson's (2003): "mathematically gifted students with sufficient motivation appear to be able to learn mathematics *much faster* [emphasis added] than students proceeding through the curriculum at a normal pace, with no harm to their learning, and should be allowed to do so" (p. xxiv).

However, the pace of mathematics presented in the typical elementary and middle school is simply not challenging enough for mathematically talented students. Assouline and Doellinger (2001) documented this statement through the results of a study with more than 6,000 academically talented third through sixth graders. As a group, these students performed well on items measuring concepts in geometry, prealgebra, and probability and statistics. In fact, these talented fifth and sixth graders outperformed the average eighth grader. "This is a strong indication that these content areas should be introduced earlier and at higher challenge levels than typically provided by a traditional elementary curriculum" (Assouline & Doellinger, 2001, p. 132; see also Lupkowski-Shoplik et al., 2003).

The choices for the delivery system for this advanced content include acceleration, enrichment, or both (see Chapter 6). When enrichment is relevant and appropriate, students benefit from studying a topic in more depth or learning about a topic that is not covered in the regular mathematics curriculum. The advantages of enrichment include giving the students more challenging work, exposing them to different topics in mathematics, or helping them to learn a topic in greater depth.

Enrichment requires that the teacher differentiate the curriculum, which requires extra work on his or her part. It also assumes that the teacher has the adequate mathematics background to give the student relevant horizontal or vertical enrichment.

In contrast to enrichment, acceleration is easy and inexpensive to implement (Colangelo et al., 2004a); in its simplest form, a student merely leaves the regular classroom to go to a higher grade level for mathematics and returns to the regular classroom after the mathematics period ends. This student is treated like all of the other students in the math class, and no extra work needs to be done by either of the teachers. For this reason, acceleration has a distinct advantage over enrichment. However, one of the problems with acceleration is that, even though a student moves up to a higher grade for mathematics, the curriculum for the older grade might still not be advanced enough or offered at a fast-enough pace. Therefore, differentiation would still be needed.

As we stated in the previous chapter, a combination of acceleration and enrichment makes the most sense when developing programs for mathematically talented students. For mathematics, relevant enrichment has some accelerative components, and appropriate acceleration includes some enrichment (Schiever & Maker, 1997; Stanley, 1979b). Sheffield (1999b) has taken this a step further. Because the typical U.S. curriculum lacks depth—it has been called a mile wide and an inch deep (Schmidt, McKnight, & Raizen, 1996)—Sheffield (1999b) suggested developing offerings for mathematically talented students that combine depth or complexity, breadth, and rate. Programs for mathematically talented students should not only increase the pace of the curriculum offered, but they also should offer greater depth and help students to see the mathematical connections among concepts.

Key Elements of a Curriculum for Mathematically Talented Students

Although textbooks have traditionally been the main (or sole) resource for mathematics classes, teachers of mathematically talented students need to go beyond the textbook and find supplementary

materials to challenge their students. Just providing an extra workbook for these students is not sufficient, however. They need a "curriculum that moves at a faster pace, is less repetitive, goes deeper into important thought processes, and covers a broader set of ideas than that encountered in a commonly used textbook series" (Center for Gifted Education, 2004, p. 2).

Johnson and others at the Center for Gifted Education at The College of William and Mary (2004) have developed criteria for appropriateness of curriculum and resources for mathematically talented students. These include:

- Selected material contains a high level of sophistication of ideas. Advanced topics such as infinity are explored.
- The material offers opportunities for extensions that challenge the most able learners.
- The material can be tailored to the individual needs of high-ability learners. A variety of activities, projects, and problems can be tailored to individual levels, interests, and backgrounds.
- Projects and problems promote higher order thinking skills, reasoning, and problem solving.
- Once mastery level is demonstrated, materials have less emphasis on basic skills.
- Opportunities for student exploration based on the student's interest are included.

Gavin, Casa, Adelson, Carroll, and Sheffield (2009) advocated using a mathematics curriculum that promotes high-level thinking. The level and depth of understanding mathematical concepts is emphasized rather than the number of skills learned. The emphasis is on engaging students using strategies more similar to those that practicing mathematicians use. The processes include communication, connections, reasoning, representation, and problem solving.

Gavin and Sheffield (2010) described a model for mathematics curriculum for talented students that includes four components: "creative and complex problem solving, connections within and across mathematical and other content areas and contexts, an inquiry-based approach, and appropriate pacing" (p. 53). The goal is to devise rigorous

curriculum that challenges students to think and act like practicing mathematicians.

No one curriculum will meet the needs of all mathematically talented youth. As discussed in the previous chapter, the instructional goals and the program options will vary, depending upon the needs of the student.

What Should Mathematically Talented Students Study?

With all of these important elements in mind, let us now turn to more specific information about the topics mathematically talented students should study while in elementary and middle school. The mathematics learned during these years forms the foundation for algebra (National Mathematics Advisory Panel, 2008); consequently, mathematically talented students need to have a thorough understanding of elementary school mathematics. The National Mathematics Advisory Panel "proposes three clusters of concepts and skills—called the Critical Foundation of Algebra—reflecting their judgment about the most essential mathematics for students to learn thoroughly prior to algebra course work" (p. 17). These include:

- fluency with whole numbers,
- fluency with fractions, and
- particular aspects of geometry and measurement.

These broad clusters correspond to a list of topics that students are recommended to study. Our list (see Table 7.1) was generated from content areas discussed by Wheatley (1983, 1988), Johnson (1994), and Sheffield (1994). Not only should students study all of these areas, but the curriculum within each area also should be enriched. And, as described in Chapter 4, once students have mastered topics in a given area, they are ready to move on to the next level.

Table 7.1
Essential Topics for Mathematically Gifted Elementary Students

- Arithmetic and algebraic concepts
- Computer programming and robotics
- Estimation and mental math
- Geometry and measurement
- Math facts and computation skills
- Probability and statistics
- Problem solving
- Ratio, proportion, and percent
- Spatial visualization
- Structure and properties of the real number system

Arithmetic and Algebraic Concepts

As discussed, the National Mathematics Advisory Panel (2008) has made strong recommendations about the role of algebra in the overall school curriculum. Its treatment of the topic addresses the foundational nature of both elementary and middle school arithmetic as preparation for algebra. Some of the important arithmetic concepts that elementary students need to learn include primes, composite, factoring, divisibility, ratio, and proportion (Johnson, 1994; Wheatley, 1983). Elementary students also need to study the concepts of variables, signed numbers, prime numbers, equations of curves in a plane, and intuitive solutions of number sentences (Wheatley, 1988). Students might find using manipulatives especially helpful when studying these topics. The standards for school mathematics set forth by NCTM (2000) encouraged teachers to present algebraic concepts starting in kindergarten. Others (see, for example, Bay-Williams, 2001) have recognized the need to provide developmentally appropriate experiences with algebra so that learning corresponds to students' natural development from concrete reasoning to the more abstract reasoning needed for algebra. Also,

Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics (NCTM, 2006) offers a strong discussion regarding the concepts and skills that are critical for preparation for algebra.

Computer Programming and Robotics

For this generation of students—and all future generations—computers are an educational standard. Rather than using computers in math class strictly for drill and practice, mathematically talented students benefit from using them to develop computer programming skills. Computer programming is valuable for its own sake, and it is beneficial in helping children to practice logic. Students can begin studying simple programming in the primary grades. These programs are useful for developing logic skills, solving problems, and exploring concepts.

For example, students in the C-MITES program at Carnegie Mellon University have enjoyed learning to program computers by using Alice. Alice is an object-oriented educational programming language developed at the University of Virginia and Carnegie Mellon University. Students use a drag and drop environment to create computer animations using three-dimensional (3-D) models. Alice is designed so students may easily create an animation to tell a story, play an interactive game, or make a video to share online. Alice was designed as a new approach to teaching introductory programming concepts and is meant to be a student's first exposure to object-oriented programming.

In Alice, 3-D objects such as people, animals, and vehicles populate a virtual world. Students may drag and drop graphic tiles to create a program. The instructions they use correspond to standard statements in much more sophisticated programming languages such as Java or C++. Students can see immediately how well their programs run, which enables them to understand how their programming statements make the objects in their animation move. By using Alice to manipulate the objects in their virtual world, students gain experience with all of the programming constructs typically taught in introductory programming

courses. Alice is made available to the general public as a free gift from Carnegie Mellon University and can be downloaded from the following website: http://www.alice.org

Caitlin Kelleher created Storytelling Alice as part of her doctoral work in computer science at Carnegie Mellon University. Storytelling Alice is a programming environment designed for middle school students (particularly girls) to learn to program computers through creating short 3-D animated movies. At C-MITES, fourth- through sixth-grade gifted students have successfully programmed computers using Alice, so Storytelling Alice should make programming accessible to even younger gifted students.

Many mathematically talented students are also intrigued by robotics. Numerous robotics kits and programs are available to help students practice their computer programming and reasoning skills. Many young students gain their first computer programming experience in working with robots.

The Robotics Academy at Carnegie Mellon University (http://www.education.rec.ri.cmu.edu/index.htm) maintains that we now have a fourth "R" in education: Robotics. Their mission is to use the motivational effects of robotics to excite students about science and technology. They have partnered with LEGO® to develop curriculum for the LEGO® Mindstorms robotics kits, and this partnership is dedicated to using robotics as the motivator to "simply teach the complex."

The C-MITES program has taken the LEGO® Mindstorms robotics kit to another level by using it with the HandyBoard controller developed by MIT. Students participating in these robotics classes learn how to add sensors and program robots to follow lines, do dances, play soccer, and go through a maze.

Estimation and Mental Math

Mental arithmetic requires an understanding of relationships among numbers, not just remembering mechanical algorithms. Mental math skills should be encouraged continuously, beginning in the preschool

years. Johnson (1994) recommended one unit each year on estimation, while encouraging estimation in all computational work throughout the year (see also Wheatley, 1988).

Even though calculators and computers are commonplace in elementary classrooms, elementary students still need to learn to estimate their answers. When students are able to estimate their answers, they can evaluate the results they get from calculators and computers and develop a sense of the "reasonableness" of an answer (Wheatley, 1983). For more information, see the books *Mental Math in the Middle Grades* by Jack A. Hope, Barbara J. Reys, and Robert E. Reys (1987, Dale Seymour) and *Estimation and Mental Computation: 1986 Yearbook of the National Council of Teachers of Mathematics* by Harold L. Schoen and Marilyn Zweng (1986, NCTM).

Specialists at the Center for Talented Youth at Johns Hopkins University, which offers the Young Students Classes for mathematically talented elementary students, recommend that children develop the habit of always checking out a problem before beginning any computation to determine approximately what their answers will be and, after working out the answer, asking themselves if it makes sense (Mills & Wood, 1988).

Geometry and Measurement

Mathematically talented students benefit from an exposure to geometry long before the time of the typical 9th- or 10th-grade geometry class. Rather than studying formal theorems or proofs, as in a formal geometry class, elementary and middle school students benefit from a study of geometry that emphasizes concepts and principles. This approach goes beyond the focus on simply defining terms and shapes that previously characterized elementary students' study of geometry. One classic text that might be helpful to young students who are not yet ready for a formal high school geometry course is *Experiencing Geometry* by James V. Bruni (1977, Wadsworth), which is an informal, intuitive introduction to geometry. It contains numerous concrete models

and illustrations and encourages readers to investigate fundamental geometric concepts through observation and experimentation.

Master teacher John Benson (2010) offered an inspirational description of algebra and geometry. Efforts in

> engaging our most able students in honors algebra as soon as they are ready—usually in seventh grade, but sometimes earlier—and then teaching them honors-level high school geometry the following year have been instrumental in getting our most mathematically able students ready for the challenges that lie ahead as well as providing an experience that keeps them excited about mathematics. Geometry enables us to provide them with continual exposure to challenging problems while consistently engaging them in important, interesting tasks. (Benson, 2010, pp. 133–134)

The C-MITES summer program offers third through sixth graders a 2-week course called Informal Geometry. Students in this class use tangrams and pattern blocks to investigate geometric concepts and vocabulary and dot-paper to make congruent and similar figures. These activities provide concrete, hands-on experiences with fractions. Students also learn about transformations and the four types of isometries (translations, rotations, reflections, and glide reflections). In addition, in the spatial visualization component of the class, students use concrete manipulatives to help them visualize and represent three-dimensional objects from two- and three-dimensional pictures. Then, they use cubes to construct buildings from plans. Students learn to distinguish different orientations, recognizing left and right views as reflections. The casual classroom visitor might perceive that students are merely playing with blocks, when actually they are struggling with mentally rotating objects and interpreting two-dimensional plans using three-dimensional objects. This is the most challenging aspect of the course for some students. Although students in this class never complete a formal proof, they learn many challenging concepts that are typically reserved for 10th-grade geometry classes. One of the resources that the Informal Geometry teachers use is Michael Serra's *Discovering*

Geometry: An Inductive Approach (1997, Key Curriculum Press), which investigates topics including inductive reasoning, using tools of geometry, transformations and tessellations, and deductive reasoning.

As the description of the C-MITES Informal Geometry course indicates, it is essential that elementary students use manipulatives to study geometry. Obviously, using manipulatives makes abstract topics more concrete and understandable. For example, students can calculate the area of a rectangle using unifix cubes. Students should also learn how to use a protractor. More information about manipulatives and useful geometry books can be found in the Materials section in this chapter.

Math Facts and Computation Skills

Good computational skills with fractions, decimals, and integers are essential for work with many higher level concepts (Johnson, 1994). Elementary and middle school students need to master the basic facts of arithmetic because further study in mathematics requires accurate calculations. We advocate the use of calculators, but we also recognize that children need to learn their basic facts. Students still need to be able to estimate and verify their answers.

Even exceptionally mathematically talented students need drill and practice. Some talented students become very good at convincing their parents and teachers that they don't need to do any drill or repeated practice. In one case, a mathematically talented girl complained of headaches when asked to practice the multiplication tables or if she didn't know the answer to a multiplication problem. Mastery tests showed that she did not have a good knowledge of basic multiplication facts, and we assumed that she was using her headaches as an excuse not to do something she did not feel confident in doing. Therefore, she needed opportunities to practice computation skills. Because school personnel recognized her level of mathematical talent, however, she was not held back in other areas because of her lack of knowledge of multiplication facts. Instead, she was given plenty of opportunities to

practice computational skills while still working with more advanced concepts (Lupkowski & Assouline, 1992).

Probability and Statistics

Before students take Algebra I, they should study probability and statistics (Johnson, 1994). At a young age, students can experience these topics by gathering data and making tables and graphs. Students in preschool and kindergarten enjoy collecting data using survey questions (e.g., "What is your favorite kind of pet?"), tabulating the responses, and graphing the results. Older students might enjoy more complex problems, such as surveying a sample of adults and predicting the results of a local election (Pratscher, Jones, & Lamb, 1982).

Nancy Pfenning used her expertise as a statistics professor to present sophisticated concepts to groups of mathematically talented fourth through sixth graders. Her book, *Chances Are: Making Probability and Statistics Fun to Learn and Easy to Teach* (1998, Prufrock Press), includes the curriculum she wrote for a 2-week summer course in probability and statistics for gifted students at Carnegie Mellon University. In her book, Pfenning teaches the students proper notation, vocabulary, and advanced concepts, but does it in a manner that is very accessible to elementary and middle school students. Students in her class learn concepts that go well beyond mean, median, and mode; the book covers concepts as sophisticated as standard deviation, the normal curve, and sampling theory.

Problem Solving

"Gifted students often report that their work outside the classroom on problems of greater depth yields benefits that go far beyond their mathematical knowledge" (Rusczyk, 2010, p. 94). Indeed, problem solving provides opportunities for higher order thinking skills in the context of real-world, interdisciplinary applications (Johnson, 1994). Children do not automatically develop the ability to solve problems

as they master computational skills; they need to be taught problem-solving skills (Lenchner, 1983). Solving problems differs from simply doing an exercise in mathematics. Mathematics exercises are used to practice a known procedure; the means to arriving at the solution of an exercise is evident. Typically, teachers teach a concept, give practice exercises, and check the exercises to be sure students have learned the procedure. In contrast to an exercise, when students are presented with a math problem, the solution or the means to solving that problem is not always apparent. Correctly solving the problem generally requires some degree of originality or creativity on the part of the problem solver (Lenchner, 1983). Thus, problem solving is a skill requiring more sophisticated cognitive skills than those required for basic computation.

George Lenchner's *Creative Problem Solving in School Mathematics* (1983) provides an excellent resource for teaching problem-solving skills. Lenchner includes classic problem-solving methods, such as drawing diagrams, making lists, finding patterns, working backward from the solution, and testing and verifying. Other helpful problem-solving books include *Reasoning and Problem Solving: A Handbook for Elementary School Teachers* by Stephen Krulik and Jesse A. Rudnick (1992, Allyn & Bacon), *Problem Solving Strategies: Crossing the River With Dogs and Other Mathematical Adventures* by Ted Herr and Ken Johnson (1994, Key Curriculum Press), and problem sets from the Mathematical Olympiads for Elementary and Middle Schools (see http://www.moems.org). Helpful problem-solving books include sample problems, activities, games, and methods for teaching problem-solving skills.

We are pleased that problem-solving activities have become more and more a part of the regular curriculum because all students benefit from them. However, it is possible for school personnel to think that using the abundant materials available on problem solving would suffice as their "gifted program" in mathematics or to enrich the regular mathematics program, thereby eliminating a need for any other special programming. Although problem-solving activities are challenging and fun for talented students, they do not provide *all* of the mental stimulation students need. In other words, problem solving is not the one answer to gifted students' needs in mathematics. In fact, no single

area can provide the challenge that talented students need. Rather, they need a complete and balanced program in mathematics.

Gavin and Sheffield (2010) suggested that:

> Moving beyond problem solving, talented students should be encouraged to pose new tasks, questions, and problems, and create new, unique solutions. In fact, these are the activities in which practicing mathematicians engage. Creativity provides both challenge and enjoyment for students with mathematical talent, and they should have opportunities to play with mathematics by creating new problems and unique, interesting solutions. This, in effect, is the way in which new mathematics is created, and providing these opportunities can offer an apprenticeship to a budding mathematician. (p. 55)

Ratio, Proportion, and Percent

Wheatley (1983) recommended studying ratio, proportion, and percent, and it is relatively simple to show students applications of these topics because proportional thought is the basis for much work in the sciences. Johnson (1994) specified that mathematically gifted students should be introduced to percents by third grade, and they should continue practicing and applying concepts of percent so that they are proficient by seventh grade. The importance of this topic is also reflected in the fact that it is among the three Critical Foundations identified by the National Mathematics Advisory Panel (2008).

Spatial Visualization

Spatial visualization means the ability to visualize spatial transformations (e.g., mentally rotating a cube). Wheatley (1988) cited a number of research studies that document the importance of recognizing spatial visualization as an ability in mathematics. Wheatley

(1988) claimed that "of all intellectual factors, spatial visualization is the most highly correlated with mathematics achievement and least addressed in the mathematics curriculum" (p. 255). This skill is important for doing mathematics, science, and engineering. Lubinski (2003) addressed the concern that individuals who have extremely high spatial reasoning abilities are a neglected population. They are neglected in terms of identification of talent as well as in development of their talent. Lubinski specifically suggested that is it ideal for spatially talented student to have "manual manipulation in the curriculum. Working with 'materials' or 'things' readily found in agriculture, architecture, computer simulation, engineering (robotics), and bench work in the physical sciences and creative arts are ideal settings for developing their talent" (p. 530).

Manipulating real objects helps students to develop an understanding of spatial relationships, which helps prepare them to manipulate objects mentally. Many commercially available computer games can help students practice this skill. Also, making three-dimensional models using tangrams, tessellations, and graphing help children learn to think visually (Wheatley, 1983). Johnson (1994) recommended including a short unit on spatial visualization each year throughout elementary and middle school. In our university-based programs for mathematically talented students, we have noticed that some mathematically talented students who were outstanding in number sense, computations, and problem solving had a relatively difficult time with spatial visualization activities, but they benefited from exposure to and practice with such activities, including the ones found in *Spatial Visualization: Middle Grades Mathematics Project* by Mary Jane Winter, Glenda Lappan, Elizabeth D. Phillips, and W. Fitzgerald (1986, Addison-Wesley).

Structure and Properties of the Real Number System

In the early grades, mathematically talented students need to learn some elements of the real number system, including whole numbers,

fractions, and decimals. In upper elementary school, they need to be exposed to a unit emphasizing the concept of a system along with the teaching of irrational numbers before they begin the formal study of algebra (Johnson, 1994).

An interesting way to enrich the study of numeration and number systems is to study the history of mathematics. By studying other numeration systems, students will better understand the base-10 system (Wheatley, 1983). For example, students can study the number systems used by ancient cultures and can even make up their own number systems. See, for example, a sample lesson described in "The Chinese Numeration System and Place Value" (Uy, 2003). Uy (2003) essentially reiterated what we have known for many years: Understanding place value and relationships among numbers and number systems is essential to forming the foundation for algebraic thinking.

Other Topics

Sheffield (1994) listed many other topics that mathematically gifted students would find challenging. These are included in Table 7.2.

Adapting and Enriching the Mathematics Curriculum in Your School

Is it an understatement to suggest that challenging mathematically talented students is a major challenge for classroom teachers? We think not. In fact, some teachers find it so overwhelming that they totally ignore the situation and do nothing. Teachers who are reading this book recognize that doing nothing is still making a decision *not* to intervene, which is a disservice to the students and to their teaching—they are able to rise to the occasion by adapting and enriching the curriculum. We value the contributions that Johnson (1994), Sheffield (1999b), and Gavin et al. (2009) have made about programming and curriculum.

Sheffield (1999b) indicated that, "A program that helps students develop their mathematical abilities to the fullest may allow them to

Table 7.2
Interesting Enrichment Topics for Mathematically Gifted Students

- Fractals and chaos
- The Pythagorean theorem and Pythagorean triples
- Fibonacci numbers
- Finite differences
- Pascal's triangle
- Golden rectangle
- Magic squares
- Other numeration systems (e.g., Babylonian, Egyptian, Chinese, Mayan, Roman)
- Figurate numbers (e.g., triangular, square, pentagonal numbers)
- Pi
- Topology
- Pendulums and other applied physics problems
- Transformational geometry
- Combinatorics
- Graph theory
- Computer programming and robotics

Note. Adapted from Sheffield (1994).

move faster in the class to avoid deadly repetition of material that they have already mastered" (p. 46).

For mathematically talented students to be challenged in mathematics, the Center for Gifted Education at The College of William and Mary (2004) recommended that students should:

- be exposed to content that is accelerated by one grade level or more, depending on the level of the student's talent and the pace at which they are ready to learn;
- be administered preassessments so they can start at the place that is most appropriate for them;

- have teachers who can raise the level of challenge by assigning the harder problems (usually found at the end of a problem set); and
- have teachers who use the textbook as support rather than the only source of curriculum.

Therefore, both the level and the pace of the mathematics curriculum need to be adjusted. These adjustments can occur in several ways including: (a) through the DT→PI model with individual students or small groups, (b) by compacting the curriculum within the regular classroom, or (c) by adjusting the curriculum and grouping talented students together for math instruction.

Project M³: Mentoring Mathematical Minds (published by Kendall Hunt) is a curriculum comprised of multiple, individual units that were developed through a partnership among gifted and talented teachers, mathematics educators, and mathematicians. The project was funded by the U.S. Department of Education's Jacob K. Javits Gifted and Talent Students Education Act and the units were developed for fourth-, fifth-, and sixth-grade students.

An extensive discussion about the goals underlying this curriculum is available at http://www.gifted.uconn.edu/projectm3/about_overview.htm. In general, Project M³ is part of the suite of enrichment-focused programming offered through the University of Connecticut's Neag Center for Gifted Education and Talent Development. As indicated on the website, the curriculum development was modeled on the Renzulli, Leppien, and Hays (2000) book *The Multiple Menu Model: A Practical Guide for Developing Differentiated Curriculum* and also on the Tomlinson et al. (2002) book *The Parallel Curriculum: A Design to Develop High Potential and Challenge High-Ability Learners.* As presented on the website, these models are used

> as the basis for the design of our curriculum, focusing on the Core Curriculum with the Big Ideas of Mathematics in each unit and the Curriculum of Practice, an outgrowth of the Multiple Menu Model. Using the Multiple Menu Model, we will help students assume the role of mathematicians as

> they develop critical and creative thinking skills in solving real problems. Projects will be included in the units and used as a way for students to pursue some of their own interests. Renzulli's Enrichment Triad Model (Renzulli, 1977; Gubbins, 1995) will be one of the instructional approaches; students will choose a topic to investigate, receive support and coaching from the teacher, and produce a product for a real audience. (Project M^3, n.d., para. 4)

A report of the research finding from an evaluation of the units (see Gavin et al., 2009) is instructive. The authors conducted a quasi-experimental study to determine the impact of the curriculum and concluded that the units were challenging; a replication of the original study demonstrated effectiveness with students from economically disadvantaged backgrounds.

The Center for Gifted Education at The College of William and Mary has also developed some mathematics curriculum units, including *Spatial Reasoning* and *Beyond Base Ten* (by Dana Johnson, available from Prufrock Press). These units are designed for high-ability learners in grades 2–4 and 3–6, respectively. The curriculum units provide an inquiry-based approach to study mathematics and higher level thinking challenges that are appropriate for mathematically talented students. For more information, see http://cfge.wm.edu/Curr_listing.html.

DT→PI Model

The Diagnostic Testing→Prescriptive Instruction (DT→PI) model was discussed thoroughly in Chapter 4. It involves pretesting students and then tailoring instruction based upon the results of the pretest to their needs. Thus, it is a highly individualized program that allows students to study mathematics at the appropriate pace and level.

Adjusting the Curriculum and Grouping Students Together

Winebrenner (2001) suggested developing a "compactor" for each student individually; assessing his or her strengths, weaknesses, and interests; and designing alternative activities for the individual student. Pretesting is an important component of this process. Once mastery is attained, the time saved can be used in a variety of ways. The most important step in this process is selecting appropriate alternate activities.

Although curriculum compacting is considered one of the 18 forms of acceleration (Southern & Jones, 2004), from our perspective, it is a very mild form of acceleration. Much has been written about curriculum compacting and a recent discussion of the topic (Reis & Renzulli, 2009) highlighted its use within the context of the Schoolwide Enrichment Model (SEM) and made the important point that a compactor is useful for individual students as well as small groups of students with similar need for academic challenge. In discussing the replacement of compacted work with acceleration and/or enrichment activities, Reis and Renzulli (2009) emphasized the importance of a student's individual interests and learning style when selecting activities. We suggest that any "replacement" activities offered for mathematically talented students be closely tied to the mathematics curriculum, yet (as also described by Reis and Renzulli) these activities should not be merely busy work that is determined by the teacher in the absence of information about the student's needs and interest. We prefer that students be allowed to move on in the curriculum (an accelerative model), yet we recognize that sometimes this is not possible. If the student does not simply move on to the next chapter or topic, it is important to choose carefully the right kinds of enrichment activities that will be meaningful and challenging. If the time saved is used for the student to go to the library, do an independent project, or play on the computer, is the student really advancing in mathematics or just doing busy work?

Another alternative to differentiating the curriculum for the individual student is to make a conscious, preplanned effort to devise a curriculum that is readily responsive to students' individual needs. This might include selecting a number of students for accelerated program-

ming, as in the school district that selects four or five students each year to take Algebra I as sixth graders, or modifying the curriculum in other ways.

The program offered by the Houston County Schools in Georgia (see Chapter 6), for example, pretests students before each unit to determine what they know and don't know. Based on the results of each pretest, students are assigned to one of three flexible groups: Concept Development, Skills, or Extension. Students in the Concept Development group are given introductory material, students in the Skills group are given somewhat more challenging work, and children in the Extension group are given enrichment. The goal of this program is to develop a curriculum that is more responsive to the students' needs. The drawback of this particular approach is that there is no specific plan for accelerating the students through the curriculum; for example, if students in the Extension group finish their unit before the 6-week time period is up, the teacher is expected to do additional enrichment activities with the students, rather than simply allowing them to move on. The planning team for this program recognizes that they need to include an accelerative component.

NCTM Standards and Their Impact on Mathematically Talented Students

The National Council of Teachers of Mathematics (NCTM) has developed a series of standards for assessing, teaching, and learning mathematics to assist school personnel in devising mathematics curricula for students in kindergarten through grade 12 (see NCTM, 1989, 1995, 2000).

The NCTM (2000) document listed six principles and themes for school mathematics. Despite the fact that the principles do not specifically address the needs of academically talented students, their message is commensurate with our focus on mathematically talented students:

- *Equity.* Excellence in mathematics education requires equity—high expectations and strong support for all students.

- *Curriculum*. A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades.
- *Teaching*. Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- *Learning*. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- *Assessment*. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
- *Technology*. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. (NCTM, 2000, p. 11)

Beginning with the publication of the 1989 Standards, NCTM has expanded its efforts to reach teachers of mathematics in grades K–12. In 2006, NCTM published *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence*. In essence, the focal points identified important mathematical topics for each grade level through grade 8. Coincidentally, the NCTM (2006) team that developed the *Focal Points* recognized the irrelevance of the question: "What should I do in class on Monday?" (p. 7), while simultaneously recognizing the need for consistency for instruction at each grade level as well as articulation among grade levels. Used in combination with the *Principles and Standards*, teachers have at their fingertips a powerful set of materials to direct their efforts for challenging mathematically talented students.

However, teachers of mathematically able students will need to recognize their students may be ready for many of the topics presented in the *Focal Points* (NCTM, 2006) well before the grade level for which the topic is typically presented. For example, the grade 8 curriculum focal point for algebra is: "Analyzing and representing linear functions and solving linear equations and systems of linear equations" (NCTM, 2006, p. 20). Thus, this focal point may be more appropriate for a

highly capable fifth grader than the grade 5 focal point, "Number and Operations and Algebra: Developing an understanding of and fluency with division of whole numbers" (p. 17).

How Do We Know When They're Ready to Study Algebra?

In the late 1980s, Wheatley (1988) addressed this question as an issue of curricular implementation. He acknowledged that the popular approach is to offer algebra at the eighth-grade level for gifted students and to open the course up to younger, exceptionally precocious students. But, that begs the question of when are exceptionally precocious youngsters ready to study algebra? Students are ready to study algebra when they have formal reasoning skills; for example, when they can think abstractly and use symbols as part of their problem-solving strategy (Baldwin, 1980). This may happen for mathematically gifted students much sooner than the age of 13 or 14, when the average adolescent has developed these skills and algebra is typically introduced. The key is not the age or the grade, but the reasoning skills of the youngster (Keating, 1973).

The work by the Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins University has been useful in helping school personnel determine when students have the formal reasoning skills needed to take a rigorous Algebra I course. SMPY defined readiness as having earned a score of 500 or above on the mathematics section of the SAT I before age 13 (see Lupkowski-Shoplik et al., 2003).

Algebra has been seen as the "gateway" to higher level mathematics and challenging science courses (National Mathematics Advisory Panel, 2008). The typical progression is Algebra I, Geometry, Algebra II, Precalculus, and Calculus. If an academically talented student waits until ninth grade to take Algebra I, he or she will be unable to study calculus until entering college unless the student takes two mathematics courses during the same year in high school (Lupkowski & Assouline, 1992).

Usiskin (1999) offered an interesting discussion about developing mathematically promising students. He noted that, 20 years ago, eighth graders taking Algebra I were seen as gifted because only 13% of the eighth-grade population took that course. In more recent years, Algebra I has been offered more routinely to eighth graders; in fact, in some schools, as many as 50% of eighth graders take Algebra I, and, "in those schools, no one sees eighth-grade algebra as a sign of giftedness any longer" (Usiskin, 1999, p. 62).

Now that the National Mathematics Advisory Panel recommends that all students study algebra by grade 8, the implications for mathematically talented students are clear. The trend described by Usiskin (1999) coupled with the National Mathematics Advisory Panel recommendation will not only impact the curriculum of the lower grades, thus requiring that teachers expose elementary students to a more challenging prealgebra curriculum but it may also mean that, finally, mathematically gifted students may be presented with more appropriately challenging curricula while in the upper elementary grades so that they can take Algebra I when they are ready for it, perhaps as early as sixth grade and, in some unique cases, as early as fifth grade.

Concepts vs. Computation

As discussed in Chapter 6, mathematically talented students may not demonstrate equal strengths in mathematics concepts and computation tasks. In fact, Rotigel (2000) documented our observation that mathematically talented students tend to perform better on measures of mathematical concepts than on measures of mathematical computations for a variety of reasons. The programming and curricular implication for this is not to hold students back from studying high-level concepts if their computational abilities are somewhat less developed. As Sheffield (1999b) stated so eloquently,

> Students who are challenged to find the answer in as many ways as possible, to pose related questions, to investigate interesting patterns, to make and evaluate hypotheses about their

> observations, and to communicate their findings to their peers, teachers and others will get plenty of practice adding two-digit numbers, but they will also have the chance to do some real mathematics. (p. 47)

Materials

Textbooks

Traditionally, textbooks have been the main teaching materials for mathematics. Although they may also use supplemental materials, teachers prefer to use a textbook as their "roadmap" for the school year. Many of the mathematics textbooks in the United States today are based upon the spiral curriculum, in which students are exposed to a topic one year, and then they return to it again in future years, presumably at a higher level. This means that first graders learn one-digit addition, second graders learn two-digit addition, and third graders learn three-digit addition. This approach is not the optimal one for mathematically talented students because they do not require this type of repetition and review in order to master a concept. Because the standard curriculum is so repetitive, students do not study much new material, especially in the middle school years. For example, in a classic study, Flanders (1987) examined three popular textbook series used in many schools and found the average number of pages offering new material for each year was quite low. He found that, for grade 4, only 45% of the material was new, while for grades 5 and 6, the percentages were 50 and 38, respectively.

When selecting textbooks, we concur with the recommendation of Johnson (2003) as well as Reys, Reys, and Chavez (2004) that educators should select textbooks that stress concepts, not computation. We also recommend using pretesting extensively, especially in the early chapters of the book, which tend to be review chapters and/or have little to no new material. Students who demonstrate mastery of those review topics should be allowed to move on (Lupkowski & Assouline, 1992).

Textbooks that are concise, contain clear explanations, provide examples of completely solved problems, and contain large numbers of practice problems that could be assigned for homework are ideal for a student who participates in the DT→PI process (described in Chapter 4). If the answers to about half of those practice problems are provided in the text, students can check their own work (Bartkovich & George, 1980).

An important point that teachers should remember is that they do not have to start on page 1 and go through each and every page. They can selectively use portions of the textbook. Pretesting students will allow teachers to determine what they already know, and they will be able to save valuable time by not spending excessive time reviewing "old" material.

Reys et al. (2004) reported that when it comes to size and amount of content covered, U.S. textbooks are unlike those used in other countries. Because textbook publishers rarely delete "old" material, despite the addition of new material in response to state and local requirements, U.S. textbooks are filled with substantially more content. However, at least in the early part of the 21st century, that content is not necessarily based on national curriculum standards. Some publishers reportedly take the NCTM (2000) *Principles and Standards* into consideration; however, as a country, our emphasis on local control over decisions tends to trump national standards.

One can only hope in the next 10 to 15 years that the next generations of textbooks are influenced by the work of national organizations such as NCTM and the National Association for Gifted Children (NAGC). If materials such as the NCTM (2006) *Focal Points* are used, then perhaps we will eliminate the current situation in which:

> U.S. mathematics textbooks have a significant amount of repetition and introduce only one or two new topics each year from 4th to 8th grade, whereas other countries introduce between six and seven new topics during this period. Excessive repetition of material leads to shallow treatment of mathematics and fails to stimulate student interest or challenge. (Reys et al., 2004, p. 63)

A Few of Our Favorite Math Books

Earlier in this chapter, we mentioned one of our favorite math books, *Creative Problem Solving in School Mathematics* by George Lenchner (1983, Houghton Mifflin, available from http://www.moems.org). C-MITES and the Belin-Blank Center have used this book for more than 15 years with great success. Written by the founder of the Mathematical Olympiads for Elementary and Middle Schools, it is an excellent problem-solving book. It doesn't require algebra, but the problems are high level and challenging. We have successfully used it with gifted third through sixth graders. Related to that are two other books: *Math Olympiad Contest Problems for Elementary and Middle Schools* by George Lenchner (1997, Glenwood Publications, available from http://www.moems.org) and *Math Olympiad Contest Problems Volume 2* edited by Richard Kalman (2008, available from http://www.moems.org).

Another favorite is Edward Zaccaro's *Challenge Math* (2000, Hickory Grove Press). This book is packed with lots of fun, tough problems for teachers and students who love challenging math. An experienced teacher, Zaccaro wrote this book for mathematically talented fourth through eighth graders. The book contains more than 1,000 problems in 19 chapters/units. The emphasis of the book is on problem solving, and solutions are available in the back of the book. There are three levels in each chapter, the most challenging level being the Einstein Level. The purpose of the book is to understand the meaning of math in the world. Zaccaro suggests that teachers use Chapter 18 as a test chapter to see if students are ready for it. The Belin-Blank Center has used this book in its Challenge Math program. It might also be useful for homeschoolers.

Finally, for both teachers and students, we really like *Mathematics: A Human Endeavor* (3rd ed.) by Harold Jacobs (1994, Freeman), which is appropriate for mathematically gifted students in middle school. Chapters include Mathematical Ways of Thinking, Number Sequences, Symmetry and Regular Figures, Methods of Counting, and Topics in Topology. We have also used this book in our teacher-training programs at C-MITES as a way of enriching teachers' own understanding of mathematics and giving them confidence in teaching mathematics.

The Resources section lists many more mathematics books for mathematically talented students and their teachers. In addition, websites, math games, puzzles, and other curriculum resources are included.

Computer Software

We try to use certain rules of thumb when selecting software for mathematically talented elementary and middle school students. First, we like software that has a variety of components so that the level of challenge can be varied. Second, we look closely at the level of challenge. Often, if a piece of software is rated as appropriate for students in grades 6–8, for example, mathematically gifted students in fourth and fifth grade might be challenged by it. When selecting software that has been designed for older students, though, it is important to look at the language level, violence, and appropriateness of content for a younger student.

We prefer software that is intuitive and user-friendly, where students can just jump in and get started without having to read a lot of directions. It should be easy to access the help mode and to get answers to questions as students work through the software. It should also be varied enough so that students remain interested in working on it. We try to find software that doesn't require a lot of reading, especially for younger students, because their reading skills may be relatively less developed than their mathematics skills. Other important aspects of the software include quality of graphics, what types of teacher materials are provided, and the overall presentation.

We have chosen only a few pieces of software to present here. The Math Forum website offers a page of links to other websites that offer software reviews (http://mathforum.org/mathed/math.software.reviews.html) so readers can obtain information about the latest software. The Hoagies Gifted Education page also offers up-to-date software reviews: http://www.hoagiesgifted.org/software.htm. Below are listed some of our favorite pieces of software.

Zoombinis Logical Journey (The Learning Company; appropriate for gifted fourth graders and older. Available through http://www.

amazon.com). Students in the C-MITES program have used this award-winning, fun piece of software for quite a few years, and it is a favorite. The Zoombinis are imaginary creatures who must escape from the evil Bloats, who have taken over their island. The Zoombinis face obstacles they must overcome using the player's logical reasoning skills. They go through 12 puzzles, with four levels of difficulty each. For example, in the "Pizza Path" section of the game, the Zoombinis must arrange combinations of pizza and ice cream toppings that are acceptable to one, two, or three evil characters. The first character might like plain pizza with ice cream topped with a cherry, while the second one prefers pizza with everything and plain ice cream. Players must figure this out by first using trial and error and then logical reasoning. Our students found this very challenging, and they used paper and pencil to make tables and charts to keep track of what was accepted and rejected. Solving Zoombinis puzzles requires good mathematical thinking. Students practice algebraic thinking, data analysis, graphing and mapping, logical reasoning, pattern finding, problem solving, statistical thinking, and theory formulation and testing. We have used this software in a classroom situation, but it is also marketed for home use. A parent's guide explains the educational approach behind each puzzle.

Descartes' Cove (http://cty.jhu.edu/ctyonline/cove/index.html). This software was developed by the Center for Talented Youth at Johns Hopkins University. Players find themselves marooned on a deserted island and begin a journey through island tunnels, volcanoes, and other surprises. They solve increasingly difficult math puzzles and problems, earn gold coins, and make entries into their journals. Their work culminates in a final quest to build a means to escape from the island. This software includes hundreds of challenging problems that include animations and interactive features (including a geoboard, 3-D rotation, and a moveable ruler and protractor). The software is suitable for home or classroom use and features math challenges based on NCTM standards.

Mathville and Mathville Middle School (Courseware Solutions; http://www.mathville.com; appropriate for gifted students in grades 2–4). Students use their math skills as they travel through Mathville

completing tasks; the harder the task, the greater the rewards. The "money" students earn can be spent in a bakery, a food store, or an art store. Skills include numbers and operations, data management, geometry/measurement, and problem solving.

Hot Dog Stand: The Works (Sunburst; http://www.sunburst.com; appropriate for grades 5 and up). Players manage a hot dog stand. They keep records, determine prices, and plan marketing strategies. Players practice mathematical problem-solving skills and communication skills, including arithmetic operations, estimation, data gathering, data analysis, interpreting graphs, recognizing patterns, working backward, writing, choosing appropriate tools, and translating information into different forms. Players analyze data to make informed purchasing and pricing decisions. They may use the estimator, electronic checkbook, franchise report, and word processing tools as they work. They also must handle unexpected events as they operate their hot dog stand.

Math Munchers Deluxe (MECC; appropriate for grades 3–6; available on http://www.amazon.com). This program gives students a chance to practice basic math skills as it simulates a TV game show of math problems. Players are presented with problems on the board and they must direct a Muncher to the correct answer. Examples of problems include "Find the factors of 3," "Find the proper fractions," or "Select the prime numbers." Players can change the game to allow bad critters to interfere with the Muncher's work. At the beginning of the game, the player is given four Munchers. If the player selects the wrong answer or allows the Muncher to run into a Troggle (the bad critter), the player loses a Muncher. Math Munchers Deluxe is highly entertaining, so children will want to play it. It is best suited for individual or small-group use.

Manipulatives

> Manipulatives are tools for problem-solving and a means to represent and embody mathematical thinking. . . . Sometimes children, particularly math-talented children, are heard to say they dislike manipulatives and find that they hinder or slow

> down their mathematical thinking. However, when manipulatives can be used flexibly and creatively to solve problems and to embody and communicate meanings, resistance to their use melts away and even very bright children become enamored with their possibilities. (Waxman et al., 1996b, p. 32)

Teachers shouldn't be discouraged by the vast array of manipulatives from which to choose. Admittedly, they are overwhelming (and expensive). At the same time, it's relatively easy to make your own math manipulatives from everyday materials. These simple materials can be just as challenging (or even more so) than expensive manipulatives kits. One book that might help teachers sort out the information about manipulatives is *Activity Math: Using Manipulatives in the Classroom, Grades 4–6* by Anne Bloomer and Phyllis Carlson (1992, Dale Seymour). Some of our favorite manipulatives include:

- cubes and blocks for studying classification, place value, area, volume, and operation on whole numbers;
- tangrams, which are geometric shapes that can be put together to form a rectangle or square, and to make designs; tangrams reinforce copying, matching, problem solving, and logical thinking skills;
- counters and items that can be sorted and grouped (such as small plastic toys, buttons, and Bingo chips) and used for studying sets, bases, and statistics;
- dice and cards for probability experiments;
- Cuisenaire rods, classic manipulatives used for counting, studying fractions, and studying different bases;
- weights;
- protractors, rulers, and compasses;
- fraction bars;
- geoboards;
- base-10 blocks; and
- dominoes.

Essential Features of Curricula for Mathematically Talented Students

The type of curricula selected depends, in part, on the type of program or delivery system in place for educating mathematically talented students. The curricula would be very different in a supplemental pull-out program that meets once a week compared to a homogeneously grouped class for mathematically talented students that meets on a daily basis. The following is a list of questions to ask when evaluating curricula and programs for mathematically talented elementary and middle school students.

- Is the mathematics program a part of the regular math curriculum (Belcastro, 1987)?
- Is there a specific scope and sequence for mathematically talented students (Lupkowski & Assouline, 1992; VanTassel-Baska, 1998)?
- Do the students have early access to high-level math concepts (Reys et al., 2004; VanTassel-Baska, 1998)?
- Are students given daily instruction in mathematics (Belcastro, 1987)?
- Are students given the opportunity to work with other mathematically talented students (Belcastro, 1987; Sheffield, 1994)?
- Do the materials contain a high level of sophistication of ideas (Johnson, 1994)?
- Do the materials contain extensions that challenge the most able learners (Johnson, 1994)?
- Is the material presented at a faster pace than in the typical classroom (Belcastro, 1987; Johnson, 2003)?
- Can the content be tailored to students' individual needs, for example, are problems coded by difficulty level (Johnson, 1994)?
- Are higher order thinking skills (analysis, synthesis, and evaluation) essential to the lesson (Johnson, 1994; Reys et al., 2004)?
- Once mastery level has been demonstrated, do the materials de-emphasize basic skills (Johnson, 1994)?

- Are students asked to go beyond the concrete, and is abstraction encouraged (Johnson, 1994)?
- Do the materials contain opportunities for exploration and extension activities based on students' interests (Johnson, 1994)?
- Do students have the opportunity for independent research and investigations (Sheffield, 1994)?
- Are there opportunities to create products that are open-ended and use advanced strategies (Belcastro, 1987; Johnson, 1994)?
- Have the teachers received specialized training in differentiating curricula for talented students (Belcastro, 1987; Reys et al., 2004)?
- Do the curricular materials encourage students to ask questions and make generalizations that go beyond the original problem (Sheffield, 1994)?
- Is achievement measured in a variety of ways, including observations, interviews, demonstrations, portfolios, and open-ended questions (Sheffield, 1994)?
- Do students have in-depth opportunities for real-world applications (Reys et al., 2004; VanTassel-Baska, 1998)?
- Do the students have the opportunity to use math concepts in other subject areas and as a tool for conducting research (VanTassel-Baska, 1998)?
- Do major themes frame the curriculum, or are they integrated into the materials (e.g., problem solving, change, models; Johnson, 1994; Reys et al., 2004)?

Conclusion

The goal when selecting a curriculum for mathematically talented students is to provide a challenging, content-filled program that allows them to move steadily through mathematics. This includes opportunities to reason, relate ideas, formulate and solve problems, become competent with computers and calculators, and learn when and how to estimate. At the same time, students establish a firm foundation

of mathematics concepts, principles, and rules. This chapter presented a rationale for developing a differentiated curriculum and resources for developing it, as well as guidance in selecting curricula from the already-existing materials for mathematically talented students.

Regular classroom teachers in elementary and middle school may not have the necessary background for developing and delivering a curriculum for mathematically talented students. Also, the fields of mathematics and mathematics education are changing, and we need teachers who can respond to those changes. All of these ideas speak to the specific point: More professional development is needed for in-service and preservice teachers. That is the topic of the next chapter.

CHAPTER 8

Teaching Mathematically Talented Students

KEY POINTS

- Teachers, along with parents, play a critical role in the academic and social-emotional development of students.
- The role of today's teachers has expanded to include mentor, counselor, content expert, and advisor.
- Despite this expanded role, or perhaps because of it, many teachers today do not feel prepared to work with mathematically talented children.
- Compared to past generations of teachers, present-day teachers are faced with many significant philosophical and pedagogical issues, including acceleration as an intervention, standards-based curriculum, and testing as it relates to the accountability required in No Child Left Behind (NCLB, 2001).
- Other issues include grade level at which math topics should be introduced, which specific techniques for teaching mathematical concepts are best, and how best to incorporate technology into the learning environment.
- The current teaching climate is impacted by both national and international reports about the success, or lack thereof, of our students in the science, technology, engineering, and math (STEM) areas, especially math and science.

The whole art of teaching is only the art of awakening the natural curiosity of young minds for the purpose of satisfying it afterwards.

—Anatole France

Every reader can quickly remember his or her favorite teacher as well as his or her worst experience with a teacher. Clearly, teachers influence their students, and we believe that teachers strive to influence their students positively, as expressed in the introductory quote ("awakening the natural curiosity of young minds"). However, the ideal is tempered by the reality that in many of today's classrooms curiosity and new learning are diminished, rather than enhanced. Because so many bright students express dissatisfaction with the current state of education, parents and other community members often become pessimistic about the education of their children and sometimes malign the role of the individual, which is unhelpful to all parties.

Despite current levels of dissatisfaction, there are significant developments in the broad field of education that may help educators attain the ideal goal of teaching: satisfying the learner's curiosity. We are optimistic about the potentially positive impact of these developments and, in our view, optimism trumps pessimism! Therefore, we will concentrate on reporting and discussing the advances that have been made in education and the ways in which these advances can influence specifically the instruction of the mathematically gifted child.

The field of teacher training is complex, and the psychological and physical demands on teachers are exacting. In this chapter, we present our views about teachers and their professional development from the multiple perspectives of parent, teacher, administrator, learner, and teacher trainer. As parents, we have firsthand experience with the fact that parents are their children's first teachers and remain teachers throughout their children's lives. As former classroom teachers, we are highly cognizant of an educator's responsibility for addressing the individual needs of students while optimizing the learning experiences of an entire class. In our present capacity as administrators for specialized university-based classes, we are responsible for finding the best teachers to teach in our programs. We also aspire to be lifelong learners, with some of our most favorable learning opportunities emanating from our interactions with talented students and their parents and teachers. It is through our work as teacher trainers (i.e., the preservice training of undergraduate students in teacher preparation programs), as well

as in-service work with classroom teachers who, in order to improve their knowledge and skills, attend classes, workshops, and seminars or graduate students who are in master's or doctoral programs, that we are able to view most optimistically the evolving landscape of teacher-student relationships.

This chapter begins with a discussion of the philosophical and pedagogical concerns for teachers of mathematically talented students enrolled in preschool through middle school. The philosophical concerns typically mirror those concerns expressed when a student is accelerated beyond his or her age-mates. The pedagogical concerns relate to teaching issues and range from specific topics such as methodology for curricular implementation, to more nebulous topics such as "sparking the joy of learning." We continue with a review of recent and contemporary educational trends, including international comparisons within the context of a standards-based curriculum, implications for the STEM workforce, and educational reform as it relates to curriculum and programming initiatives designed to address the needs of all students. We offer observations on the effects of these trends or movements on the instructional goals for mathematically talented students.

The Preschool Years and the Impact of Parents and Teachers on the Mathematically Gifted Child

The years before a child enters kindergarten are magical years, during which time parents rediscover the joy of learning something new through their child's first experiences. Parents of a mathematically precocious child often are astounded with their child's early fascination with numbers. This fascination typically precedes any interest in numbers displayed by a child's age-mates, and often will exceed that of the child's peers throughout the school years. In the next chapter, we describe Rosie's propensity to count and arrange things by groups well before kindergarten. Christopher also preferred stories that involved numbers and counting. Many parents respond to their child's curiosity similarly to Rosie and Christopher's parents: They encourage and

provide for individualized learning opportunities. For example, Sarah was first introduced to division on a napkin in a donut shop when she was 3 because, as her mother said, "That's where it came up!"

Preschool teachers, like parents, are usually attuned to the developmental needs of their students and have the flexibility to individualize and personalize the curriculum so that the mathematically precocious preschool student will typically experience much satisfaction with the preschool setting. Combined, our work with mathematically talented students spans nearly 50 years and, to date, we find that preschool teachers are more responsive to their students' needs for individualized and developmentally appropriate curricula than their elementary and middle school colleagues. The phone calls—mostly from parents, but some from teachers—begin with the first day of kindergarten, when the child (and parent) is first introduced to the standardized, grade-level curriculum. If the kindergarten curriculum requires that students learn to count from 1 to 100, then that is what they are required to do, even if they are already doing addition, subtraction, and multiplication. It is almost as if a child enters an elementary school, and *bam!*, the door to curiosity, exploration, and individualization slams shut.

An extensive discussion concerning curricula for mathematically talented students was presented in Chapter 7. Our major point here is that teachers have been trained to follow a set curriculum developed for implementation as a lock-step sequence by grade level (rather than ability or aptitude level). When efficiency (i.e., teaching large numbers of students) is the goal, the standardized curriculum meets the purpose. However an effective standardized curriculum for large numbers of students comes at a cost to both the individual students and the teacher. Csikszentmihalyi et al. (1993) highlighted the cost:

> the role of the teacher has largely evolved from a practitioner in a domain into that of a transmitter of information . . . modern curricula . . . tend to depersonalize even the transmission of information. In the name of diversity and efficiency, relations between teachers and students are kept highly specialized, pro-

> grammatic, and brief. The rapid transfer of facts to masses of learners is stressed, not the slow cultivation of a unique individual's diverse gifts. The roles and goals available to teachers within these standardized curricula are in turn impersonal. They emphasize external mass performance standards, the delivery of uniform services, and the insulation of the curriculum from the interests of the particular person conveying it. (p. 178)

This observation captures the transition from the free-spirited, curiosity-oriented years of the preschooler, where the teacher's role is as much guided by the child's interests as it is to guide the child's interests, to the lock-step, standardized years of elementary and middle school, where the child's interests may be superseded by a predetermined curriculum. In the elementary years, the student's exposure to curricula is not guided by his or her needs; rather, a curriculum committee predetermines it.

The Elementary Years and the Impact of the Teacher on the Mathematically Gifted Child

In a landmark, now-classic, study of the development of talent, Benjamin Bloom (1985) provided a retrospective look at the influence of the elementary classroom teacher on the future of individuals who became prestigious mathematicians: "In general, the mathematicians were not at all enthusiastic about their elementary school experiences" (p. 289). Bloom explained this finding as a symptom of the condition that the average elementary school teacher is not trained in methods for identifying or instructing students with exceptional abilities. Therefore, the special abilities of these mathematicians remained undiscovered and, therefore, underdeveloped during their elementary years. The mathematicians in Bloom's study considered that the "best [elementary] teachers were seen as the ones who would supply books or materials so that the mathematicians could work on their own" (Bloom, 1985, p. 293). We do not interpret these results to mean that independent work in

lieu of an appropriately challenging curriculum is the optimal situation; rather, the students Bloom studied were so far advanced compared to their classmates (and their teachers) that the independent work was better than sitting through a curriculum that was not challenging.

Since the publication of Bloom's (1985) study, several generations of students and teachers have gone through the U.S. educational system. How much has changed for these subsequent generations? Apparently, the changes have been minimal. The report, *Foundations for Success* (National Mathematics Advisory Panel, 2008) thoroughly examined mathematics education in the U.S. and among their conclusion is the negative finding that:

> International and domestic comparisons show that American students have not been succeeding in the mathematical part of their education at anything like a level expected of an international leader. Particularly disturbing is the consistency of finding that American students achieve in mathematics at a mediocre level by comparison to peers worldwide . . . only 23% [of American students] are proficient at Grade 12. Consistent with these findings is the vast and growing demand for remedial mathematics education among arriving students in four-year colleges and community colleges across the nation. (p. xii)

Keep in mind that this significant report concerns *all* mathematics education; through hypothetical extrapolation, it is easy to conclude that the mathematical education of the mathematically gifted student is seriously at risk. Recommendation #30 (out of 45 recommendations) addresses the needs of mathematically gifted students: "Mathematically gifted students with sufficient motivation appear to be able to learn mathematics much faster than students proceeding through the curriculum at a normal pace, with no harm to their learning and should be allowed to do so" (National Mathematics Advisory Panel, 2008, p. xxiv).

The National Mathematics Advisory Panel's (2008) message about content acceleration in mathematics echoes K. B. Rogers (2007) meta-analytic-based conclusion that, "For specific curriculum areas

[mathematics] instructional delivery must be differentiated in pace, amount of review and practice, and organization of content presentation" (p. 390).

Seven recommendations from the National Mathematics Advisory Panel (2008) relate to teachers and teacher education. Recommendation #19 states:

> The mathematics preparation of elementary and middle school teachers must be strengthened as one means for improving teachers' effectiveness in the classroom. This includes preservice teacher education, early career support, and professional development programs. A critical component of this recommendation is that teachers be given ample opportunities to learn mathematics for teaching. That is, teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach. (p. xxi)

The aim of enhancing teacher expertise in mathematics waxed and waned during the latter half of the 20th century. When the former Soviet Union launched Sputnik in 1957, the U.S. government expanded programming and funded numerous grants that resulted in improved math, science, and foreign language curricula. Funding was also available to encourage a generation of individuals in science and math professions to become teachers. However, for a number of political and sociological reasons, the level and energy associated with post-Sputnik funding for professional development all but disappeared during the late 1960s and 1970s. Twenty-first century thinking, as reflected in publications like the *Foundations for Success* (National Mathematics Advisory Panel, 2008) or *A Nation Deceived: How Schools Hold Back America's Brightest Students* (Colangelo et al., 2004a, 2004b), lend optimism that we are in a period of increased attention to the needs of mathematically gifted students *and* their educators.

Expertise in mathematics content knowledge of teachers matters; however, content expertise is not the entire story when it comes

to effective pedagogy. Neither is an exclusive focus on the type of instruction (e.g., small group or individualized, teacher centered or teacher directed). In Chapter 9, you will meet Zach, who is currently a graduate student in physics. In his interview, he made the following comment, which is relevant to the discussion in this chapter:

> One area in which I feel as if the performance of my elementary teachers was relatively uneven was how they treated gifted children in their personal interactions. I think it is generally counterproductive for teachers to give the impression to the student that they are ignoring or disrespecting the student's abilities. Perhaps the best way is to congratulate students on their accomplishments and to suggest that more is expected of them. I think that by far what upset me the most was when I felt (rightly or wrongly) that a teacher was condescending to me. Teachers can't treat students like adults because they aren't, but I think they ought to lean further in that direction when dealing with gifted students than they might suspect. Children obey teachers they respect, and they respect teachers [who] respect them. It's probably inescapable that the gifted student will think they're smarter than the teacher, but the teacher shouldn't take it personally.

In the next section, we describe a study in which students teach educators some important lessons about teaching and the relevance of a teacher's attitude.

The Middle School Years: How the Teacher and the Classroom Culture Make or Break the Student

Csikszentmihalyi et al. (1993) wrote a powerful book, *Talented Teenagers: The Roots of Success and Failure*. This book is the report of a quasireplication of Bloom's (1985) seminal, retrospective study of talented individuals. Just as Bloom's study is a classic in the field of

gifted education, so too is the Csikszentmihalyi et al. study. The focus of this study was a group of bright adolescents who were talented in one of five areas: math, science, art, music, or athletics. Even though the subjects of the Csikszentmihalyi et al. study represent a narrower age range than the focus of our book and the talent domains represent a broader picture than our specific focus on mathematics, we have read this book many times and found it to be a tremendous resource, both practically and philosophically, for individuals interested in developing the mathematical talent of elementary and middle school students.

This book is potent because it answers the following question: Why do some gifted teenagers cultivate their talent while others, who are equally talented, give up? To answer the question, data were analyzed from 200 talented teenagers to identify major factors involved in engagement and disengagement with the teens' talent areas. Because understanding the factors connected with involvement or dropping out is relevant for teachers who participate in the academic lives of talented teens and talented preteens, we briefly describe some of the results from the *Talented Teenagers* chapter devoted to the complex relationship among talent development, schools, and teachers (Chapter 9).

Csikszentmihalyi et al. (1993) made many poignant points regarding the role of teachers in the lives of their students and introduced the relationship between the student's talent and the role of the teacher as follows:

> Natural ability is a great advantage in learning to enjoy a field of talent. It is the key that unlocks the potential for *flow* [emphasis added] in activities that others experience as difficult, tedious, or boring . . . a time comes when the exercise of pure ability must give way to the guidance of adults. (p. 177)

Flow, as described in Csikszentmihalyi's (1990) opus on the psychology of optimal experience, is the interaction between ability and passion. Flow happens when an individual is so totally engrossed in an activity that time seems to be suspended. Csikszentmihalyi et al.'s (1993) discussion about the role of the teacher in the academic lives of students continues with the paradoxical observation that math teachers

are often "industrious and even inventive purveyors of knowledge across diverse domains [however] they are rarely practicing mathematicians" (p. 177). This finding is commensurate with Bloom's (1985) findings from the previous generation of talented mathematicians, and, in our opinion, this gap between the practice of teaching mathematics and the practicing mathematician may partially explain why so many teachers are neither interested in discovering mathematically talented students nor in developing the mathematical talent of students whose learning needs are clearly outside of the parameters of the regular standardized curriculum.

Thus, the fact that today's mathematics teachers are rarely mathematicians is important to understanding how to improve the skills of the teachers of talented students (i.e., increase their content expertise). However, even that observation, according to Csikszentmihalyi et al. (1993), is not the main reason that talented students remain unmotivated in the typical classroom setting. The main reason, which will be described below, is due to a concept that is less tangible than curricular content: teacher attitude regarding the role of the teacher in his or her students' lives.

Rather than concentrate on what it is about schools that disengage students, Csikszentmihalyi et al. (1993) focused on what it is about teachers who are able to engage their students. The critical concept is that teachers who are able to motivate their students have a great deal of content expertise, but equally—or perhaps more importantly—they have the ability to transcend the traditional attitude about the teacher's role of competent information-giver to that of an individual who serves as "a daily model of the deeply interested life, a life of full participation in the world of challenges afforded by the domain [mathematics]" (Csikszentmihalyi et al., 2003, p. 178). Thus, while we strongly support a teacher's pursuit of additional expertise in a subject area as a means to more effective teaching, we maintain that adding more knowledge about math will only superficially improve the learning environment for mathematically talented students. In fact, expertise that is translated into a strict, rigid, authoritarian teaching style will do little to enhance the motivational culture of a learning environment.

The key to motivating students is being an effective role model who demonstrates a passion for learning that is translated into action. An effective role model is comfortable with a shift in power from the traditional role of teacher as an impersonal provider of information to the personal approach that ultimately empowers the student to a lifelong love of learning within the talent area. An effective role model establishes a positive classroom culture that motivates students.

The culture of the classroom also impacts the motivation of students. With such a simple statement, it may seem as if we are overstating the obvious, but that is not our intent. It is important to understand the more subtle psychology that underlies a motivating environment. Skill, challenge, and ability are closely intertwined in the psychology of the student, and unmotivating learning environments occur when there is an absence of both challenge and the "sense of exercising skill." So, why not insist that all classrooms have extremely rigorous content and be done with the discussion? Csikszentmihalyi et al. (1993) found that it takes more than rigorous content to motivate students. This is not to say that the talented teens were not focused when the content was rigorous; compared to average teens, they were very focused. However, focus—or attention—is but one component of motivation that needs to be considered when discussing the talented teen. The companion to focus is volition.

Volition, or desire, is also necessary for the positive energy associated with motivation. In other words, students have to concentrate on the curriculum, and they have to desire to concentrate on the curriculum. In many traditional classrooms, especially when the content is rigorous, attention and volition are viewed as mutually exclusive. Students are expected to focus on rigorous content because of its rigor and challenge. *Why* they desire to focus on this content is not often considered. A rigorous curriculum is important for getting the students' attention, but the way in which the teacher presents the curriculum is critical to their volition. Interestingly, extracurricular activities represent the one learning situation in which volition and attention are equally and consistently high among talented teenagers. The moral of the story is not to schedule regular classes as extracurricular activities.

Rather, structure the environment of the classroom similarly to that of extracurricular activities where volition and attention are companions.

Is there a formula for an excellent middle school teacher? At this point, we ask the reader to stop for a moment and try to recollect your feelings about school during your adolescent years. We suspect that many recollections will parallel the findings of the *Talented Teenagers* (Csikszentmihalyi et al., 1993) study: Teens are rarely in a focused and positive frame of mind when they are in class. There are many factors that contribute to this condition, most of which are beyond the control of an individual teacher. However, teachers do have control over their teaching style and the content and pace of the curriculum.

Now, pause again and recall your favorite teacher during your teen years. Why was that person your favorite teacher? Because of the rigor of the content? Because of the teacher's passion for the content? What did that teacher do to make you recollect him or her now? The teens in *Talented Teenagers* (Csikszentmihalyi et al., 1993) were uniquely sensitive to the style and quality of teaching. They were very dynamic when referring to their most and least favored teacher, and they offered specific criticism: "Math and science students in our study complained most often of the standardization and rigidity of the curriculum and of their teachers' reluctance to deviate from highly structured programs" (Csikszentmihalyi et al., 1993, p. 194). They also had specific examples of their preferences:

> [T]eenagers—talented and average—are captivated by examples of adults, such as star athletes and entertainers, who enjoy what they do and achieve fame and riches in the process. More surprising is the ability of some exceptional teachers to find a permanent place in their students' memories. What most intrigues students about these teachers is their enthusiasm for subjects that seemed boring and purposeless in other teachers' classes. Memorable teachers challenge students to expect more than just recognition or a paycheck from the work they choose . . . memorable teachers might be thought of as alchemists of consciousness whose art lies chiefly in transmuting abstract symbol systems into problems that matter to students . . . teach-

> ers who enjoy their subject have a distinct advantage in being able to focus the attention of their student. (Csikszentmihalyi et al., 1993, pp. 184–185)

Memorable teachers present a complex curriculum that is matched to their middle school students' abilities while stretching their students' skills beyond their current grasp of content. Contrary to popular opinion that students want to take easy courses so that they are not too stressed, Csikszentmihalyi et al. (1993) found that students had

> an avid willingness to accept challenges and overcome obstacles when the problems were interesting and the necessary skills were within the individual's reach. Becoming engaged in such challenges yielded useful feedback about ability, and permitted deep and sustained immersion in activities that began to become rewarding in their own right. (p. 186)

Students expressed sentiment such as, "I like math most when there is a hard problem and I can figure it out. When it's really hard at first and then I look at it and see the light—that's when I like it" (Csikszentmihalyi et al., 1993, p. 187). This student was expressing what Robert Browning so aptly phrased: "A [person's] reach should exceed his [or her] grasp or what's a heaven for?"

In sum, effective classrooms are free from extraneous distractions, students are exposed to challenging curricula, and teachers provide informed and timely feedback concerning student performance. But, that is not the entire formula for effective teaching:

> [W]e assume that if the material is well organized and logically presented, students will learn it. Nothing is farther from the fact. Students will learn only if they are motivated . . . Unless a person enjoys the pursuit of knowledge, learning will remain a tool to be set aside as soon as it is no longer needed. Therefore we cannot expect our children to become truly educated until we ensure that teachers know not only how to pro-

> vide information, but also how to spark the joy of learning. (Csikszentmihalyi et al., 1993, p. 195)

Sparking the Joy of Learning Within the Current Culture of Schools

Teachers play an essential role in satisfying the natural curiosity of young minds by sparking the joy of learning, but teaching does not happen in isolation. In this section, we address the effects of several separate, yet related topics on teachers of mathematically gifted students. First, we consider the relationship between regular and gifted education teachers and highlight some essential features of educators who can successfully differentiate the curriculum. This is followed by a brief discussion of standards-based curricula and international comparisons. Next, we touch on the highlights of the extensive discussions surrounding the dwindling workforce in the STEM careers. Space limitations have kept our remarks on educational reform to a minimum. Finally, we will comment on the ubiquitous reliance on technology.

Classroom Teachers, Gifted Educators, and the Mathematically Talented Student

Croft (2003) highlighted the critical role of the teacher in the lives of gifted students, including the fact that the role of a teacher of the gifted must extend far beyond that of the typical classroom teacher. For one thing, many elementary and middle school educators have limited or no training in either gifted education or mathematics instruction. Therefore, it is often the gifted education teacher who will assist both the classroom teacher and parents in their efforts at advocacy for challenging math. Often, the gifted education teacher is the individual who coordinates student involvement in competitions and other academic activities.

It is not uncommon to assume that classroom teachers can provide for the needs of the gifted student through differentiation of the curriculum. Westberg and Archambault (1997) described teachers who were already successfully implementing curricular differentiation practices for high-ability students. The following factors were evident:

- Teachers who successfully differentiated for gifted students had advanced training and knowledge. This training included graduate degrees in gifted education or special education, frequent participation in professional development experiences, and a continued interest in improving their practices.
- Teachers were willing and ready to embrace change. These teachers experimented with new strategies and were willing to take risks and spend extra time and effort to make changes in their practices.
- Teachers collaborated with colleagues, including other grade-level teachers, gifted education specialists, and curriculum specialists. These teachers were either provided time within the school schedule or they made the time outside of school to collaborate with others.
- Teachers viewed their students as individuals who would benefit from differentiated instruction. The teachers were aware of their students' strengths, and they did not expect all of their students to complete all of the same work within the same period of time. They focused on organizing learning opportunities instead of dispensing knowledge.
- School administrators took on a leadership role in advocating for differentiated programming for gifted students. For example, superintendents spoke publicly about providing programs for gifted students.
- Teachers were given both autonomy and support in implementing new practices. The culture of the schools was described as "supportive" and "collaborative."

Standards-Based Curriculum and International Comparisons

Since 1989, when the National Council of Teachers of Mathematics (NCTM) published the first iteration of the *Curriculum and Evaluation Standards for School Mathematics*, teachers of mathematics have had a "map" to guide them in establishing standards for their students, as well as for their profession. Fast-forward to the beginning of the 21st century and the promotion of the new NCTM (2000) standards, which were generated under the shadow of a looming concern for the performance of U.S. students in math and science as compared to international students. That shadow only darkened as new evidence continued to paint a gloomy picture about U.S. students' performance in math.

The report, *International Outcomes of Learning in Mathematics Literacy and Problem Solving: PISA 2003 Results From the U.S. Perspective Highlights* (National Center for Education Statistics, 2004) is revealing. The Program for International Student Assessment (PISA) was first implemented in 2000 by an intergovernmental organization, the Organisation for Economic Co-operation and Development (OECD), which is comprised of 30 member countries.

> In 2003, U.S. performance in mathematics literacy and problem solving was lower than the average performance for most OECD countries . . . The United States also performed below the OECD average on each mathematics literacy subscale representing a specific content area (space and shape, change and relationships, quantity, and uncertainty). . . . (National Center for Education Statistics, 2004, p. iii)

These findings hold true for the top 10% of U.S. students. The average score (scores were reported on a scale from 0 to 700) of the U.S. students in the upper 10% was 607 on the PISA assessment. However, the average score for the upper 10% across the OECD countries was 628. "Scores for the top 10% of students within countries ranged from 466 or better in Indonesia and Tunisia to 672 or better in Hong Kong-China" (National Center for Education Statistics, 2004, p. 16).

Results in the *Trends in International Mathematics and Science Study* (TIMSS; Gonzales et al., 2009) are more encouraging. The TIMSS scale ranges from 0 to 1000, with an average score of 500 and scores are comparable across testing administrations. In 2007, the average mathematics scores of U.S. fourth (529) and eighth (508) graders were higher than the average score of 500. Also, the 2007 scores were higher when compared to the 1995 average scores (518 for fourth graders and 492 for eighth).

These improvements are good news; however, the rate at which U.S. students improved from 1995 to 2007 is significantly below the rate of improvement of some higher performing countries. For example, Hong Kong's fourth graders improved from 557 in 1995 to 607 in 2007. Cautious optimism is also necessary when looking more closely at the scores of higher performing students:

> In 2007, the highest-performing U.S. fourth-graders (those performing at or above the 90th percentile) scored 625 or higher . . . This was higher than the 90th percentile scores for fourth-graders in 23 countries and lower than the 90th percentile score for students in 7 countries. The countries in which the 90th percentile cut point score was higher than the cut point score for U.S. are the same as those that outperformed the United States as a whole [Singapore, Hong Kong, Chinese Taipei, Japan, Kazakhstan, England, and the Russian Federation, respectively] . . . with the exception of Latvia where the 90th percentile score of 628 is not significantly different from 625 in the United States. The 90th percentile scores ranged between 371 (Yemen) and 702 (Singapore). The difference in the 90th percentile score between Singapore, the highest performing country, and the United States was 77 score points. (Gonzales et al., 2009, p. 15)

What does this mean for U.S. teachers? We cannot say for sure, because no experimental studies were conducted, but we can suggest that (a) perhaps the hard work is starting to pay off, and (b) there is still much more work to do.

The Dwindling STEM Workforce

This topic and the previous one are strongly connected. The National Mathematics Advisory Panel (2008) eloquently expressed the relationship:

> The eminence, safety, and well-being of nations have been entwined for centuries with the ability of their people to deal with sophisticated quantitative ideas. Leading societies have commanded mathematical skills that have brought them advantages in medicine and health, in technology and commerce, in navigation and explorations, in defense and finance, and in the ability to understand past failures and to forecast future development . . . During most of the 20th century, the United States possessed peerless mathematical prowess—not just as measured by the depth and number of the mathematical specialists who practiced here but also by the scale and quality of its engineering, science, and financial leadership, and even by the extent of mathematical education in its broad population. . . (2008, p. 1)

The pressure on today's educators to produce students who achieve in mathematics and sciences is significant. There is likely no one approach to teaching and/or teacher training. Our purpose is to raise the reader's awareness of the issues, especially as they relate to mathematically talented students and their teachers. The National Mathematics Advisory Panel (2008) determined that "The effects of teachers on student achievement compound dramatically if students receive a series of effective or ineffective teachers" (p. 35).

Educational Reform

Educational reform has the potential to be oxymoronic. Nevertheless, since the days of Horace Mann—and even before—reform in general has impacted positively the educational landscape even though the

process is not necessarily pretty. In the early 90s, buzzwords included equity and excellence (still not resolved!) as well as middle school and cooperative learning, each of which led to "movements." Newer terms include differentiation, constructivism, core curriculum, and Response to Intervention (RtI). Because differentiation has been addressed in a previous section as well as in Chapters 6 and 7, we will focus our discussion on the contrasts between the core curriculum and constructivism as well as on the connection between the core curriculum and RtI.

How do core curriculum and constructivism compare? To obtain a definition of the core curriculum, we Googled the words, which led to the Funderstanding website and the posted definition and brief discussion:

> In a core curriculum, a predetermined body of skills, knowledge, and abilities is taught to all students. . . . The core curriculum movement assumes there is a uniform body of knowledge that all students should know. . . . there is growing conflict about what topics a core curriculum should contain. Some advocates would limit the core to basic academic subjects like English, math, science, and government, while others would include general learner outcomes such as problem solving, critical thinking, teamwork, and community service. (Funderstanding, 2008b, para. 1–2)

At the same website, we see the following description regarding the impact of constructivism on teaching and student learning: "Constructivism calls for the elimination of a standardized curriculum. Instead, it promotes using curricula customized to the students' prior knowledge. Also, it emphasizes hands-on problem solving" (Funderstanding, 2008a, para. 3). A guiding principle of constructivism is that, "In order to teach well, we must understand the mental models that students use to perceive the world and the assumptions they make to support those models" (Funderstanding, 2008a, para. 2).

Given the theoretical foundation of a "predetermined body of skills and knowledge" (Funderstanding, 2008b, para. 1), the concept

of the core curriculum seems to be in direct contrast to constructivism. Although one could argue that the broader definition of the core curriculum, which includes problem solving and critical thinking, could be perceived to subsume at least some of the principles of constructivism. Regardless of an educator's proclivity toward one or the other, awareness of all curriculum reform efforts is relevant to students with high ability in mathematics because their teachers, administrators, and other support personnel, including school counselors and curriculum directors, make school- and classroom-based decisions within the context of the latest reform efforts.

How are RtI and the core curriculum connected? There is a national push toward standards; therefore, it seems logical that the core curriculum is going to dominate the field. Also of note is the connection between the core curriculum and RtI. What is the connection?

RtI was discussed in Chapter 3, and you may recall that it was discussed within the context of assessment, especially as it relates to special education. Iowa has been a leader in promoting both RtI (referred to as Instructional Decision Making [IDM]) and the core curriculum, and the following description demonstrates the connection.

> The IDM process begins with each student having access to, and an opportunity to demonstrate mastery of, a guaranteed and viable curriculum which [*sic*] demonstrates rigor and relevance. This core instruction (tier 1) is provided through the combination of sound instructional practices, learning objectives, materials and strategies provided in the general education classroom. When data suggest that students require additional instructional supports, they are provided through either 'supplemental' or 'intensive instruction. (Burdette & Etemad, 2009, p. 3)

Educators of gifted students have tried to make the best out of this "reform" by demonstrating ways in which gifted education best practices represent "intensive instruction." Some educators of gifted students have acquiesced to the necessity of a core curriculum, which we believe jeopardizes opportunities for systematic progression in content.

Two experienced gifted facilitators and advocates for high-ability learners from Kansas, Marcia Law and Cindy Sheets, developed a PowerPoint presentation, "RtI and the Gifted: Rising Above the Core Curriculum" that spells out for regular educators how the needs of the mathematically gifted learners may be addressed within the RtI context. In their excellent PowerPoint presentation (see http://sheetsgiftedpages.com/NAGC.htm or http://www.cindysheets.weebly.com), they present a version of the ubiquitous pyramid, which offers intervention options for mathematically able students. Their pyramid has three levels. The level for "all" students represents core components of instruction; the level for "some" children represents supplementary forms of instruction and includes accelerative options; and the level for "few" students represents forms of acceleration that are more radical. The pyramid in Figure 8.1 represents all of the programs and strategies addressed in this book, including radical acceleration, talent search, competitions, and increased pace. In fact, the Sheets and Law pyramid has many similarities with our Pyramid of Curricular Options (see Chapter 5). Teachers of gifted and talented students, especially students with high math aptitude, should find this presentation to be helpful as they advocate for programming options that go beyond the minimal approaches of supplementing the core curriculum to actually addressing the needs for systematic progression.

Technology

Television, computers, distance learning over the Internet, access to the Internet via cell phones, e-mail, word processors, color laser printers, fiber-optic transmission of televised broadcasts, CD-ROMs, listservs, blogs, Twitter, and Facebook are all aspects of the seemingly boundless frontier of technology. It is an understatement to suggest that technology will play an increasingly large role in the lives of all people. The reason we mention it in this chapter is that, through technology, students and teachers have greater access to curricula and resources than ever before. If a teacher of gifted students has a question about curricula or assessment, he or she can e-mail that question to a

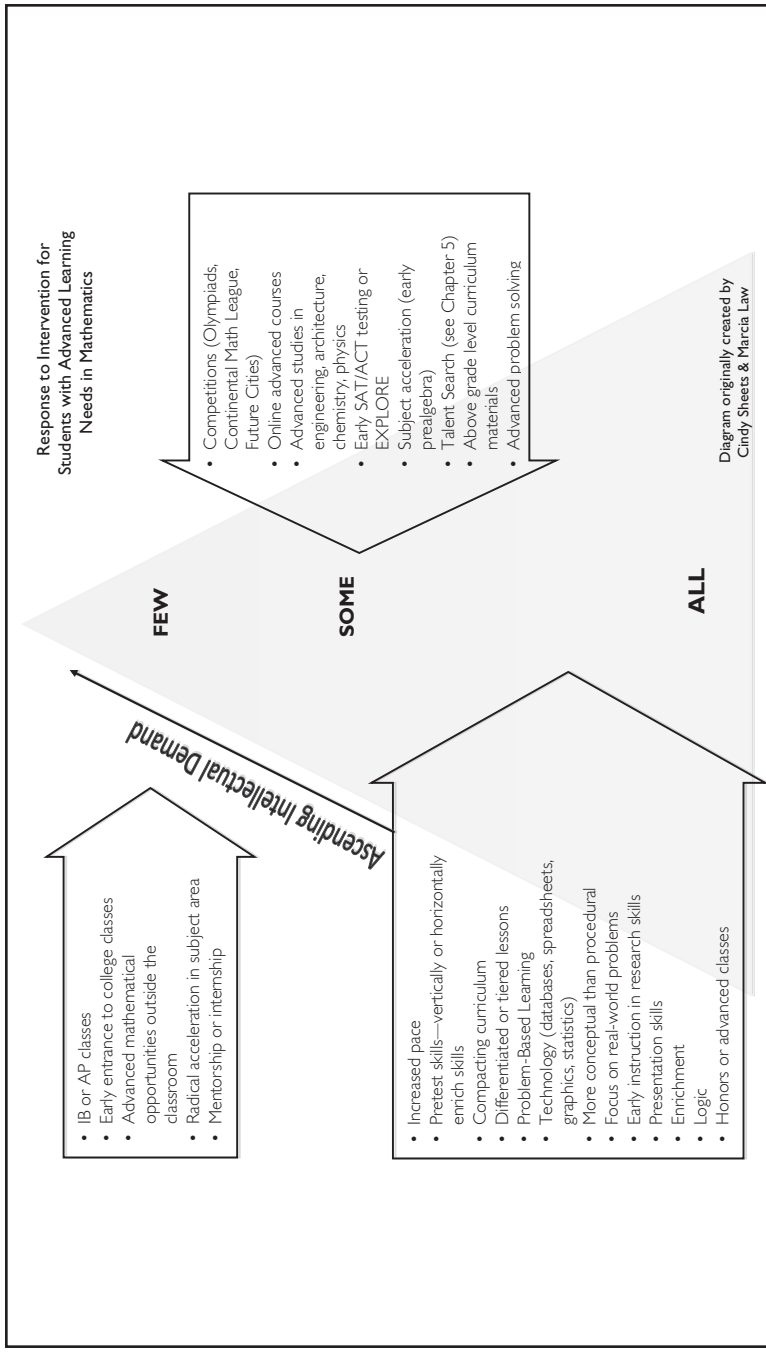

Figure 8.1. Sheets and Law pyramid. Diagram originally created by Cindy Sheets and Marcia Law.

listserv (see the Resources), and, within minutes, the question will be answered, often prompting other discussions. Parents of students who are ready for an advanced curriculum that is not easily available through the school will often pursue acquisition of the advanced course through electronic correspondence courses (see the Resources).

Technology has eliminated some of the geographic and psychological barriers that impeded the academic development of gifted students, especially those who live in rural or isolated areas or have extraordinary talents. However, technology cannot serve as a substitute for peer interaction and the teacher-student relationship. Therefore, we encourage the use of technology, but strongly recommend that it be considered a component of, not a replacement for, the classroom learning environment and the student-teacher relationship.

Modifying Curriculum and Classroom Practices

With the seemingly constant bombardment of new information, reform, and innovations, how does a teacher know which ones are effective for modifying the curriculum and which classroom practices will meet the needs of mathematically talented students? Single professional-development events seldom cause changes in the ways teachers teach. It is particularly difficult to achieve transformational changes, those that require numerous, significant alterations in the classroom (Johnsen, Haensly, Ryser, & Ford, 2002; factors that facilitate change are delineated in Table 8.1). Johnsen et al. (2002) suggested the following components for professional development activities:

- Involve all of the stakeholders who will be affected by the change, including teachers, counselors, the community, and administrators. Strong leadership is necessary when these changes are being developed and implemented. These factors are helpful in developing a positive attitude, which is essential to effecting change.
- The professional development must simulate the desired practices so that the participants will identify with the innovation and be stimulated to make changes.

Table 8.1
Factors That Facilitate Change

- Strong leadership and involvement of the stakeholders
- Professional development simulates the desired practices
- Clear definition of new practices and follow-up support
- Teachers should have a voice in the type and degree of change in their classroom
- Material and human support:
    - Collaboration with peers
    - Mentoring
    - Resources
    - Time to implement
    - Time to evaluate the process

Note. Adapted from "Changing General Education Classroom Practices to Adapt for Gifted Students," by S. K. Johnsen, P. A. Haensly, G. R. Ryser, & R. F. Ford, 2002, *Gifted Child Quarterly*, 46, 45–63.

- The practices need to be clearly defined so that teachers will be able to transfer new practices to the classroom. Research also shows that teachers need follow-up support as they are implementing the changes.
- The teachers should have a voice in the type and degree of change they will incorporate into their classrooms. This freedom both empowers teachers and builds positive attitudes toward the change.
- Teachers need ongoing material and human support to make the changes. These types of support include staff-development days, peer and mentor support, leadership support, materials, and time to implement.

Conclusion

The field of education and the role of the teacher have evolved dramatically throughout the 20th century. We are well into the 21st century, and the rate of change seems to be accelerating exponentially! Throughout this chapter, we have presented major educational issues confronting parents and teachers. Our purpose was not to be discouraging; rather, our main message is this: Teachers are the catalyst to the development of their students' talents. Teachers must think globally while acting locally. Teachers have at their fingertips the resources to educate and be educated, and we know that they are ready for the challenge!

CHAPTER 9

Case Studies

KEY POINTS

- Mathematically talented students can be identified at a young age, and their parents are often the first to recognize their talents. Parents play a critical role in advocating for appropriate challenges for their children.
- Objective information is essential in helping parents to be effective advocates for their children.
- The Diagnostic Testing→Prescriptive Instruction model helps ensure that exceptionally mathematically talented students study mathematics at a steady rate.
- Long-term planning is essential so that students are always studying an appropriately challenging level of mathematics.
- Mathematically talented students benefit from finding an intellectual peer group; this is important for both boys and girls.

All parents worry about their child's social–emotional development and Zach's were no exception. Now that their highly gifted son is in graduate school, they have the luxury of knowing that he has grown into an individual who, as described by his dad, is a "good citizen." Teachers and other parents may find it useful to know that Zach's father believes that, if he had it to do all over again, he "would have worried less about the social aspects of Zach's development."

We have worked with thousands of students over the past several decades. From this work, we have identified case studies that highlight the main points presented throughout this book. We are grateful to the families and students who so willingly shared their experiences and

insights with us in order to help other mathematically gifted students, their families, and their educators.

The case-study approach is extremely useful for the families with whom we work as well as in our own research. The stories we relate in this chapter help to illustrate the points we have made in earlier chapters. Although case studies are anecdotal in nature, they offer a general model that can apply to individual students. Both parents and teachers have told us that they like reading case studies because they realize that others have blazed the trail, and they do not have to "go it alone." We think our case studies are somewhat unique because they integrate specific individual information with a robust theoretical approach that has been proven to be effective with hundreds of thousands of students (see Chapter 5).

Rosie: It's April, and It Is Time to Plan for Next Year[1]

Rosie is a cute 7-year-old who loves taking gymnastics and swinging on her swing in the back yard. She skipped several grades and is currently in third grade. Her parents first contacted us in October, when they were thinking of having her take the EXPLORE test. Her father gave some examples of the math they were doing together at the time:

> Recently, we've been plotting simple functions at home like $y = x^2$. She can solve equations like $3x^2 - 25 = 50$, and do the basic operations on fractions and decimals. We're trying to convince the school that she can do more than fourth-grade math (the current accommodation beyond her grade skip), but she makes so many mistakes on the pretests that they can't tell the difference (she makes errors in execution, doesn't read the problem properly, and sometimes even leaves items blank).

[1] Most of this case study was written by Rosie's dad. We thank him for his comprehensive notes and attention to detail.

Rosie demonstrated her intellectual abilities at a young age. She read her first sight word on a billboard advertisement for the local supermarket at 18 months, although her parents didn't think much of it at the time. Rosie's speech was so advanced by age 2 ½ that her father, a software developer turned stay-at-home dad, decided to research early reading. He found that some children had begun reading at her age, and so he set about trying to instruct her. After a few lessons in a formal system, she let him know she was not interested by throwing the book across the living room. Her father decided to try again 6 months later when she turned 3. Rosie negotiated a reward of a cookie-milkshake for completing the remainder of the instructional book, which she finished in 6 months.

At this point, Rosie could read second-grade books. She attended a Montessori preschool and demonstrated that not only could she count objects (a skill she had acquired at a young age), but she also was ready for addition, subtraction, and counting by twos, threes, fours, and all the digits through 10. She also appeared to understand the basics of algebra (which her father explained as a letter "pretending to be" a number) and could solve simple equations like 5 + b = 10. One day after preschool, she said, "I told my friend Caitlyn that a letter could pretend to be a number." Her father asked, "What did she say then?" "She just looked at me weird," Rosie replied.

Rosie was developing a comprehension of place values through thousands by the time she finished her second year of preschool at age 4 ½. The following summer she was tested at a university school psychology program, which her parents elected to use as a trial before investing a lot of money in testing. Rosie scored a Full Scale 141 on the Stanford Binet V, with a Nonverbal IQ of 144 and a Verbal IQ of 135. The university tester made a comment to Rosie's parents that that, in the experience of their school psychology program, they had "never seen anything like her." The written report recommended that, "She may be considered for placement in first grade." For Rosie's parents, this was the first professional call for radical acceleration.

Rather than send her to school for kindergarten at the age of 4, she was homeschooled with her father as her teacher. In that year, she read more than 15,000 pages of chapter books, completed eight spelling

workbooks, performed science experiments on nearly a daily basis, completed a world history course, and stepped through second- and third-grade Singapore math as well as *Daily Word Problems (DWP): Grade 3* and part of *DWP: Grade 4*.

Her parents had heard of the Davidson Young Scholars program (http://www.ditd.org) and thought that membership would benefit both Rosie and themselves. With only a four-point gap between Rosie's IQ score and the Davidson cutoff, they decided to have Rosie retested by someone experienced with high-achieving girls, Dr. Sylvia Rimm, at her Family Achievement Clinic in Cleveland, OH. Rosie's WISC-IV results placed her IQ at "146+," with a verbal Comprehension of "155+." Dr. Rimm suggested further testing to determine Rosie's "actual" IQ, but her parents declined, feeling that it would provide no additional information that would impact their decisions. Rosie was accepted as a Davidson Young Scholar. Her father describes the Davidson resources as "invaluable" and would encourage the parents of any highly advanced child to apply.

Dr. Rimm recommended that the family place Rosie in a typical school setting, with at least a double grade skip, for Rosie to have an opportunity to work with peers and exchange ideas. Although opposed to the plan, her father agreed to proceed on a trial basis. The reaction from the two leading local private schools was, "We don't accelerate." One local parochial school did not return the parents' call. The principal of the leading local parochial school ridiculed them with the response, "What does she do, suck her thumb and read a book?" Similarly, the principal of that school implied that Rosie's parents were lying about her abilities, while he boasted that his school's curriculum was 6 months ahead of the public school (little comfort to a child 4 or more years ahead). Although Rosie's father had been openly critical of the local public school's gifted services and felt that he had burned bridges, Dr. Rimm encouraged him to try there. Initial opposition to the idea was high, and to make matters worse, less than 2 weeks remained before the start of the school year. However, the district agreed to allow Rosie to go directly into second grade after its first-time use of the *Iowa Acceleration Scale* and a conference call with Dr. Rimm. This was a double skip based on Rosie's October birthday. The administration

wisely consulted the second-grade teachers to see who was most receptive to instructing Rosie; luckily, someone stepped forward to meet the challenge.

In Rosie's state, Individualized Education Programs (IEPs) for gifted students are required by regulation, but there is no enforcement behind the regulations. In fairness to the school, her parents report that the entire year was an experiment to answer the question, "What would happen if this student went to regular school but skipped two grades ahead?" Rosie's IEP focused mostly on her integrating into to school life in second grade, but there were other accommodations. Rosie was placed in the highest guided reading group in her grade (which read fourth-grade books) and was given access to many fifth-grade books for pleasure reading (her teacher formerly taught fifth grade). Rosie received a specialized "challenge" spelling list, which her teacher eventually shared with all four second-grade classes. Although allowed to work ahead on math speed tests as she was able, this was an inconsequential accommodation as Rosie was not a rapid calculator. Similarly, she was allowed to work ahead in the district's computerized adaptive math curriculum (Compass), but she made so many computational mistakes that the system never caught up with her conceptual knowledge. Perhaps most helpful, Rosie engaged in weekly pull-out sessions for math problem solving with other advanced students. Rosie continued her work at home on *DWP: Grade 4*, as well as supplementing it with the HeyMath! website (http://www.heymath.com).

Rosie was allowed to bring various extension projects into school. For the 100th Day of School celebration, in which the number 100 was discussed, Rosie and her father taped together 20 sheets of graph paper with 1 mm squares to give a visual representation of 1,000,000 in a 1,000 by 1,000 grid. (She got a kick out of making the class guess what she had a million of in her backpack.) The diagram also included a 10 x 10 square indicating 100, and a 10 x 100 rectangle indicating 1,000. The teacher loved it and kept it for future classes. Other outside projects included a magnet experiment, a large drawing and brief biography of Marie Curie, and a handmade mini electrical generator (which might have produced all of one watt). On entering second grade, Rosie's

weakest area was writing because of physical limitations; however, it ended up being her area of greatest growth.

She was given the school's standard placement tests and screening tools for gifted programming at the end of second grade and she earned scores that placed her above the 90th percentile on the Stanford Achievement Test for reading and math. She scored in the 90+ percentile on the Stanford 9 for reading and math among third graders and on the Otis-Lennon School Ability Test (OLSAT) she scored in the 99.9th percentiles on both the verbal and nonverbal scales.

Because Rosie had forgotten some of her math facts over the prior summer, math and reading became daily activities over the next summer. She completed the HeyMath! website through the fifth-grade unit, and when given *DWP: Grade 6*, she worked (by her own choice) through the first 3 weeks of problems in one sitting. For reading, she conquered *The Lord of the Rings* and many other books. Additionally for language arts, she read a book on grammar entitled *Woe Is I.*

Within the first couple of weeks of third grade, an IEP meeting was convened. School personnel suggested starting Rosie's math by trying to fill in third-grade "holes." Her parents looked at each other and literally burst out laughing. They reiterated the points they had presented in an e-mail in advance of the meeting (that Rosie had completed a fifth-grade online course and sixth-grade word problems) as well as pointing out that the district's own testing in the prior year had placed her in high fourth-grade performance. An agreement was reached to use math contracts with Rosie in a fashion akin to the DT→PI model and to attempt to complete both the fourth- and fifth-grade curricula. There were two major shortcomings. At first, any errors she made in the pretests were simply added to Rosie's contract without further probing into whether she actually understood the concepts and had simply made computational errors. This lengthened the duration of fourth-grade material, but was corrected with the fifth-grade work. Unfortunately, throughout the year, Rosie worked by herself, or in a small group of children (working on different contracts), who received instruction only when they did not understand the material and needed to ask a question. No lessons explaining the material were prepared for

her. On nights when Rosie had no homework, her father taught her advanced concepts from prealgebra as well as problem solving.

Other school accommodations included placement in the highest guided reading group, spelling extensions from Merriam-Webster's Spell It! site (http://www.myspellit.com; which also serves as the Scripps National Spelling Bee study site), and quarterly extension projects alternating between social studies and science. One social studies project involved creating a small travel guide to four landmarks in Washington, DC. A science extension entailed creating a crude yet functional wooden xylophone and PVC flute. Rosie also took part in a gifted pull-out program that covered subjects like algebraic concepts using blocks and dominoes, creating and testing structures, solving word puzzles, and studying a unit on a pioneer schoolhouse. One of the most helpful accommodations was a weekly 5-minute phone call from the teacher to Rosie's parents; it kept communication lines open and reduced anxiety on the parents' side.

In February of grade 3, Rosie took the EXPLORE test. When Rosie got home after taking the test, she told her parents she thought she did well; she was able to answer 5/6 of the math questions before running out of time. Her EXPLORE Math score was 17, which is at the 75th percentile when compared to eighth graders, students who were 6 to 7 years her senior. That score also placed her at the 99th percentile when compared to other gifted third graders taking the test, the vast majority of whom had not skipped two grades as she had. Her other scores were English 14, Reading 15, and Science 14, all excellent scores for such a young student. Somewhat amusingly, she left the test during the science section in order to use the restroom (obviously not giving the testing the same weight as her parents). Her parents forwarded the scores to her school and as of this writing are negotiating next year's IEP.

For fourth grade, Rosie's parents are considering several options regarding math. Their highest priority for fourth grade is to move from a system in which Rosie asks questions to initiate instruction to a system in which she receives regular instruction with the curriculum. In other words, they are looking for a planned curriculum for Rosie. A related priority for the next year is for Rosie to be instructed with

one or more peers. School personnel have promised to administer the Orleans-Hanna Algebra Prognosis Test, usually given in her district to fifth graders to see who is ready for prealgebra or possibly algebra in middle school (grades 6–8). Based on the results, Rosie could take sixth-grade math or higher at the middle school, which raises issues in scheduling, transportation, and social adjustment. Alternately, she could be instructed two times per week by a mentor (again, preferably with another student) and work independently the other three days. This could involve the sixth grade or higher curriculum or the M^3 gifted units from the University of Connecticut and/or gifted math units developed by the Center for Gifted Education at The College of William and Mary. The parents are also considering a year's worth of problem solving from the Olympiad books and participating in a math team coached by her father. Secondary options include online instruction via ALEKS, independent study with *Teaching Textbooks*, or some sort of cyber-school hybrid. Rosie has expressed a strong desire to remain with her friends and not be whole-grade accelerated again.

The story so far takes us to the spring semester of third grade. Her parents fully realize that they will need to continue to advocate for their daughter and attended an end-of-year conference with the following goals, all of which were met:

- Help school personnel recognize the extraordinary level of Rosie's mathematical talent.
- Discuss the need to plan for a steady diet of appropriately challenging mathematics throughout Rosie's school years.
- Recognize the importance of allowing Rosie to take tests such as the Orleans-Hanna or Iowa Algebra Aptitude Test in order to answer the question, "Is she ready for algebra?"
- Advocate that she be allowed to advance to a prealgebra class that is appropriately paced and advanced to algebra when she is ready, even though she'll be quite a few years younger than many "peers" in the mathematics classroom.
- Help school personnel recognize that, for Rosie, there are options that are far preferable to studying math alone in the back of the classroom. These options require putting together an appropriately planned mathematics program for Rosie that

might include placement in a regular class with students several years older, working with a mentor, or participating in online math classes—or a combination of all three options.

- Address with Rosie's educators the common phenomenon of gifted students' superior understanding of mathematics concepts, relative to acquisition of basic computation skills.

In a conversation with the district's Director of Student Services, there was broad agreement that although Rosie is accelerated by 2 years, she interacts well with other students in her class. To eliminate any concern about possible gaps in her mathematics background, she is scheduled to take the cumulative fifth- and sixth-grade math tests. Also, Rosie's parents assured her educators that they need not be concerned about socialization in math because she has many social experiences outside of math. All-in-all, this is a wonderful example of how good communication between parents and educators, coupled with both parties' willingness to be flexible, can address the needs of all parties. Most importantly, the student is appropriately challenged and educators have input into the decision-making process.

Rosie's dad had one final comment about his advocacy for his daughter's education. He experienced some disdain from other parents because, as he put it, he "actively attempted to challenge her in math." But, he put it all into perspective with this statement: "Some parents will drive their children one hour each way to take part in 'travel sports'—and they think I'm a maniac for encouraging my kid to do 15 minutes of math every day and to read for a half an hour. You make the call." He summarizes his daughter's progress relative to parental effort with these words: "My original goal was to have Rosie reading before kindergarten . . . oops!"

Christopher: "He Wants to Learn Math as Much as He Wants Air"

Christopher's parents knew from a young age that he was bright. By the age of 1, he had memorized books and knew the numbers 1–10, and

by the age of 2, he knew the alphabet. At this young age, Christopher became obsessed with numbers. He made his mother sit with him for hours to practice writing numbers and letters. He memorized house numbers, added and subtracted two-digit numbers at the age of 3, and was fascinated with mechanical things.

At the age of 3, he figured out negative numbers by playing with a calculator, and he asked his parents, "Zero is a non-number, isn't it?" It was at this point that his parents first contacted Julian Stanley at Johns Hopkins University, founder of the talent search model. Dr. Stanley kindly forwarded the inquiry to us (see Figure 9.1 for examples of Christopher's handwritten work done at the age of 3).

Christopher was also talented at writing. He liked writing stories, but they were always about numbers. He dictated his stories to his parents, who typed them out on the computer. His stories always ran along the lines of, "Once there was a 10. And then there was a 15. One day they found a 12." When he was 3, he woke up in the middle of the night crying. On his way to respond to Christopher, his father jokingly said to his mother that Christopher was probably having a nightmare that someone came and took away all of the numbers, because "numbers are his life." It turned out that Christopher had been dreaming that he couldn't find the number 79.

When Christopher was 4, he could read, multiply, and divide (he said he preferred multiplication). He understood squares and square roots, played with numbers on the computer (he liked *Math Blaster*), and figured out powers on his own. About this time, he showed an interest in infinity and negative infinity.

By the time Christopher entered kindergarten at the age of 5, he was reading at the third-grade level. As a programming option for his advanced reading level, he was pulled out of kindergarten for a special reading program. The reading teacher reported to Christopher's mother that his "attention span was short," but his mother believed that his behavior reflected that he didn't like being separated from his kindergarten class.

When Christopher entered first grade, he demonstrated such superior understanding of mathematics that he clearly did not fit into the regular mathematics classroom. He needed a formal assessment

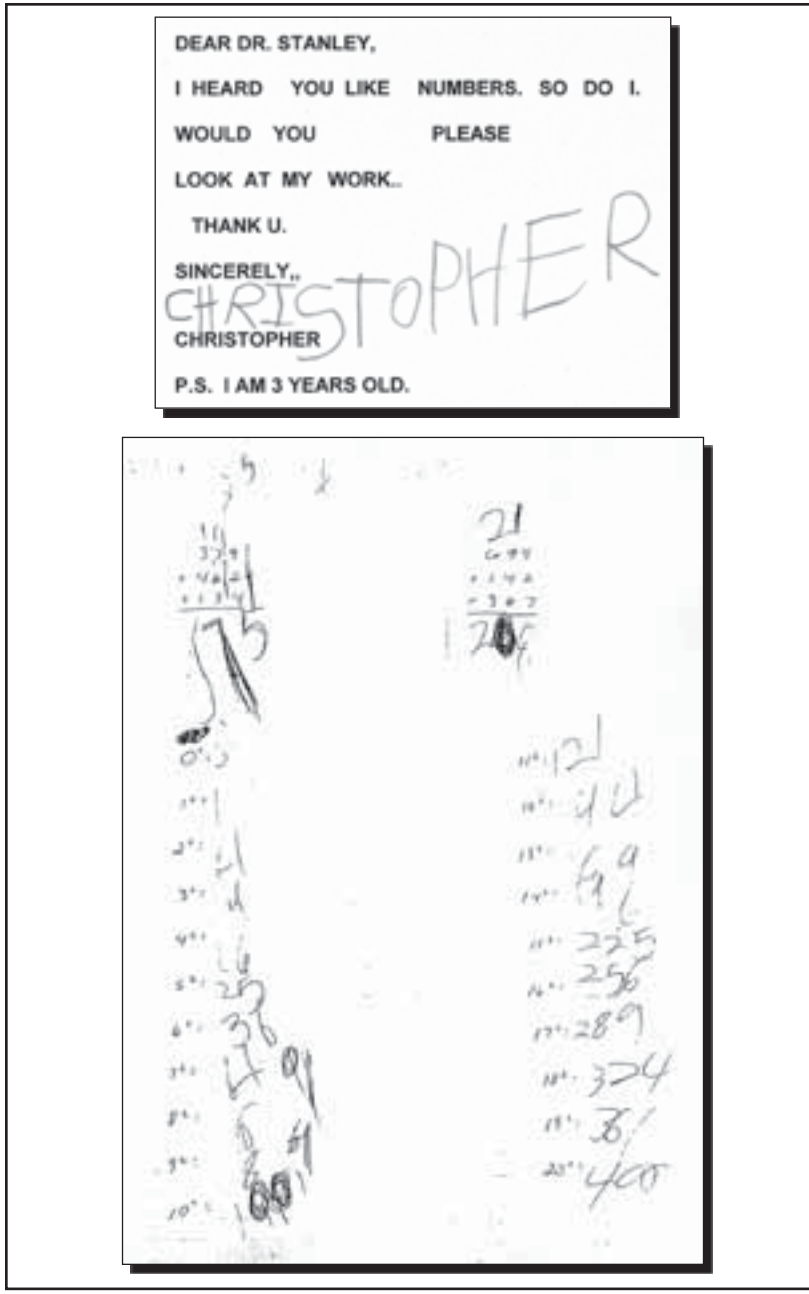

Figure 9.1. Examples of Christopher's work, age 3.

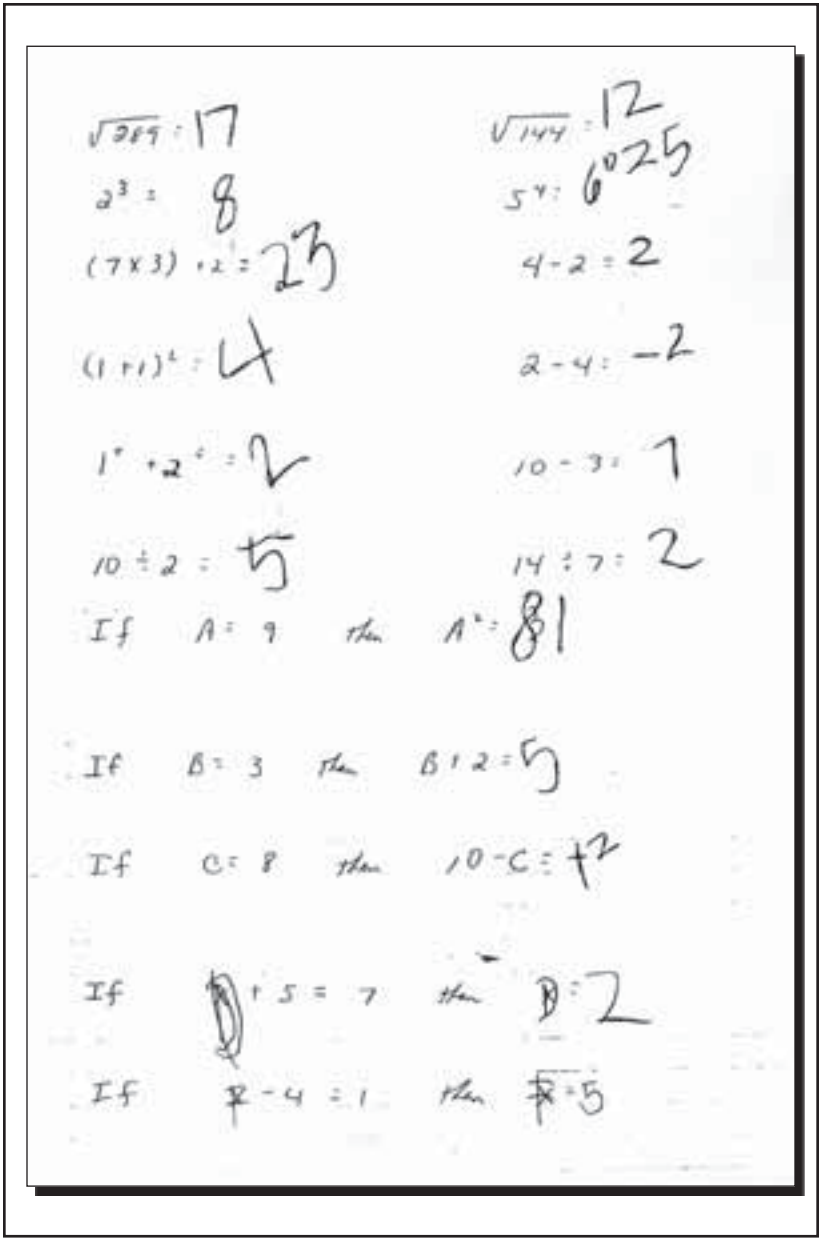

Figure 9.1, Continued

of his mathematical abilities so that a clear educational program could be developed. His parents asked us to do that assessment. Midway through his first-grade year, we gave him the Quantitative section of the Lower Level of the Secondary School Admission Test (SSAT-Q; previously, we had used this test with gifted third through fifth graders; it was normed on private school students in grades 5–7). The administration of the SSAT-Q occurred as the first step of the Diagnostic Testing→Prescriptive Instruction process that we have found to be so helpful in devising appropriate programs for mathematically talented youth (see Chapter 4). On the SSAT-Q, Christopher (a first grader) earned a score that placed him at the 60th percentile when compared to the fifth-grade national norms. When compared to norms that we had gathered on gifted third graders, his score was at the 93rd percentile.

His advanced quantitative reasoning skills were clearly not a fluke, nor were they the result of external "pushing" on the part of his parents. The attitude that a child's performance is primarily attributable to pushy parents or chance behavior is often expressed by educators; however, the fact that, as a first grader, he earned a score that was higher than the scores earned by 60% of the national group of fifth graders for whom the test was normed and his score surpassed 93% of the SSAT-Q scores earned by gifted third graders would refute any arguments of "chance" or "pushy parents" attributed for Christopher's success.

The next step in determining the appropriate curriculum for him was to administer a test that provided more specific achievement information. This was accomplished by administering the Mathematics sections of the Sequential Tests of Educational Progress (STEP). On the most basic level of the STEP Basic Concepts test, he earned a score at the 82nd percentile when compared to first-semester fourth graders. On the same level of the STEP Computation test, he earned a score at the 80th percentile compared to first-semester fourth graders (he was compared to fourth graders because this was the grade level for whom that level of the test was developed). Clearly, Christopher had a great deal of mathematical aptitude and achievement. Although his exceptional performance did not indicate that he had mastered all of elementary school mathematics or even all of fourth-grade mathematics,

it pointed to the fact that, if Christopher was going to be challenged in mathematics, he would need a program that was substantially different from that of the average first grader and even that of a gifted third or fourth grader. No specific accommodations were made for Christopher's mathematics education at that time; however, a plan was formulated for the beginning of second grade. The idea was for the gifted education teacher to teach him math. Although this seemed like a good idea, it was not a good student-teacher match. His mother recalled,

> The teacher had a difficult time accepting [Christopher's] abilities and often insisted that he solve word problems her way. She would scold him for working problems different ways even when his way was logical and resulted in the correct answer. She kept insisting that the third-grade math was beyond him. As proof that he was working on material beyond his abilities, she reported that one time he had slapped her hand. When we questioned [Christopher] about the incident, he told us that, every time he started to work a problem (and before he had a chance to complete it), the teacher would push his hand away and tell him he was doing the problem incorrectly and to start over. He apparently got so frustrated that he finally slapped her hand.

At the beginning of second grade, school personnel suggested having Christopher skip at least one grade, but his parents thought he was not emotionally or physically ready for that. Together, his parents and school district personnel decided to place Christopher with three advanced fourth-grade boys. Together, the four boys studied fifth-grade mathematics with a high school mathematics teacher whose salary was paid by the district. Although this was a highly unusual situation, it seemed to work well for Christopher and the other students. Sometimes, Christopher got a little "teary" when he didn't get the right answer immediately, but his parents recognized that it was good for Christopher to have other children with him.

This arrangement continued for both Christopher's second- and third-grade year. As the end of third grade approached, Christopher's

parents began making plans for the next school year. (Planning ahead is an excellent strategy, as it gives all parties involved a lot of time to work out solutions.) Christopher did well with the advanced math material. At the beginning of the next year, though, the fifth-grade boys (his peers) would be moving on to middle school, leaving Christopher with 2 more years in his elementary school without any math peers. The high school teacher continued to be concerned that Christopher wasn't grasping all of the material being taught. She acknowledged that he seemed to understand what was being presented to him, but she admitted that she did not know for sure what he understood because she had never taught so young a student.

By the end of third grade, Christopher had finished all of the sixth-grade math curriculum plus most of the seventh. His father noted that Christopher benefited from being with the other boys because it allowed him to see that he was not always the best. Two of the fifth graders usually did a little better than Christopher, and he usually did better than one of them. When they took their end-of the-year sixth-grade math test, Christopher correctly answered 89% of the material. He took the prealgebra test at the end of the year and did well. The school district administrators usually require a raw score of 60 to select students for advanced programming, and Christopher earned a 59. Christopher's father noted that it was a timed test, and Christopher didn't finish 8 or 9 of the questions. Christopher's father said that he sensed that the high school math tutor

> could never accept that Christopher could understand this work. She told me that she expected Christopher to do well on the prealgebra test, not because he really understood the math, but simply because he had learned it. I am still trying to figure that one out.

In January of third grade, Christopher took the EXPLORE test and earned excellent scores, especially when using the eighth-grade percentiles as a comparison. His scores and corresponding eighth-grade percentiles were: English 20 (92nd percentile); Math 16 (72nd percentile); Reading 15 (60th percentile); Science 15 (60th percentile);

and Composite 17 (75th percentile). Christopher's excellent scores once again confirmed his extraordinarily high abilities.

During fourth grade, Christopher covered the first half of the Algebra I book. Although the high school teacher wanted Christopher to finish Algebra I by the end of fourth grade, Christopher's parents thought she was requiring too much homework, preventing him from doing other things they thought were important. As a consequence of their request, Christopher did not finish Algebra I until the end of fifth grade. His mother reported that, "He is currently in the sixth grade and taking Algebra II with the advanced eighth graders. He has done very well, earning A's both terms thus far. Next year, he will take Geometry with the eighth-grade class. (Algebra II and Geometry are only offered in alternating years at his school.) His teacher tells me that in his eighth-grade year, she will teach him trigonometry and calculus privately since it is not offered in the middle school."

Christopher's family was successful in devising an appropriately challenging program in mathematics for him for a number of reasons. First, they recognized his abilities at a young age. Second, they did a great deal of research on their own to find the needed resources and experts. Third, they worked with their school district and kept the school district informed. Fourth, they constantly looked at Christopher's future mathematics education as they made plans for the current school year. It also helped that Christopher's parents were well educated and lived in an affluent school district that could afford to send a high school teacher to the elementary school for mentoring, not to mention that district personnel recognized Christopher's extraordinary abilities and were willing to devote special resources to meeting his needs.

Even though they were in such a good situation, they still experienced difficulties. For example, the high school math teacher demonstrated the hesitation that so many math teachers show: She could hardly believe that Christopher could do the math, even though he demonstrated his knowledge to her over and over.

Billy: "If He Does Third-Grade Math in Second Grade, What Will He Do in Third Grade?"

According to his mother, Billy had an "unquenchable" thirst for learning and he constantly asked for something harder and more challenging. He was an "extremely intense and driven child." At the age of 2, Billy became utterly fascinated with numbers. He counted everything, everywhere. He enjoyed doing dot-to-dot books because he was able to follow the numerical order of the numbers. For additional challenge, he began creating his own dot-to-dot puzzles.

Billy also showed an interest in letters and reading at a very young age. For example, at 18 months, he recognized all of the letters of the alphabet. He loved playing with his ABC puzzle. After removing all of the letters and mixing them up in a pile, he would complete the puzzle in correct order with no hesitation. Throughout the day, he searched for letters, pointing to and announcing letters wherever he happened to be (e.g., in the store, in the car). Because of his enjoyment of letters, he loved watching *Wheel of Fortune* from his playpen.

Billy taught himself to tell time and to calculate minutes on a traditional clock at the age of 3. One evening at dinner—out of nowhere—he looked up at the clock and told his parents it was 5:22 and, in 38 minutes, it would be 6:00. That was the first time they realized he could tell time and that he was adding and subtracting. After that day, he began to add and subtract minutes on a daily basis. He became obsessed with calculations. He would calculate anything he could. He also became obsessed with television game shows. He would rush home from preschool so that he could watch *The Price Is Right*. At church, he would take all of the numbers of the songs from the songbook and add them together.

One day, Billy's mother found him on his bed surrounded by supermarket ads. He was adding up all of the prices with a calculator. One of his favorite activities was going grocery shopping. By the age of 4, he could deduct coupons from the grocery bill total and calculate the change his mother should receive before the cashier did. One incident in particular stood out in his mother's mind. Her total was $207.60. She had $175 in cash, and was planning to write a check for the balance.

Before the cashier could calculate the difference on the cash register, Billy said the check needed to be for $32.60. The cashier was amazed.

At age 4, Billy knew the correct locations of all 50 states on a United States puzzle. He also knew all of the capitals. He played Yahtzee daily. It was one of his favorite games because of the numbers. He also liked to calculate all of the players' scores in his head. His parents realized he could read when, one night during his bath, he read the back of a shampoo bottle. From then on, he read everything he could and phonetically sounded out what he could not. He began reading first-grade level books at home.

When Billy was 5, he entered kindergarten at a local parochial school. It quickly became evident that he was well beyond his peers and the course material that was offered. His parents were frustrated by his first report card. It showed "satisfactory" performance. It did not show any advanced knowledge or abilities beyond those of an average kindergarten student. The report card stated that he could recognize numbers 1–12, when, in fact, he could recognize numbers in the thousands. He was adding, subtracting, and doing simple multiplication and division problems at home. His report card also stated that he was reading at Level 1, which was the expectation for the average student. Billy's mother took two mathematics workbooks with her to the first parent-teacher conference to show what Billy was working on at home. She attempted to show those books to the teacher as evidence of his ability, asking if the teacher could provide some first-grade work. The teacher would not even look through the workbooks, and she said she couldn't make any notes of advanced ability or advanced work on his report card other than what was expected or "satisfactory." She said she would not give him any extra work and that maybe they should look into sending him to a public school. When Billy's mother met with the school principal, she received the same responses. Like his teacher, the principal wouldn't even look at the workbooks, and it seemed as though she didn't believe that Billy was exceptional.

Standardized testing that spring showed that Billy was performing in the 99th percentile for his grade (Cognitive Abilities Test: Verbal, 85th percentile; Quantitative, 99th percentile; Nonverbal, 99th percentile; and Composite, 99th percentile). Billy's parents became

aware that the public school district had a gifted program in which Billy could participate. He was tested using the Stanford-Binet and earned a Composite standard score of 148 (in the Very Superior or Gifted range), a Verbal Reasoning Standard Score of 127, an Abstract/Visual Reasoning score of 143, and a Quantitative Reasoning standard score of 163. On the Wechsler Individual Achievement Test, Billy's Basic Reading standard score was 149, and his Math Reasoning standard score was 160.

Clearly qualified, he started the district's gifted program in the fall of first grade. He continued to attend his parochial school and was pulled out for 30 minutes twice a week to attend the public school gifted program. Billy's regular first-grade class was filled with worksheet after worksheet, and he was happy to participate in the gifted program because he did a lot of above-level reading (third- and fourth-grade level), math work, and projects.

At home, Billy worked on third-grade math workbooks on his own initiative. He enjoyed a subscription to a magazine filled with math puzzles and logical thinking activities. He especially enjoyed board games involving money and geography. He continued to enjoy grocery shopping—still deducting coupons and calculating change—but now also figuring out the best buy for mom. On one shopping trip, he instructed her to buy a certain brand of raisins because the "price per ounce" made it a better deal.

Second grade was a turning point for Billy and his family. He sat through regular classes at school. He woke up early most mornings in order to work on his fourth- and fifth-grade math workbooks before getting on the bus for school. He still attended the pull-out gifted classes and was very excited by them. For those classes, he had an excellent teacher who realized his potential and need for challenge. Billy enthusiastically worked on several independent study projects, mostly in math. He also developed an intense interest in the solar system. He attended his first C-MITES weekend workshop at Carnegie Mellon University, which he thoroughly enjoyed.

At the first parent-teacher conference of the year, his parents were presented with Billy's Iowa Tests of Basic Skills scores, which indicated he performed at the 99th percentile in many areas. Billy's mother asked

his teacher about giving him more advanced work. They discussed the possibility of having him sit in on third-grade math classes, but the idea was vetoed by the principal, who said, "If he does third-grade math in second grade, what will he do in third grade?" The only challenge he was given was a sheet of third-grade math homework on a fairly regular basis.

Up to this point, the parochial school had done very little to support Billy's exceptional abilities. Instead of the administration being proud to have a student like Billy in the school, they thought of him as a problem. In turn, his parents were viewed as problems for being his advocates. School personnel didn't know how to handle the situation or what to do with Billy. His parents found this lack of school support and interest quite unexpected. They quickly realized that the role of playing advocate for their son wasn't going to be an easy one.

Several years before, Billy's mother had contacted Dr. Lupkowski-Shoplik at Carnegie Mellon University and had received a lot of information about mathematically talented youth, some of which discussed mentoring. His parents decided that, if Billy could be placed in a mentoring program at his school, it might solve their problems. His mother discussed this idea with the public school gifted teacher, who thought it sounded like a great idea and even volunteered to accompany Billy's mother to the meeting scheduled with the principal and Billy's second-grade teacher.

Although the principal said she previously had not heard of a math-mentor program, she was willing to discuss the idea with the Diocese (the supervisory body for the parochial schools in the area) and gain approval from her supervisors. As a second grader Billy then took the STEP Mathematics Basic Concepts test. His score was at the 46th percentile when compared with the sixth-grade norm group. He scored at the 37th percentile on STEP Computation when compared to sixth graders. Billy also took the second- and third-grade end-of-the year math tests. The results of these tests showed his mastery of the second- and third-grade material. Clearly, Billy was performing well above grade level in mathematics.

Testing occurred in February, and Diocesan approval was received in early May (a lag of 2 or 3 months between testing and the

administrative decision is not unusual). With only about 6 weeks of school left, Billy was finally able to begin working with his mentor, with whom he met twice a week in lieu of his regularly scheduled math classes. On the remaining 3 days of the week, he worked on mentor-assigned homework and e-mailed his mentor with his progress or to ask questions. He loved it. Billy and his mentor worked on permutations, scientific notation, associative and commutative properties, and probability games.

Billy's mother remembers that he had experienced some social difficulties in school during second grade. He got the distinct impression that it wasn't "cool" to be smart. The other students seemed to resent his intelligence and his enthusiasm for learning. In one math class, another student beat him at flashcards, and the whole class cheered. He was crushed and couldn't understand why kids who used to be his friends were treating him this way.

Although the mentor program was wonderful and Billy and his family were very pleased with the results, it targeted only mathematics for Billy. They thought he needed acceleration and challenge in other areas, as well. They decided to look into other schools. Billy was excited about the idea, saying that it would be nice to attend a school that would be challenging and also where there would be more "kids like me." They found a local private school that seemed to be a good fit, and he began attending that school in third grade.

Billy's mother said,

> We are very content with our decision. As parents, we were searching for a school that would provide challenge without being overwhelming and where [Billy] could be the enthusiastic, inquisitive student he needs to be. From the time he was a toddler, he has had an unquenchable thirst for knowledge. We believe his new school provides him the atmosphere in which he can thrive. Our beliefs were confirmed when, at our first parent-teacher conference at the new school, his teacher said that [Billy] was at the top of his class and was "made" for the school. Because of his personality and his intense desire to learn, the transition to his new school has been a very smooth

one, as indicated by all of his teachers. He is finally happy and content. He is with other children like himself where it is acceptable to be smart, and he is able to accelerate within the class. For example, in reading, his teacher is recommending he read fifth-grade-level books for his monthly book reports. In math, he is in a small group that is working on the same subject matter as the rest of the class, but at a higher level and a quicker pace. Billy has his own math folder, as do other students, which is filled with math activities geared toward his ability: critical thinking, logical thinking, algebra, and so forth. There are several centers in the classroom, and students are able to choose a center at which to work. The key here is that he is not being restricted by anyone, and an entire class is not being rushed ahead because of him. The administration and teaching staff are supportive, and they are thrilled to have Billy and students like him attend their school. What made him so difficult at his former school has impressed his new school.

Our parting with the previous school was amicable. They understood our position and said they couldn't structure their curriculum to accommodate students like Billy. They had to target the average student. We believed our son had so much potential and that it would be an injustice to let him remain in a school where it wasn't possible for him to develop to his full potential.

Billy took the EXPLORE test in third grade and earned the following scores and percentile ranks, which are comparisons to eighth graders: English 17 (73rd percentile) Math 15 (62nd percentile) Reading 12 (44th percentile), Science 17 (76th percentile), and Composite 15 (61st percentile). His performance on the EXPLORE test was exceptional. Specifically in mathematics, third grader Billy had outperformed the average eighth grader, who had 5 more years of experience in mathematics than he did. Once again, Billy's above-level test scores indicated his tremendous abilities and potential in mathematics and his need for differentiated programming.

Personnel at Billy's new school went to great lengths to create a mathematics curriculum that would satisfy both Billy and his parents. Billy's parents met with the school headmaster, the fourth-grade teachers, the upper school math teacher, and other parents to discuss ideas for the upcoming years with regard to Billy's mathematics curriculum. For fourth grade, Billy and five other students were separated into an accelerated mathematics group and were taught fifth-grade math. It worked well.

For fifth grade, after careful consideration, school personnel moved this group of six fifth-grade students directly into the sixth-grade accelerated math class. It was also decided that this acceleration plan would follow this group of students through the eighth grade. Again, this plan worked out extremely well. The sixth graders accepted the group of fifth graders immediately, both socially and academically. Billy was at the top of his class. He also participated and placed in several math contests at the sixth-grade level.

Presently, Billy is in the sixth grade, and he attends the seventh-grade accelerated prealgebra classes. He is at the "right" grade level for his age, although he is accelerated in math by one year. In addition to his prealgebra class five times a week, he attends a math computer class weekly. He is at the top of the class and has participated in several math contests at a seventh-grade level (e.g., Continental Math League, American Math Competition). He has earned straight A+'s in math since he started attending the private school in third grade.

Next year, when Billy is in seventh grade, he will attend eighth-grade Algebra I and plans to participate in several math contests offered at his school. He will also be eligible to try out for the school's MathCounts team. He expects to take Algebra II in eighth grade.

Billy's parents are very pleased with his progress. They believe that their decision to move him to a different school was critical. Billy is now in an environment where his talents are valued and nurtured. His mother commented,

> It is not easy to parent a gifted child. Over the years, we have had to hurdle many educational roadblocks because of the guidelines established by standard curricula. Most of the time,

> parents are the only advocates a gifted child has. We also learned that standing up for your gifted child is not always the "popular" thing to do. In fact, when we made the decision to move our son from the parochial school to the private school, certain parents [from the parochial school] started to act differently toward us. One comment was, "He can't be that smart. Who do they think they are?" Why should we feel the need to apologize for our son's exceptional abilities? But, that is exactly how many people have made us feel. . . . These thoughts of mine are important, I think, to other parents who are just beginning their journey into the world of educating the gifted child. I know that, when I first read *Jane and Johnny Love Math*, [*Developing Math Talent* is a follow-up to *Jane and Johnny*] it was comforting to realize that our family was not alone.

We are pleased that Billy and his parents are content with the decisions they have made and with the curricula that have been made available to Billy at the private school. We think it is interesting, however, that the private school made such an issue out of accelerating their mathematically talented students by one year. Clearly, the students were ready for that type of acceleration, and Billy might have benefited from even more radical acceleration in mathematics. In relation to that, we wonder why there was any question about what the students should do after completing fifth-grade math. What other choice should they have except to go on to sixth-grade math? Should they have repeated fifth-grade math? This lack of logical thinking doesn't surprise us because we hear these stories frequently, but we are always disappointed by it.

Arthur: "Is He Being Challenged Enough?"

Mrs. A. first contacted us when Arthur was a first grader in a suburban school district. She explained that he was in the school's gifted program and had demonstrated that he could do fourth-grade work. Arthur had shown high ability in mathematics since he was quite

young. For example, he could tell time at the age of 2. When he was 4, he understood sophisticated concepts such as, "There are 18 minutes left until I can go to preschool" or "I got home 23 minutes ago."

Arthur's mother explained that he didn't learn anything academic in kindergarten, with the possible exception of how to make 7s the right way. One day, in kindergarten, the students were talking about numbers, and the problem 10 – 14 came up. The teacher told the students that, "You can't take 14 from 10." Arthur replied, "Yes, you can. It's –4!"

Arthur was identified for the district's gifted program with a gifted IEP in kindergarten. Although Arthur's academic abilities were recognized at this young age, his school did not provide a special program for him in mathematics. He was in the district's gifted program, but it met only once a week for half a day. Arthur enjoyed the projects they did in the class, but the rest of the week, his math program was unaffected.

In the spring of second grade, Arthur was permitted to work on some third-grade mathematics problems in his regular second-grade classroom. His parents began considering whether or not he should move up a grade in mathematics and start taking fourth-grade math when he was in third grade. They did not have to make that decision, however, because they moved to another state in the summer following second grade.

The new district had much lower academic standards than their previous district. Arthur and his sister found they were repeating information they had previously learned. At this point, Arthur's parents insisted that he be given the opportunity to study fourth-grade mathematics in third grade. So, his teacher tested him and determined where his gaps were in third-grade math. She tutored him a bit on rounding, and he began studying fourth-grade math. Unfortunately, he studied this advanced math in the back of his third-grade classroom on his own while his teacher continued to teach the rest of the third graders in the regular mathematics class. An aide and a special needs student shared the same table with Arthur, and other students in the class often asked him questions and interrupted his work. His teacher reported that he "had trouble focusing" on his work. He earned B's

in math for most of the school year, although he got an A in the final quarter. Arthur felt he was being punished for being smart. He recognized that he could have received A's if he had stayed in third-grade math.

Arthur did find that he was challenged by the fourth-grade math he was studying. He learned how to do long division that year, but he found it difficult to learn on his own. He was used to knowing how to do math instinctively, so not knowing how to do something was a new experience for him.

After Arthur completed third grade, he and his family moved back to his former school district. We checked in on him when he was in fourth grade. His new school grouped students for mathematics. Arthur was in the "high" group, and, within that group, he was grouped with a smaller cluster of mathematically talented students. On the same day Arthur attended the half-day gifted program, the gifted teacher went into his math class and offered enrichment. So far that year, the students had been studying "Hands-On Equations" (an algebra program for elementary students), which Arthur enjoyed very much. This weekly math program ended after the fall semester and was replaced by a Great Books class for the second half of the year.

Once every 3 or 4 weeks, Arthur joined a group of mathematically talented students in a districtwide math program. Students qualified for this program through a two-step testing process: First, they had to score at the 98th percentile on the California Achievement Test, then they had to score 90% or above on a seventh- or eighth-grade math test. In this special program, students were bused to a central location for half a day every 3 weeks to do enrichment in mathematics, such as studying the Fibonacci sequence, which Arthur loved.

In the regular classroom, Arthur repeated the fourth-grade math material he had taught himself in the back of his third-grade classroom, partly because his parents were concerned that he might have some gaps in his background from last year's experience. The gifted coordinator in the district also advocated for not advancing Arthur. She said, "What's the rush? He'll end up in 11th grade with nothing to take in mathematics." His parents decided he could be accelerated in math later, if needed.

Arthur loves problem solving, and he loves to tinker with things. During the summer before fourth grade, he participated in Camp Invention (a program designed to promote creativity and problem solving) at a local school. One of his assignments was to take a broken clock radio apart and make an imaginary invention out of it. He thrived in this open atmosphere, saying, "I wish school could be like that."

Arthur's father gives him brain teasers to think about before he goes to bed. When Arthur comes in for breakfast the next morning, he has solved the problem. He loves having something to think about.

Arthur sometimes solves math problems in unconventional ways. For example, when doing three-digit addition, he always starts in the hundreds column and works toward the right. He says, "But on paper, I do it the right way, so I don't get in trouble."

Arthur still loves math. Is he challenged right now? "I don't know," replied his mother. "He gets 100s on most of his tests, and he doesn't study." Arthur and his family know he didn't like the way he studied math in third grade. He doesn't want to repeat that experience. They are pleased that he has been identified for a special math class in the district. Although they would like the class to meet more often than once every 3 weeks, they feel that being identified for special programming in math is a great benefit of the program. They believe that this current identification will lead to being recognized as mathematically talented in the future, and it will help draw attention to him for math programs as he moves on to high school.

Arthur took the EXPLORE tests in fourth grade and earned the following scores: English 17, Math 16, Reading 8, Science Reasoning 16, and Composite 14. These scores are exceptional, and Arthur clearly has the aptitude for accelerated study of mathematics.

Arthur's parents ask, "What are the advantages to accelerating Arthur, given [our] circumstances?" They know that he is not being stretched in math this year, but they think he needs to work on his language arts. He got a B in the subject in the first quarter, and he hates to read and write. They have decided to focus on that this year, in the hopes of making Arthur a little more well-rounded. They also believe he needs to practice math facts such as 7 x 8 and 6 x 7, so they don't think there is any harm in letting him have an extra year of computing.

Maintaining a balance in his life and being well-rounded are very important to the members of this family. Arthur is active in soccer, baseball, church youth group, and piano lessons. They see Arthur's strength in mathematics in contrast to his relative "weakness" in English and reading, and they are interested in helping him to achieve more in that area.

Arthur's parents are taking a conservative approach. They are not interested in "rushing" Arthur through school. They do recognize that he is repeating material he learned last year, and they are disappointed that he isn't being challenged more, although they appreciate the efforts their school district has made to group students and to offer periodic enrichment classes in mathematics.

School personnel have correctly diagnosed Arthur as being talented in mathematics and needing special programming. However, our belief is that Arthur's programming does not progress in a systematic fashion. Although they correctly identified Arthur as mathematically talented, the program offered to him is inadequate. It is simply not enough for him to participate in a challenging math class just once every 3 or 4 weeks.

We checked in on Arthur again when he was in fifth grade. By this time, his school district had developed a special accelerated mathematics program. Arthur and other gifted students were tested and identified for special programming (using the Diagnostic Testing→Prescriptive Instruction approach described in Chapter 4). Arthur is grouped with seven other advanced students for a math class that meets first period every day (three of the students are bused in from another school in the district). This mathematics class covers material that is 2 years above grade level. Next year, Arthur will be eligible for another advanced mathematics class that will result in his being accelerated in mathematics by 3 years. According to his mother, Arthur's performance in language arts has improved. She said it seemed as if he learned how to work harder as a result of the advanced math class, and the study skills he is beginning to learn have helped him in other areas. We are pleased that Arthur is finally being challenged.

Elizabeth: A Scientifically/Mathematically Gifted Female With Parents Who Did Not Want Her to Be Different

We became aware of Elizabeth because of a phone call from her mother, which occurred after many years of discussions (but little action) with school personnel. In those discussions, the most specific issue concerned whole-grade acceleration and its pros and cons. At that time, Elizabeth was in fourth grade and her mother was concerned about the lack of academic challenge in her daughter's education. Elizabeth's mother was worried that an unchallenging curriculum would have a negative impact on Elizabeth's self-esteem, possibly leading to boredom and eventually to acting-out behaviors. At the same time, Elizabeth's mother expressed concern that any recommended placement options not make her daughter "different from her classmates."

Our initial response to her request for our consultation was to highlight the need for specific, objective information prior to making any educational programming decisions, including acceleration. We also suggested that it would be best for the school to conduct any assessments (with our consultation) so that they would feel more ownership of the results, as well as the recommendations. We thought it was important to have collaboration among the school, the parents, and us. Fortunately for Elizabeth, the collaborative process was possible because of the very capable gifted education teacher who worked with her in the school's enrichment-focused pull-out program.

Elizabeth was one of the most able students in her school's pull-out program, which consisted of an hour of enrichment per week. However, her teacher for this enrichment program was the first to recognize that this level of programming was not providing Elizabeth with the systematic, high-level curriculum she needed in mathematics and science.

The first step in identifying Elizabeth's educational needs was to see how she compared to others in her grade. Elizabeth had consistently scored at the 99th percentile on the Iowa Tests of Basic Skills (ITBS); a careful examination of the item profile for her ITBS results indicated that she never missed an item. It was obvious that she was talented—

Table 9.1
Elizabeth's EXPLORE Math and Science Scores in Fifth Grade

EXPLORE test	Standard score	Talent search percentile	Eighth grade percentile
Math	13	48	48
Science Reasoning	21	93	93

achieving at a level that was higher than others in her grade—but the grade-level assessment could not reveal the level for which curriculum/instruction was most appropriate for her. The only "given" was the fact that Elizabeth and her parents were convinced that she needed a challenging mathematics curriculum. We decided to administer the quantitative sections from an above-level test. On that test, while in fourth grade, Elizabeth scored at the 79th percentile when compared to sixth graders.

We then used the Sequential Tests of Educational Progress (STEP) tests that were normed for grades 5–8, and compared Elizabeth's scores to spring-semester sixth graders. Elizabeth earned a score at the 87th percentile on the STEP Basic Concepts. On the same level of the STEP Computation test, Elizabeth earned a score at the 25th percentile—a score that was considerably lower than her Basic Concepts score. This did not surprise us because we had already anticipated that her computation skills, like those of many mathematically gifted students, lagged behind her conceptualization of mathematics (Rotigel, 2000). An interesting observation concerned the fact that Elizabeth was able to earn a high Computation score when she was allowed to complete the items in an untimed setting. Nevertheless, it was apparent that she needed to have a firmer foundation in computation. From the results of the diagnostic testing, we recommended that Elizabeth begin prealgebra as soon as possible. A late-spring meeting in Elizabeth's fourth-grade year that included teachers, parents, the principal, and us was set to identify instructional goals for systematically working through the prealgebra curriculum during fifth and sixth grade.

During these discussions, we addressed the parents' concerns about Elizabeth being "different" from her classmates. Interestingly enough, this was not and still is not a concern for Elizabeth. We also discussed the parents' concerns that their daughter might be stressed by doing advanced work. We shared with them our perspective on the differences between "stress" and "stretch." Because the parents felt the need for further discussions regarding their daughter's giftedness and its impact on her and their family, they were referred to a family counselor who had specific training in working with issues unique to families of gifted students.

Our impressions of Elizabeth were that she seemed like many academically able girls: self-effacing with respect to her abilities and simultaneously anxious to please her teachers. Fortunately, she had both a classroom teacher and an enrichment program teacher who very much wanted to see Elizabeth appropriately challenged and were willing to work together to differentiate the curriculum to do so.

These two teachers made a 1-year curriculum last for 2 years by "stretching" the prealgebra curriculum through fifth and sixth grade. Also, when Elizabeth was in fifth grade, she participated in the Elementary Student Talent Search. Her scores on EXPLORE were superior (see Table 9.1) and demonstrated exceptional scores in both math and science reasoning. Because she seemed to be appropriately "stretched" in science through an enrichment project in which she built a camera, less emphasis was placed on providing specific programming in this subject area.

When Elizabeth was in seventh grade, she was allowed to take algebra, which at that time was a rare occurrence for her district, because mathematically able students were only accelerated into algebra in grade 8. Unfortunately, she didn't have any math as an option in her school when she was in eighth grade, and she did not have the transportation to get to the high school for additional math classes. Because the district was not willing to provide programming for her, she played with a computer in the back of the mathematics classroom during her eighth-grade year. At that time, there was still not much available for summer programs in her area, so she was unable to take advantage of that sort of opportunity.

However, once she entered high school, Elizabeth took full advantage of the opportunities. She took as many science classes as were available, often "doubling up." She completed the college-entrance requirements fairly quickly and participated in the postsecondary enrollment option available in many districts. This opportunity permitted her to do research with a physics professor throughout her junior and senior years. More importantly, working with a college professor while still in high school gave her added confidence. She also got back on track in mathematics and completed calculus as a senior. When it came time to consider colleges, she applied and was accepted at prestigious technical institutes on both coasts, but chose the institution that fully covered her tuition (and is located on the warmer of the two coasts). Her declared major was physics. Later, she earned a master's degree in earth science and as a young adult has found employment with an environmental consulting firm.

In the first conversation with Elizabeth's mother, the concern about her daughter being "different" as a result of a program intervention was a major point of discussion. We clearly remember advising the mother that, with respect to a need for a very challenging curriculum, Elizabeth was different from her classmates and that our consultation would provide objective information that could be used to develop such a curriculum.

If we knew then what we know today about mathematically (and scientifically) talented students, we would have done things slightly differently in three areas. First, we would have addressed Elizabeth's talent in science, as well as in math. Additionally, we would have strongly insisted on long-term planning in both of her talent areas. This would have avoided the lost year in which Elizabeth sat at the back of the classroom and worked on the computer. Finally, we would have tried to "normalize" Elizabeth's talents. That is, we would have encouraged the mother's efforts at advocacy for her daughter and emphasized that her daughter is normal with some unique academic needs. Placing Elizabeth with other gifted students (such as in special classes, summer programs, and academic competitions) would also have given Elizabeth and her parents the chance to get to know other

talented youngsters and helped Elizabeth to realize there are other students who have the same academic interests and needs as she does.

Zach: The Preschool to Graduate School Journey: What Parents and Teachers Need to Know

Zach is a doctoral student in physics and likely on the path to an academic career where he will continue doing what he does as part of his doctoral program: reading papers, consulting with colleagues, working calculations and thinking about physical models, drafting papers and presentations, and attending presentations. Zach and his parents are, understandably, the heroes of his story, but so too are his elementary gifted education teacher and other educators and administrators who made decisions or used teaching strategies that positively influenced Zach's academic and personal life. His journey from preschool to graduate school serves as an inspiring testimony for the necessity of parents and educators to work together to ensure a differentiated and challenging curriculum.

Zach is the firstborn of three boys, all gifted. His precocity with letters and numbers began when he was a toddler and was apparent to his father, a university professor and administrator, and his mother, a physician. Zach has always been an avid reader; when in elementary school he especially enjoyed novels and comics and thought he would grow up to become a novelist or cartoonist.

Information from his parents coupled with his outstanding performance on standardized tests validated Zach's self-awareness of his academic giftedness. Erringly similar to observations made by the expert mathematicians in Bloom's (1985; see previous chapter) classic study, Zach's comments about elementary subjects—especially math—are an important part of his story:

> Elementary math is learning long division, in science one learns the parts of a flower or an ant, and in social studies one memorizes the countries of South America. In reading and writing however we had much more opportunity to explore

> material that we found interesting, and that was at our level. . . .
>
> Looking back on my elementary years now, I am forced to conclude that I spent a great deal of time learning facts, most of which I cannot now recall. That's not to say that students shouldn't learn factual information, but I don't think that that should be a goal in and of itself (which I think is how some teachers genuinely regarded it). It seems to me that the most important things my teachers did were to motivate, socialize, and to teach certain skills.

Outside of school, reading and the game of chess would prove to be important leisure activities for Zach. In school, Zach recalls that his fifth- and sixth-grade homeroom teacher provided substantial personal feedback on his reading journal and also engaged with him in intellectual discussions. When he was in fifth grade, his gifted education teacher became his prealgebra teacher and in sixth grade, he traveled to the middle school building to take algebra with seventh graders. Contrary to the implicit theories of some educators, which either assume that elementary students do not want to be in classes with middle school students or that middle school students will make fun of the younger student, Zach had a very positive experience with subject acceleration.

> In sixth grade I traveled by van to [the junior high school] to take Algebra with Mr. D. I was quite proud of it, and I believe I found math increasingly interesting, probably because the subject matter in math simply was becoming more interesting.
>
> I didn't like taking the van, which meant getting up earlier and missing other activities at [the elementary school]. Other than that, I don't recall finding the homework or other aspects of junior high education particularly burdensome. *I believe now, and believed then, that it was easily worth the extra effort* [emphasis added]. It was inconceivable to me that I would not have a math class, and I was told that taking the van to [the junior high school] was the best way to do it.

Zach's parents were continuous advocates for their son and, as Zach was finishing elementary school, his parents and educators, led by his elementary gifted and talented teacher, advocated for whole-grade acceleration in addition to the single-subject acceleration in math already in place. In fact, Zach's recollection of the process was that he did not actively pursue the option or even had input on the final decision, but he was kept informed during the process. Zach's insightful comment about whole-grade acceleration, as it pertained to him, captures the rationale generally expressed by gifted education professionals:

> I also believe my parents said that the motivation was not so much that [junior high school] would be especially challenging but that the acceleration would get me closer to the high school honors classes. I agreed with their reasoning, and certainly preferred more challenges to being bored in school. I recall feeling that I was being pressured to do something hard, but that I agreed with the reasoning.

Zach skipped the first year of a 2-year junior high school program and went from grade 6 (elementary school) to grade 8 (junior high school). This approach is supported by the *Iowa Acceleration Scale* (Assouline et al., 2009) process, which stresses that the final year in a building before going to the next level, known as the transition year, is critical to the success of an accelerated student. Zach remembers feelings of social isolation at the beginning, which he attributed both to the newness of the setting and the fact that he was younger than most of the students in that grade.

> The quasi-gifted class [with a focus on leadership and career exploration] was unmemorable academically, but did introduce me to a number of people with whom I would be good friends for years to come. My favorite class was Geometry. I spent the whole class working on math problems with other students. The teacher helped us when we got stuck and occasionally told us interesting stories.

When regular educators or school administrators are unsupportive of academic acceleration, the most common reason is rooted in the concern that children who are accelerated will not adjust well socially to the new class (Colangelo et al., 2004a). The research countering this argument is robust (see, for example, Colangelo et al., 2004a). Zach eloquently supports the research with his personal experiences:

> Most socializing in school happens without direct intervention by teachers. In class, I think it's important for students to work in groups. I think it's very important for a gifted student to not always be the smartest person in the room (or group). Placing a gifted student with other gifted students and/or older students seems to me beneficial for this reason. In high school I often had class with students two or three years older than me [*sic*], and generally preferred their company to that of my age-mates (who were by then a grade behind me). *On the whole I think that acceleration eventually helped me socially by allowing me to meet similar students. I think that an educational program can't be considered particularly successful if it has a negative impact on the child's social development* [emphasis added].

With respect to whole-grade acceleration, it is impossible to say with certainty that such a decision, at that moment in time, is the right one for any particular individual. That's why it is so important to hear from individuals regarding the long-term impact of acceleration. Extensive research (see Lubinski, 2004) on groups of accelerated individuals reveals that the long-term impact is positive, and the only "regret" is that individuals didn't accelerate sooner or experience more accelerative opportunities. Zach gives voice to the group data:

> I definitely think that skipping seventh grade was the right decision. Accelerating further would probably have been possible, but not [necessarily] beneficial. Even knowing where I would end up (and if I had to do it over again, I would end up right where I am now), I don't think there is any reason to narrow my academic focus earlier than I did. There are cer-

> tain classes I could see myself skipping, and I do wish there had been some way to take another math course in the spring semester of 12th grade. Ideally, I would have loved honors classes stretching all the way into junior high and perhaps elementary school, though I understand there are downsides to such "tracking." It would be bad if I only interacted with gifted students, but it's not clear to me what the downside would be if my academic interactions were more restricted to other gifted students, relegating interaction with a broader spectrum of my age-mates to nonacademic activity.

Because Zach took geometry as an eighth grader, he was on a trajectory for subject acceleration that would place him in calculus his junior year in high school. Zach's comments about this class and his teacher once again support our points about the need to have students in challenging classes that are taught by teachers who themselves are lifelong learners (see Chapter 8).

> Calculus was by far the most enjoyable of my many math classes, before or since. It answered so many questions that I had (tangent lines, areas, maximization), and gave the impression of being extremely useful. The teacher was an enthusiastic doctoral candidate [in math education] with the improbable name of [Demi Pi]. Perhaps it was because he himself was focused so hard on math and so enthusiastic about it (as opposed to teachers with only mild interest in the subject matter) that the class was so interesting.
>
> My senior year I took Honors Linear Algebra at [the university] through the school district's Post-Secondary Enrollment Option. I think many more students than actually utilize this program would benefit from it. Linear algebra was more abstract than I was used to, and also much more difficult. I had had homework before, but working for 10 or so hours a week and still not being able to solve many of the problems was shocking. Later, the freshman linear algebra course at [my undergraduate university] was a breeze. I felt

> good about taking the course, although I might have been bitter if the generous curve hadn't saved my class grade in the end.
>
> Unfortunately, there was no way to schedule an appropriate math class around my high school schedule in the second semester. I ended up taking a programming class in C+ +, which I haven't used since then. The material was similarly difficult, but not interesting or useful. I was also somewhat bitter about my first B+, which I attributed to a rounding error on the part of the [university] professor (he acknowledged it, but refused to correct the record).

Zach's recounting of his high school years is more detailed than space permits. In brief, he indicated that throughout high school he remained an avid reader but also spent a lot of time on chess tournaments and math club. In high school, he became very competitive in adult chess tournaments—he was the state champion. However, he did not continue with competitive chess during college for two main reasons: (a) his undergraduate courses were more challenging than high school, and (b) Ultimate Frisbee, which had been suggested by his mother, became his main leisure activity. In reflecting on the benefits of participation in Ultimate Frisbee during his undergraduate years, Zach commented that he enjoyed the competition, exercise, and social aspects of the game, even though he acknowledges that his team was more often on the losing side than the winning side. Also, the Ultimate Frisbee teams were slightly "less nerdy" than the chess competitors. He continues to play Ultimate Frisbee.

High school math club was a different story. Zach describes his high school's math club as a "serious force," with opportunities for regional math competitions where he made a number of friends and saw that others were sometimes equally talented or even more so. Also, the math club provided an important opportunity for leadership. In reflecting upon the role of competition in his cognitive and social development, he commented that perhaps he was "too competitive," and suggests that teachers should not use competition to motivate their gifted students, but to challenge them to do their best, "which is often

better than they [the students] think they can do rather than to focus on doing better than their classmates."

By his senior year in high school, Zach recognized that math and science were areas in which he was more able and more interested, relative to language arts. Although he applied to both general liberal arts colleges and technical universities, his visit to a private technical university sealed the decision to attend that school. He started out with chemistry as his major and—almost by chance—took a class in physics that he might have passed out of. Zach credits the excellent physics professor with making the experience much more enjoyable than that of his chemistry classes. He also declared a second major in economics, but realized that it was an area that would not be pursued in a graduate program.

> My two majors used math to describe broad sections of the physical and human worlds, which is what appealed to me about them. From that, we might reasonably say that it was physics' methodology, rather than the particular physical systems involved, that attracted me. I like solving problems and explaining things, and my best tool is math.

Zach's teachers and parents have provided nearly constant support, which has resulted in the maximization of his academic experiences. Zach clearly articulates the distinction in roles between parents and educators:

> Schools have a responsibility for the education of the student body as a whole, but it's up to parents to hold schools accountable for educating an individual child, particularly one [who] would benefit from special programming.
>
> I would say that it is important for a child to be challenged in school, if for no other reason than that they [*sic*] need to become accustomed to facing challenges. In that sense a gifted student enduring regular programming not only misses the opportunity to acquire advanced skills, but fails to receive the normal training that an average student experiences. Further,

> many careers which would challenge a gifted student (I think in particular of my own) expect the skills that can only be developed through advanced coursework. That is to say, the benefits of a personalized curriculum are helpful, if not essential, to success in a gifted student's likely life goals.

These statements provide solid support for parents and educators who advocate for program change to ensure that a curriculum is appropriately challenging. If a parent doesn't advocate for the child, who will? But advocacy cannot take place in a vacuum—the application of a differentiated, challenging curriculum must come from the educational system and is maximized when in-school and out-of-school experiences are combined.

Lessons Learned

As a result of working with Rosie, Christopher, Billy, Arthur, Elizabeth, Zach, and many other students, we have learned numerous lessons.

1. Parents know their kids. Their advocacy is a critical aspect of their child's academic development.

The parents of the students in our case studies noticed their young child's unusual facility with numbers. Christopher was 3 years old when his parents noted his precocious ability with numbers. As soon as he began expressing his observations about numbers, they began to take note, and their observations proved extremely useful in alerting educators to Christopher's abilities.

2. Mathematical ability can be recognized at a young age. Some of the early indicators of mathematical ability include:
    - Telling time early and showing an early interest in numbers (e.g., counting objects; Waxman et al., 1996a).

- An early sustained interest in manipulating numbers (e.g., 3-year-old Matthew spontaneously added the numbers he saw on a truck, 369 + 369, and shouted out, "738!").
- An ability to construct their own understanding of mathematics without direct instruction (e.g., Peter demonstrated an intuitive understanding of multiplication at the age of 2. He was counting out three raisins for each gingerbread man he and his mother were making. When he was told they would make four more cookies, he said, "Then we need 12 more raisins!").

These early indicators of mathematical ability are first observed typically by the parents, which again illustrates our earlier point that it is critical to listen to parents' stories about their children. However, the precocity of some mathematically talented students is not necessarily realized before the student starts school. At the time, Zach's parents did not think the things he did at a young age were especially remarkable because he was their firstborn and they didn't have any other young children with whom he could be compared.

3. Because school personnel often take a "wait and see" attitude or just say "no," parents need to be persistent in their advocacy efforts.

Over and over again, we hear stories about parents being dismissed by school personnel or told to "give it more time." Some of our favorite quotes include: "All kids need enrichment and that enrichment will satisfy his or her need." "We don't do acceleration here." "We accelerate only 1 year." "No student is ready for algebra before age 13 or 14." These statements have been made by real school personnel, and these ideas represent educational myths that we have attempted to dispel throughout this book.

We've also heard interesting stalling tactics, such as, "Let's put Anthony in the regular classroom this fall and see how it goes. If he has trouble, we'll test him in November. Then, it'll be Christmas vacation, and we shouldn't start any special programs. Let's plan to

start something in January." This is why it is important to plan in the spring for the following school year. Then, the principal and guidance counselor (and whoever else is involved with scheduling) can take into account a child's special needs when doing the scheduling. Christopher's parents did an excellent job using this strategy.

We have found that parents and gifted education teachers are usually the driving force behind discovering mathematically talented students and generating appropriate programming for them. Both parents and gifted educators need to be persistent at creating an appropriately challenging program. For Zach, his gifted education teacher was also his fifth-grade prealgebra teacher and that made all of the difference to his program. The circumstances with Zach evolved into a positive learning environment with his best interests in mind. Oftentimes teachers have less-than-adequate support from their administrator or from colleagues. We have sometimes experienced the situation where the teacher is willing to give a student more advanced material, but the principal will not support it. This is a difficult situation that can require the negotiating skills of a professional diplomat! It is always helpful to have an advocate within the system, but it is not appropriate to put that advocate at risk in his or her job situation.

4. School-based assessments are typically grade level in nature, and many educators are not aware of the need or procedure involved in above-level testing.

Billy's teachers used the standards for the primary grades as indicators of his achievement ("can recognize numbers 1–12"), but didn't attempt to measure what he could do beyond these minimum expectations or what he was ready to learn next. The information we presented in Chapters 3, 4, and 5 should be helpful in determining what level of test to give to a student and how to interpret the resulting scores. As we have stated repeatedly in this book, the goal is to determine what the student already knows and then move on from there. Rather than using grade-level tests that tell us what we already know (the student is performing very well compared to age-level peers), we offer students the opportunity to take a much more challenging test and

demonstrate their exceptional abilities. Although the approach we advocate is common sense, few educators have had specific training in providing this type of testing and programming for gifted students.

5. All assessment (testing) needs to be driven by a question.

In our case, the question is, "What is the extent of the child's mathematical ability and what types of programming modification does he or she need?" In this situation, IQ testing may not be necessary. Although an IQ gives a sense of overall ability and provides guidance regarding the pacing, it does not give the specific information needed to develop a systematic mathematics program. Billy's testing in kindergarten and first grade pointed out that he was an exceptionally bright child, but it didn't yield specific recommendations for his math program. The only obvious benefit from his IQ testing was that it helped to identify him for a gifted program, and this resulted in his working with an understanding teacher who became one of his advocates.

6. The Diagnostic Testing→Prescriptive Instruction model is useful for helping mathematically talented youth study mathematics at the appropriate level and pace.

The skills and content students have mastered in mathematics must be assessed carefully to ensure that instruction begins at the proper point, and the DT→PI approach provides the systematic means of doing so.

Both Zach and Elizabeth exemplified some of the issues that mathematically talented kids have to deal with in a district that focuses primarily on enrichment. They were lucky because they had teachers who were willing to go outside of the imposed enrichment curriculum and seek assistance through additional testing (DT→PI) to get the right "dosage" of mathematics. With Elizabeth, it wasn't perfect because there was no programming in eighth grade. For Zach, possible gaps in his math program were not an issue because the system was already in place when he needed it. (As Zach noted, the exception to this was the second semester of his senior year.)

7. Even extremely talented students need time to develop the cognitive structures that characterize mathematical maturity.

We are strong advocates of acceleration because it is often the only way in which a student can be truly challenged in mathematics. At the same time, we recognize that students (especially very young students) need time to develop intellectually. We are concerned that students who rush ahead before they are ready might learn advanced mathematics by rote without the benefit of deep understanding.

It is important for students to understand the logic underlying elementary mathematics problems before advancing to abstract concepts. We are also concerned that students not have large gaps in their mathematical backgrounds. Thus, while acceleration is appropriate for many mathematically talented students, at the same time it is being planned it is critical to look at how the time saved will be used. We also advocate using the DT→PI model to ensure that students do not have gaps in their mathematics background (see Chapter 4).

8. Mathematically talented students should study mathematics at a steady rate.

If a mentor-based program is implemented, it is not necessary for a student to meet individually with a mentor for several hours each week to advance in mathematics. Two 1-hour sessions with a mentor, with homework assigned between sessions, might be all of the challenge and stimulation a student needs. Students who do accelerate in mathematics also have the luxury of more time for studying enrichment topics in mathematics. This extra time should be put to good use. We encourage students to study enrichment topics that might not be a part of the regular curriculum (see Chapter 7).

Continuous study of mathematics 12 months out of the year is not necessary, and students do not need to race through the standard sequence of mathematics. What happened to Zach should be avoided: He could not take a math class his second semester of his senior year.

Long-term, continuous planning is essential for these students whose needs are very different from the norm.

In a similar vein, students do not need to continue an intensive study of mathematics during the summer. Consider other opportunities, especially participation in summer programs, or activities that allow the child to interact with intellectual peers. These programs and activities might be completely unrelated to mathematics or may be connected. For example, the student may find that an enrichment course in science is not only interesting and fun, but actually enhances the study of math!

9. Choices made when students are in elementary school may affect their high school mathematics program. Educators often fear that "students will run out of math" before they graduate from high school.

If the level and pace of instruction are matched to the abilities of mathematically talented students, by definition they will be accelerated. They may run out of mathematics courses that are offered by their school district before they finish high school, but there is plenty of mathematics to be learned. Students may need to take college courses part-time, work with a mentor, complete distance-learning or online courses, or enter college early in order to have access to an appropriate level of mathematics (see Chapter 6 and the Resources). For example, a young student, like 7-year-old Rosie, is well on her way to radical acceleration in mathematics. Therefore, it is critical to implement long-range planning so that (a) she develops the cognitive maturity to thrive in accelerated coursework and (b) she, her parents, and the educators in the system understand the ways in which her program will need to be adjusted to accommodate her needs.

10. Students may need to look outside of their school system for appropriate programs in mathematics and/or general enrichment opportunities.

These opportunities include summer and weekend programs offered at universities, online programs, mentor-paced programs, and competitions. All of the students included in our case studies took

advantage of programs offered by local universities, zoos, museums, and other educational groups, and their parents were constantly on the lookout for new and interesting opportunities for their children. (The Resources section provides information about many of these opportunities.)

Zach's retrospective on his experiences led to the recognition that such programs provided

> challenge [for] them as well as contributing to their social development and emotional maturation. The standard should not be, however, to present the student with the greatest academic challenge they [*sic*] might possibly meet. Too much schoolwork can discourage or unduly stress a student, or crowd out other important parts of life. There are of course resources for students outside of school. I heartily recommend intellectual pursuits such as chess, and private academic programs such as the ones offered by [university-based precollege programs].

11. Concerns about a student's social development shouldn't stop educators from making pedagogical adjustments.

There are a couple of ways to think about the relationship between social-emotional adjustment and academic challenge. In some cases, when students are appropriately challenged, their social-emotional adjustment seems to fall into place. It is also important to help mathematically talented students achieve a balance in their lives while also making sure that they study mathematics at the appropriate level. One of the most effective ways to enhance social development is to ensure that students have time to study other subjects, to play with friends, and to be alone.

Also, most accelerated students still have the opportunity to be with same-age peers in many other situations, such as scouts, religious youth groups, recess, and lunch, even if they are not grouped with their age peers for any of their academic activities. We encourage students to try out different types of activities, including sports and music, not

in an effort to be "well-rounded," but in an effort to be exposed to different situations and different people.

12. Mathematically talented girls may have special needs.

Research and personal experience has shown that it is important that mathematically talented girls be placed in a challenging, supportive atmosphere so that their talents can be fully realized. As we have mentioned elsewhere, we find many more mathematically talented boys than girls. Many more parents of boys than girls call us seeking our advice. This seems especially true with the youngest students. We have also noticed that parents of girls seem to be more concerned about social issues, stress the fact that they don't want their daughters to be different (as in Elizabeth's case), and are more willing to allow school personnel to talk them out of seeking accelerative options or other special educational programming.

We recommend that parents of mathematically talented girls be especially aware of the social pressures that encourage girls not to demonstrate their exceptional abilities. These students benefit from opportunities to explore mathematics in a supportive atmosphere, work with female role models, and study mathematics with other similarly talented girls. Experiences provided by programs such as those listed in the Resources may be especially important for their mathematical development. (A more detailed discussion about issues affecting mathematically talented females is presented in Reis & Gavin, 1999).

13. Talented students benefit from finding an intellectual peer group.

This is why academic summer programs and weekend programs are so valuable. Talented students can be placed together with others who have similar interests. They enjoy the rewards of being challenged intellectually while also becoming friends with students like themselves. Christopher and Zach both benefited from this type of experience during their regular school day; they were grouped with other mathematically talented students who also happened to be older than they were. The benefits of this type of grouping include the realization

that they are not the only ones who like and are good at math. This experience may be especially important for mathematically talented girls.

Although mathematically talented elementary and middle school students may not have many intellectual peers during the K–12 years, they are usually able to find peers once they attend college. This may also be possible while they are still in high school, as was the case with Zach. All parents worry about their child's social-emotional development and Zach's were no exception. Their son is now in graduate school and they have the "luxury" of knowing that their highly gifted child has grown into an individual who is a "good citizen," as Zach's dad says. Teachers and other parents may find it useful to know that Zach's father believes that, if he had it to do over again, he "would have worried less about the social aspect of Zach's development."

14. Most parents of talented students we have met are not "pushing" their children.

Most of the parents with whom we work are simply responding to their child's needs and interests. These parents have not been showing their children flashcards from birth. Billy's case illustrates this point clearly. He spontaneously demonstrated his capability to tell time, calculate, and read; his parents did not teach him these feats directly. Instead of pushing their children, these parents are being pulled by their children, who consistently demand, "More math!"

Conclusion

The case studies in this chapter illustrate many of the points we have made throughout this book. Mathematically talented students can be identified at a young age, and they benefit from programming that matches the curriculum and pace of instruction to their specific academic abilities and needs. Talent searches, first pioneered by Julian Stanley in 1972 for seventh graders and now offered by several universities for elementary, middle school, and junior high school

students, provide a systematic method for identifying precocious youth and providing appropriate programs for them. The Diagnostic Testing→Prescriptive Instruction (DT→PI) model, used by the talent searches and by a number of schools, has been effective in helping exceptionally talented students to study the level of mathematics appropriate for them.

Resources

This section presents general information and resources relevant for mathematically gifted students, their parents, and their teachers. It includes a listing of talent searches and academic programs for talented youth. Next, we indicate programs that provide for early entrance to college and distance-learning opportunities.

The section continues with an alphabetical listing of major competitions and contests, as well as a listing of printed materials, including periodicals and books and math games. This section concludes with a list of other resources, websites, and listservs.

Talent Searches and Academic Programs

Academic Talent Search (http://edweb.csus.edu/projects/ATS)
California State University, Sacramento, CA
Offers talent searches, as well as summer and weekend programs, for students in grades 4–9.

ADVANCE Program for Young Scholars (http://www.advanceprogram.org)
Northwestern State University, Natchitoches, LA
Offers Louisiana's only residential program for gifted youth in grades 8–12.

The Connie Belin & Jacqueline N. Blank International Center for Gifted Education and Talent Development (http://www.education.uiowa.edu/belinblank)
The University of Iowa, Iowa City, IA

Offers talent searches (Belin-Blank Exceptional Student Talent Search; BESTS) for students in grades 2–9 and commuter and residential programs for students in grades 3–12.

Canada/USA Mathcamp (http://www.mathcamp.org)
Cambridge, MA
Offers a summer program for mathematically talented students ages 13–18.

Carnegie Mellon Institute for Talented Elementary and Secondary Students (C-MITES; http://www.cmu.edu/cmites)
Carnegie Mellon University, Pittsburgh, PA
Offers a talent search for students in grades 3–6 and commuter programs throughout Pennsylvania for students in grades K–9.

Center for Bright Kids (http://www.centerforbrightkids.org)
Westminster, CO
Offers talent searches for students in grades 3–9 and residential and commuter programs for students ages 11–16.

Center for Talent Development (http://www.ctd.northwestern.edu)
Northwestern University, Evanston, IL
Offers talent searches for students in grades 4–9 and summer and weekend programs, commuter and residential, for students in grades K–12.

Center for Talented Youth (CTY; http://www.cty.jhu.edu)
Johns Hopkins University, Baltimore, MD
Offers commuter and residential programs for elementary and secondary students.

Frances A. Karnes Center for Gifted Studies (http://www.usm.edu/gifted)
The University of Southern Mississippi, Hattiesburg, MS
Offers programs for students in grades pre-K–12.

Gifted Education Resource Institute (GERI; http://www.geri.soe.purdue.edu)
Purdue University, West Lafayette, IN
Offers programs for students in grades pre-K–12.

Gifted Students Institute (http://smu.edu/education/gsi)
Southern Methodist University, Dallas, TX
Offers summer and academic-year programs for students in grades 7–11.

Hampshire College Summer Studies in Mathematics (http://www.hcssim.org)
Hampshire College, Amherst, MA
Offers programs for mathematically talented and motivated high school students.

Office of Precollegiate Programs for Talented and Gifted (OPPTAG; http://www.opptag.iastate.edu)
Iowa State University, Ames, IA
Offers summer classes in a variety of subjects for students in grades 3–11.

Program in Mathematics for Young Scientists (PROMYS; http://math.bu.edu/people/promys)
Boston University, Boston, MA
Offers a challenging program designed to encourage ambitious high school students to explore the creative world of mathematics.

Research Science Institute (http://www.cee.org/programs/rsi)
Boston, MA
Offers a mentor program in the sciences for rising high school seniors.

The Ross Mathematics Program (http://www.math.ohio-state.edu/ross)
Ohio State University, Columbus, OH
Offers programs and intense math courses for 14- to 17-year-olds deeply interested in math and science.

Summer Program for Verbally and Mathematically Precocious Youth, The Center for Gifted Students (VAMPY; http://www.wku.edu/gifted)
Western Kentucky University, Bowling Green, KY
Offers a summer program for verbally and mathematically talented students in grades 6–11.

Talent Identification Program (TIP; http://www.tip.duke.edu)
Duke University, Durham, NC
Offers summer programs and other services for students in grades 4–12.

University of Minnesota Talented Youth Mathematics Program (UMTYMP; http://mathcep.umn.edu/umtymp)
University of Minnesota, Minneapolis, MN
Offers a program in accelerated mathematics for students in grades 6–12.

Wisconsin Center for Academically Talented Youth (WCATY; http://www.wcaty.org)
University of Wisconsin-Madison, Madison, WI
Offers programs for academically talented students in grades 4–12.

Early Entrance to College Programs

Accelerated College Entrance Program (http://www.educ.csus.edu/projects/ace)
California State University, Sacramento, CA
Allows students in grades 11–12 to enroll in college courses while completing their high school studies.

Advanced Academy of Georgia (http://www.westga.edu/~academy)
University of West Georgia, GA
Allows students to complete 11th and 12th grade and the first 2 years of college simultaneously.

Bard College at Simon's Rock (http://www.simons-rock.edu)
Bard College at Simon's Rock, Great Barrington, MA
Offers early entrance to college for students who have completed 10th grade.

Clarkson School (http://www.clarkson.edu/tcs)
Clarkson University, Potsdam, NY
Offers early entrance for students who have completed 11th grade.

The Davidson Academy (http://www.davidsonacademy.unr.edu)
University of Nevada, Reno, NV
Provides a free public school for exceptionally talented middle and high school students; students attend middle and high school classes at the academy and have access to university courses for advanced studies.

The Halbert and Nancy Robinson Center for Young Scholars (http://www.depts.washington.edu/cscy)
University of Washington, Seattle, WA
Offers early entrance to students younger than 15 years old.

The National Academy of Arts, Sciences, and Engineering (http://www.education.uiowa.edu/belinblank/old/programs/naase)
Offers early entrance to The University of Iowa for high school students who have completed the equivalent of 11th grade.

Program for the Exceptionally Gifted (http://www.mbc.edu/peg)
Mary Baldwin College, Staunton, VA
Offers a program to girls as early as the eighth grade; students generally complete their bachelor's degree within 4 years.

Texas Academy of Math and Science (http://www.tams.unt.edu)
University of North Texas, Denton, TX
A 2-year residential program for Texas high school students. Students simultaneously complete high school and the first 2 years of college.

Distance Learning

Advanced Placement Program (http://apcentral.collegeboard.com)
The College Board, Princeton, NJ
Offers online courses through APEX Learning Corporation (http://www.apexlearning.com/Solutions/AP.htm) to students who do not have access to AP courses in their high schools (many states have state-sponsored grants to pay for online AP courses through APEX).

Education Program for Gifted Youth (EPGY; http://epgy.stanford.edu)
Stanford University, Stanford, CA
Offers courses in mathematics, computer science, chemistry, physics, biology, languages, history, and music for academically talented students in elementary school through college.

The University of Nebraska Independent Study High School (http://highschool.unl.edu)
University of Nebraska–Lincoln, Lincoln, NE
Offers more than 80 online high school courses (no age or grade requirement to take the courses).

Contests and Competitions

American Mathematics Competitions (http://amc.maa.org)
The Mathematical Association of America
Offers a series of national contests available to any student who has not graduated from high school; high-scoring students move on to the American Invitational Mathematics Exam, USA Mathematical Olympiad, and International Mathematical Olympiad.

American Regions Mathematics League (ARML; http://www.arml.com)
ARML is held at four sites: Penn State University; The University of Iowa; University of Nevada, Las Vegas; and The University of Georgia.

It offers an annual national mathematics competition for high school students.

Future Problem Solving Program International, Inc. (http://www.fpsp.org)
Offers curricular and co-curricular competitive and noncompetitive activities in creative problem solving.

Intel Science Talent Search (formerly the Westinghouse Science Talent Search; http://www.sciserv.org/sts)
Society for Science & the Public, Washington, DC
Offers a prestigious science competition for high school seniors who submit independent research projects; winners receive college scholarships.

Junior Engineering Technical Society (JETS; http://www.jets.org)
Offers competitions and programs to high school students to promote interest in engineering, science, mathematics, and technology.

Math Olympiads for Elementary and Middle Schools (MOEMS; http://www.moems.org)
Offers an in-school academic year competition for students in eighth grade and younger; two divisions: E for grades 4–6, and M for grades 6–8.

MATHCOUNTS (http://www.mathcounts.org)
Offers a series of competitions designed for middle school students; the competitions have both written and oral rounds, as well as individual and team components.

National Merit Scholarship Corporation (http://www.nationalmerit.org)
Offers an opportunity for students scoring high on the PSAT (taken in 11th grade) to advance to other levels in the competition; winners receive college scholarships.

Science Olympiad (http://www.soinc.org)
Offers competitions, classroom activities, and training workshops used to improve the quality of science education, increase students' interest in science, and provide recognition for outstanding achievement in science education by both students and teachers.

U.S. National Chemistry Olympiad (http://portal.acs.org/portal/PublicWebSite/education/students/highschool/olympiad/index.htm)
Offers a multitiered science competition for high school students.

U.S. Physics Team (http://www.aapt.org/physicsteam/2010)
Offers a high school competition sponsored by the American Association of Physics Teachers and the American Institute of Physics.

Printed Materials

Books

Assouline, S. G., Colangelo, N., Lupkowski-Shoplik, A. E., Lipscomb, J., & Forstadt, L. (2009). *Iowa acceleration scale* (3rd ed.). Scottsdale, AZ: Great Potential Press.

This guidance instrument provides a systematic and thorough method of decision making for educators and parents who are considering whole-grade acceleration for students in kindergarten through eighth grade.

Colangelo, N., & Davis, G. A. (Eds.). (2003). *Handbook of gifted education* (3rd ed.). Needham Heights, MA: Allyn & Bacon.

This is a text commonly used in graduate courses in gifted education.

Center for Gifted Education. (2004). *Resource guide to mathematics curriculum materials for high-ability learners in grades K–8*. Williamsburg, VA: Author. (Available from http://cfge.wm.edu/

Documents/Resource%20Guide%20to%20Mathematics%20Curriculum%20Materials.pdf)

This resource provides guidance to teachers and parents about selecting mathematics curriculum.

Georgiev, V., Ulovec, A., Mogensen, A., Mushkarov, O., Dimitrova, N., & Sendova, E. (2008). *Meeting in mathematics*. Sofia, Bulgaria: Demetra.

This provides guidance to teachers and parents about selecting mathematics curriculum.

Karnes, F. A., & Riley, T. L. (2005). *Competitions for talented kids: Win scholarships, big prize money, and recognition*. Waco, TX: Prufrock Press.

This book lists many different contests and competitions for students.

Lupkowski-Shoplik, A. E., & Assouline, S. G. (n.d.) *Building gifted programs for math talented students: Tools for teachers*. Author.

This resource is a "how-to" manual that explains how to apply the theories described in *Developing Math Talent*.

Peterson's. (2009). *Summer programs for kids & teenagers*. Lawrenceville, NJ: Author.

This publication is a source of information about summer programs and is updated annually. One can order copies through a local bookstore or online.

Saul, M., Assouline, S., & Sheffield, L. (Eds.). (2010). *The peak in the middle: Developing mathematically gifted students in the middle grades*. Reston, VA: National Council of Teachers of Mathematics.

This resource is written for teachers. It summarizes the current research and program options for mathematically talented students in middle school.

Waxman, B., Robinson, N. M., & Mukhopadhyay, S. (1996). *Teachers nurturing math-talented young children* (RM96228). Storrs: University of Connecticut, The National Research Center on the Gifted and Talented.

This book focuses on mathematically talented young students in preschool and primary grades.

Curriculum Resource Books

Fitzgerald, W., Winter, M. J., Lappan, G., & Phillips, E. (1986). *Middle grades mathematics project: Factors and multiples*. Menlo Park, CA: Addison-Wesley.

This book is one in an excellent series of hands-on math books. We highly recommend the Middle Grades Mathematics Project series. We have used these books extensively in C-MITES classes.

Johnson, C. V. (1994). *Packaging and the environment: Real-world mathematics through science*. Menlo Park, CA: Addison-Wesley.

This is another book in the Middle Grades Mathematics Project series.

Johnson, D. T. (2008). *Beyond base ten: A mathematics unit for high ability learners in grades 3–6*. Waco, TX: Prufrock Press.

This is a curriculum unit developed by the Center for Gifted Education at The College of William and Mary.

Johnson, D. T. (2008). *Spatial reasoning: A mathematics unit for high-ability learners in grades 2–4*. Waco, TX: Prufrock Press.

This is a curriculum unit developed by the Center for Gifted Education at The College of William and Mary.

Kleiman, A., Washington, D., & Washington, M. F. (1996). *It's alive!* Waco, TX: Prufrock Press.

This is a fun book of real-life math problems, such as "Have you ever wondered how many earthworms are in a football field?"

Kremer, R. (1995). *From crystals to kites: Exploring three dimensions*. Palo Alto, CA: Seymour.

This is a great curriculum resource.

Kremer, R. (1989). *Exploring with squares and cubes*. Palo Alto, CA: Seymour.

This book was used in an informal geometry class for grades 3–5.

Kroner, L. R. (1997). *Slides, flips and turns*. Palo Alto, CA: Seymour.

This book was used in an informal geometry class for grades 3–5.

Lappan, G., Fitzgerald, W., Winter, M. J., & Phillips, E. (1986). *Middle grades mathematics project: Similarity and equivalent fractions*. Menlo Park, CA: Addison-Wesley.

This is another book in the Middle Grades Mathematics Project series.

O'Connor, V. F., & Hynes, M. C. (1997). *Mission mathematics: Linking aerospace and the NCTM standards*. Reston, VA: National Council of Teachers of Mathematics.

C-MITES teachers use this resource in several math classes.

Pentagram. (1989). *Puzzlegrams*. New York: Simon & Schuster.

This is a hard-to-find book available in used bookstores, but it's worth the trouble to find it. Pentagram has also developed *More Puzzlegrams*, which, like its predecessor, is packed with interesting math puzzles and gorgeous graphics.

Pfenning, N. (1998). *Chances are: Making probability and statistics fun to learn and easy to teach*. Waco, TX: Prufrock Press.

This book was written by a C-MITES teacher, and it contains the curriculum she has used in her extremely successful and hands-on Probability and Statistics summer class, taught to gifted fifth through seventh graders.

Phillips, E., Lappan, G., Winter, M. J., & Fitzgerald, W. (1986). *Middle grades mathematics project: Probability*. Menlo Park, CA: Addison-Wesley.

This is another book in the Middle Grades Mathematics Project series.

Posamentier, A. S., & Salkind, C. T. (1988). *Challenging problems in geometry.* Palo Alto, CA: Seymour.

This book was used in an informal geometry class for grades 3–5.

Roper, A. (1991). *Cooperative problem solving with tangrams*. Sunnyvale, CA: Creative Publications.

This resource was used in several classes for mathematically talented students.

Serra, M. (2002). *Discovering geometry: An investigative approach* (3rd ed.). Emeryville, CA: Key Curriculum Press.

This book is used in the Center for Talented Youth geometry course.

Seymour, D. (1971). *Tangramath*. Sunnyvale, CA: Creative Publications.

This is a great curriculum resource.

Seymour, D., & Britton, J. (1986). *Introduction to tessellations*. Palo Alto, CA: Seymour.

Tessellations give mathematically talented students a chance to show their artistic capabilities.

Sherard, W. H., III (1995). *Cooperative informal geometry*. Palo Alto, CA: Seymour.

This is a great curriculum resource.

Shroyer, J., & Fitzgerald, W. (1986). *Middle grades mathematics project: Mouse and elephant: Measuring growth*. Menlo Park, CA: Addison-Wesley.

This is another book in the Middle Grades Mathematics Project series.

Stenmark, J. K., Thompson, V., & Cossey, R. (1986). *Family math*. Berkeley: University of California, Lawrence Hall of Science.

This book contains activities for parents to use at home with their children. It is also an excellent resource for school personnel who want to organize a "family math night."

Thompson, F. M. (1994). *Hands-on math!: Ready-to-use games and activities for grades 4–8*. San Francisco, CA: Jossey Bass.

This publication provides some popular math games.

Wickelgren, W. A., & Wickelgren, I. (2001). *Math coach: A parent's guide to helping children succeed in math*. New York, NY: Berkley Books.

This is a resource directed at parents.

Winter, M. J., Lappan, G., Phillips, E., & Fitzgerald, W. (1986). *Middle grades mathematics project: Spatial visualization*. Menlo Park, CA: Addison-Wesley.

This is another book in the Middle Grades Mathematics Project series.

Willoughby, S. S. (1991). *Mathematics for a changing world*. Alexandria, VA: Association for Supervision and Curriculum Development.

This is a great curriculum resource.

Zaccaro, E. (2001). *Real world algebra*. Bellevue, IA: Hickory Grove Press.

Ed Zaccaro provides a series of books aimed to help teachers challenge their mathematically talented students.

Zaccaro, E. (2005). *Challenge math for the elementary and middle school student* (2nd ed.). Bellevue, IA: Hickory Grove Press.

Fun lessons in this book teach difficult concepts in areas such as statistics, algebra, physics, and astronomy.

Zaccaro, E. (2006). *Becoming a problem solving genius*. Bellevue, IA: Hickory Grove Press.

This book provides math students with problem-solving strategies for solving word problems.

Periodicals

Advanced Development Journal (http://www.gifteddevelopment.com/ADJ/adj.htm)
Institute for the Study of Advanced Development
Offers the first journal on adult giftedness.

C-MITES News (http://www.cmu.edu/cmites/news.html)
Carnegie Mellon Institute for Talented Elementary and Secondary Students.
Provides information about C-MITES programs and other opportunities for gifted students.

Gifted Child Quarterly (http://www.nagc.org/index.aspx?id=979)
National Association for Gifted Children
Contains articles of interest to professionals and those with some background in the field of gifted education.

Gifted Child Today (http://journals.prufrock.com/IJP/b/gifted-child-today)
Prufrock Press
Directed at teachers and parents, this journal avoids jargon and provides practical advice on working with gifted, creative, and talented children.

Imagine (http://cty.jhu.edu/imagine/index.html)
Center for Talented Youth, Johns Hopkins University
Provides information for academically talented students.

Journal of Advanced Academics (http://journals.prufrock.com/IJP/b/journal-of-advanced-academics)
Prufrock Press
Focuses on research that supports and enhances advanced academic achievement for students of all ages (formerly *The Journal of Secondary Gifted Education*).

Journal for the Education of the Gifted (http://journals.prufrock.com/IJP/b/journal-for-the-education-of-the-gifted)
The Association for the Gifted (TAG), a division of the Council for Exceptional Children
Offers a journal aimed at the experienced reader of the literature in gifted education.

National Council of Teachers of Mathematics (http://www.nctm.org/publications/default.aspx?id=218)
Publishes journals including *Teaching Children Mathematics*, *Mathematics Teaching in the Middle School*, and *Mathematics Teacher*.

Parenting for High Potential (http://www.nagc.org/index.aspx?id=1180)
National Association for Gifted Children
Offers an excellent magazine designed for parents.

Roeper Review (http://www.roeper.org/RoeperInstitute/roeperReview/index.aspx)
The Roeper School
Offers a publication designed for professionals; the articles are research-based and often deal with both theoretical and practical issues concerning gifted individuals.

Teaching for High Potential (http://www.nagc.org/index.aspx?id=1498)
National Association for Gifted Children
Offers an excellent publication designed for educators.

Understanding Our Gifted (http://www.our-gifted.com)
Open Space Communications, Inc.
Offers an online journal for parents, educators, and counselors.

Vision (http://www.education.uiowa.edu/belinblank/about_us/newsletter.aspx)
The Connie Belin & Jacqueline N. Blank International Center for Gifted Education and Talent Development, The University of Iowa
Offers a newsletter filled with information about the Belin-Blank Center.

Mathematics Games and Puzzles

The Soma Cube (http://www.fam-bundgaard.dk/SOMA/SOMA.HTM and http://en.allexperts.com/e/s/so/soma_cube.htm)
The Soma Cube, first discovered by Piet Hein, is composed of 27 cubes that can be glued together to form various shapes. These shapes can then be arranged to make a cube with side length 3. This is a fascinating puzzle, and it can be solved 240 different ways.

Other Logic Games
Some of our favorites include: Battleship, checkers, chess, Connect Four, dominoes, GO, Mastermind, Othello, and Pente. These games

require players to think in a logical, organized fashion and to use their reasoning skills.

Sources of Math Enrichment Materials: Books and Manipulatives

AMSCO School Publications
http://www.amscopub.com

ETA/Cuisenaire
http://www.etacuisenaire.com

Free Spirit Publishers
http://www.freespirit.com

Hickory Grove Press
http://www.hickorygrovepress.com

The Math Forum @ Drexel University
http://mathforum.org/resources_tools_landing.html

National Council of Teachers of Mathematics
http://www.nctm.org

Pearson
http://www.pearsonschool.com

Prufrock Press
http://www.prufrock.com

The Zephyr Catalog
http://www.zephyrpress.com

Computer Software

See Chapter 7 (Curricula and Materials) for a discussion about selecting computer software.

Math Forum (http://mathforum.org/mathed/math.software.reviews.html)
Offers a page of links to other websites that offer software reviews.

Hoagies' Gifted Education Page (http://www.hoagiesgifted.org/software.htm)
Provides up-to-date software reviews.

Organizations

Most states have a state organization to promote advocacy for gifted and talented students at the state and local level; provide preservice and in-service training in gifted education; and support parent/community awareness, education, and involvement. See the National Association for Gifted Children's website (http://www.nagc.org) for specific information by state.

American Association for Gifted Children (http://www.aagc.org)
This is an association housed at Duke University.

The Association for the Gifted (TAG; http://www.cectag.org)
TAG is a special interest group of the Council for Exceptional Children (CEC).

Davidson Institute for Talent Development (http://www.davidsongifted.org)
Provides resources to families and educators of exceptionally talented students, advocates for gifted children, and maintains a comprehensive database of articles and resources pertaining to gifted students.

The Hollingworth Center for Highly Gifted Children (http://www.hollingworth.org)
Offers a network for highly gifted children, their families, and their schools and communities.

National Association for Gifted Children (NAGC; http://www.nagc.org)
NAGC hosts an annual convention and publishes three periodicals: a magazine for parents (*Parenting for High Potential*), a magazine for teachers, (*Teaching for High Potential*), and a journal for professionals (*Gifted Child Quarterly*). As an organization, its purpose is to serve parents, educators, community leaders, and other professionals who work on behalf of gifted children.

National Council of Teachers of Mathematics (NCTM; http://www.nctm.org)
NCTM is an organization that supports mathematics teachers.

Supporting Emotional Needs of the Gifted (SENG; http://www.sengifted.org)
SENG focuses on enhancing the social and emotional lives of gifted children and adults.

TAG Families of the Talented and Gifted (http://www.tagfam.org)
Maintains several different e-mail lists encouraging families of gifted children to interact with each other.

Websites, Listservs, and Social Networking

Our Favorites

Art of Problem Solving (http://www.artofproblemsolving.com)
Offers online resources and classes specifically designed for mathematically talented students. We highly recommend investigating this comprehensive website! For example, the organization administers

an online resource, the AoPSWiki, for supporting educational content useful to avid math students. The AoPSWiki is meant to be easily readable by even the least advanced student (see http://www.artofproblemsolving.com/Wiki/index.php/Main_Page). It also provides a series of articles of interest to "students of problem solving" (http://www.artofproblemsolving.com/Resources/articles.php).

The Davidson Institute for Talent Development (http://www.davidsongifted.org)
This is a national nonprofit organization dedicated to recognizing, nurturing, and supporting profoundly gifted young students under the age of 18. The Davidson Institute website is filled with helpful information for parents and teachers, especially the link to the Davidson Database.

Hoagies' Gifted Education Page (http://www.hoagiesgifted.org)
This is a classic resource for gifted education. One of the pages is specifically for mathematically talented students (http://www.hoagiesgifted.org/math.htm). It contains links to many other math-related websites that students will enjoy.

Institute for Research and Policy on Acceleration (http://www.accelerationinstitute.org)
Hosted by The University of Iowa's Belin-Blank Center, this website is a comprehensive resource dedicated to the study and support of curricular acceleration for advanced students. The website provides resources, including a free slide presentation about academic acceleration, an annotated bibliography, individual stories of acceleration, guidelines for establishing acceleration policies, and access to the nine translated versions of Volume I of *A Nation Deceived*.

The Math Forum @ Drexel (http://mathforum.org)
This is an excellent resource for math students and teachers. Students can find answers to their burning math questions in the Ask Dr. Math section. The Internet Mathematics Library offers lesson plans, resources, and games. The Problems of the Week are designed to provide creative,

nonroutine challenges for students in grades 3–12. Other sections on this site include mathematics appropriate for kindergarten through graduate school.

Other Recommended Resources

American Statistical Association (http://www.amstat.org/education/publications.cfm)
This organization offers teachers resources for teaching statistics, grades K–12.

Center for Gifted Education, The College of William and Mary (http://cfge.wm.edu/curr_math.htm)
This site offers mathematics materials, links to math resources, and other information pertinent to parents and educators.

EarlyEntrance.org (http://earlyentrance.org/Home)
This is a comprehensive site developed by a student who entered college early that is very informative for both parents and students.

KidSource (http://www.kidsource.com/kidsource/pages/ed.gifted.html)
This site offers a list of resources and links focused on the education of gifted and talented students.

Mega-Mathematics (http://www.ccs3.lanl.gov/mega-math)
This site discusses fun mathematics topics and provides other interesting links.

FREE—Federal Resources for Educational Excellence: Math (http://free.ed.gov/subjects.cfm?subject_id=33)
This site offers resources for algebra, geometry, data analysis, and number and operations.

Mathematics Education at Northern Kentucky University (http://www.nku.edu/~mathed/gifted.html)
This site offers resources for parents, teachers, and students.

National Association for Gifted Children (http://www.nagc.org)
This site includes resources for educators and parents, webinars (online seminars), books, tools for advocacy, and other similar materials.

National Research Center on the Gifted and Talented (http://www.gifted.uconn.edu/nrcgt.html)
The National Research Center on the Gifted and Talented is funded under the Jacob K. Javits Gifted and Talented Students Education Act and is housed at The University of Connecticut.

The Math Open Reference Project (http://www.mathopenref.com/site/about.html)
This site provides excellent resources in geometry. Follow the links to the section on convex polygons, for example, and you'll find an interactive page where the student can move the vertices and notice what it takes to make the polygon convex or concave.

Project M^3 Home Activities and Student Resources (http://www.gifted.uconn.edu/projectM3/students_activities.htm)
This site provides fun math activities for students to do at home.

Renzulli Learning Systems (http://renzullisystemsvm.renzullilearning.com)
Developed by the University of Connecticut Research and Development Corporation, this search engine offers individualized enrichment options based upon the student's interest area and learning style.

Social Networking
University programs for gifted students often maintain websites and social networks to encourage their students and parents to interact with each other. See, for example:

- Center for Talent Development at Northwestern University (http://www.ctd.northwestern.edu)
- Center for Talented Youth at Johns Hopkins University (http://cty.jhu.edu)
- Talent Identification Program at Duke University (http://www.tip.duke.edu)

Teachers (http://www.loc.gov/teachers)
This Library of Congress site offers classroom materials, professional development, and other resources for teachers.

In addition to the websites described above, there are many listservs that are designed to keep people informed of current events. Many educators have indicated that they find the listserv hosted by the professional development program at The University of Iowa's Belin-Blank Center to be very informative. To subscribe to the Gifted Teachers e-mail list, send an e-mail to LISTSERV@LIST.UIOWA.EDU and, in the text of your message (not the subject line), write: SUBSCRIBE GIFTED-TEACHERS First-Name Last-Name.

Glossary

Ability grouping: Students are identified according to their cognitive ability or according to their ability in a specific content area (e.g., mathematics) and are grouped accordingly. Ability grouping is sometimes referred to as homogeneous grouping, which is in contrast to heterogeneous or mixed groupings of students with a range of ability levels.

Ability test: Usually refers to a standardized test that assesses an individual's competence in a domain that has been defined as relevant to the purpose of training. This may be a cognitive domain (e.g., general intelligence), a physical domain (e.g., swimming), or a creative domain (e.g., composing music).

Above average: If 100 students' scores are plotted on a graph, a bell curve would likely be generated. Most of the scores will fall in the middle of the curve. Average scores are those in the middle two thirds of the curve. Students who earn scores above the average scores are "above average." Compared to the other 100 scores, their scores are better than 85%; they are at the 85th percentile or above. The top 3%–5% of scores earned by a group of 100 typical students of a particular age and/or grade—those whose scores are at or above the 97th or 95th percentile—are sometimes considered for gifted programming.

Above-level test: A test designed for older students that is given to younger, academically talented students. The above-level test is presumably more difficult than grade-level tests. Having more difficult items helps us to measure the students' abilities more accurately and to devise more appropriate programs. The concept of

above-level testing is the foundation upon which the talent search model was built.

Above-level testing: Giving a test to younger students that was developed for students who are several years older. Also called out-of-level testing or beyond-level testing.

Acceleration: Moving at a faster pace through the curriculum than is typical. This can be accomplished by entering kindergarten, first grade, or college early (early admission); skipping an entire grade (grade skipping); or advancing in a subject area more rapidly than is typical (subject-matter acceleration).

Achievement test: Test that measures what a student has learned. It is usually given to students after completing a course or topic. Achievement test results are also useful for adapting instruction to individual needs because this information helps instructors determine what topics a student does or does not know.

Age-equivalent: Students' scores on tests are sometimes compared to the performance of a typical child at a given age level. A student's score on a test will correspond to the highest year/grade level that he or she can successfully complete. However, caution needs to be used in drawing conclusions regarding most age-equivalent scores, as scores at the extremes are usually extrapolations and not actually based upon performance. For example, a 6-year-old who earns an age-equivalent of 14 years on a reading test has actually earned an age-equivalent score that is based upon an extrapolation of data, not upon an assessment of 14-year-olds.

Age-in-grade grouping: Placing a student with other students of the same chronological age and grade level.

Aptitude test: Test that is used to predict performance in specific areas, such as verbal comprehension, mathematical ability, mechanical aptitude, spatial relations aptitude, or nonverbal reasoning ability.

Aptitude tests have more specific content than intelligence tests do (i.e., they measure only one or a few abilities, rather than a wide variety of abilities).

Assessment: A comprehensive and systematic approach to acquiring detailed information about an individual in order to generate data that can be used to formulate recommendations about the individual's educational needs.

Breadth/depth approach: Typically used in conjunction with discussions about enriching the curriculum. The goal of the breadth/depth approach is not only to make a quantitative change in the curriculum, but also to ensure that qualitatively the curriculum reaches deeper and broader than the curriculum developed for general education students.

Ceiling effect: When a test does not have enough difficult items to measure a student's abilities accurately, the test is said to not have enough "ceiling." Students who correctly answer all or almost all of the items on the test are said to have reached the ceiling of the test.

Chronological age: An individual's age based upon birth date, which is usually represented in years and months.

College Readiness Standards: A series of statements developed by ACT that connect score ranges earned on ACT tests (e.g., EXPLORE, PLAN, ACT) with skill levels.

Constructivism: A philosophical approach to teaching and learning that assumes that the learner will "mentally construct" knowledge by fitting new information with previously acquired information. Following this approach, mathematics teachers present activities that encourage students to construct their own understanding of the mathematics being studied.

Core curriculum: The underlying principle of a core curriculum is that there is a uniform body of knowledge that all students should know.

Curriculum-based assessment: Measurement of student progress is based upon direct observation of student performance with the curriculum used in the classroom.

Curriculum compacting: A method developed for eliminating unnecessary repetition of material already learned. By pretesting or some other means, what a student already knows is determined and then he or she is allowed to move on. The time saved can be used for engaging in enrichment activities or for studying more advanced material.

Curriculum scope and sequence: The range of topics of subject-matter content and the order in which those topics are covered during an academic year and from grade to grade.

Diagnostic Testing→Prescriptive Instruction (DT→PI) model: A model for educating talented youth that is useful for determining specific strengths and weaknesses in a subject. This information is then related to instructional prescriptions. DT→PI model participants are first given aptitude tests. High-scoring students are recommended for further testing using beyond-level achievement tests. Instruction is prescribed based upon those test results. Students work on the topics they do not fully understand, and they spend little time on topics they already know well. After studying a topic, students are retested to assess their level of mastery.

Differentiated curriculum: A curriculum that is designed specifically for gifted students and is defensibly different from the curriculum offered to average students. It can refer to a medley of educational programs, including acceleration and enrichment.

Distance learning: An approach to delivering and/or receiving instruction in a manner that doesn't require students and teachers to be on site in a traditional classroom or school building.

Enrichment: Students are given opportunities to study subjects or content not covered in the typical school curriculum or an in-depth exploration of topics covered in the regular curriculum. Students participating in enrichment activities study richer, broader, and more varied content.

Grade-equivalent: A standard score that is designed to indicate growth from one academic year (i.e., 10 months) to another academic year. For example, a score of 5.0 refers to average performance compared to other students at the beginning of fifth grade. One year later, those same students would be expected to have an average growth reflected in a grade-equivalent score of 6.0. However, for populations of gifted students, because they tend to gain more than 10 months in one year, grade-equivalent scores can be misinterpreted. A specific grade-equivalent score does not necessarily mean that the student is ready to begin instruction at that grade level, nor does it mean that a student will be challenged by material at the grade-equivalent grade.

Individualized Education Program (IEP): The federal law known as the Individuals with Disabilities Education Improvement Act (IDEA) mandates that every student with a diagnosed disability must have a program designed that permits him or her to receive a free and appropriate public education (FAPE).

Intellectual peer group: Students with similar cognitive abilities and interests, regardless of age.

Intelligence test: Designed to sample a wide variety of functions in order to estimate the individual's general intellectual level.

IQ (Intelligence Quotient): The score earned on an intelligence test. The average score on an IQ test is 100. A score of 130 is at the 98th percentile, which represents the top 2% of a typical group of individuals within an age range. Because this score is earned by no more than 2% of a population, it is often considered a score earned by a gifted individual.

Jacob K. Javits Gifted and Talented Students Education Act: The only legal act that has been passed by the U.S. federal government to support funding for the development of gifted and talented students. The emphasis of this funding is on serving students from groups traditionally underrepresented in gifted and talented programs.

Magnet school: Public school that offers specialized courses in certain areas (e.g., science, math, or performance in the arts).

Manipulatives: Objects that can be handled; they are used in mathematics classrooms to present abstract concepts in a concrete fashion.

Mastery: The point at which the student demonstrates a thorough understanding of the subject-matter content.

Mentor: Mentors serve as instructors for talented students and are well trained in the subject matter beyond what they are expected to teach. Typically, mentors work with only one or two students at a time. They present new information, rather than reviewing already-studied material the way a tutor would. Mentors help their students (sometimes called "mentees") to move through material at a challenging pace. Mentors also clarify and extend the material to be learned. The goal of mentoring is to help students learn self-teaching skills, not to spoon-feed them information.

***A Nation Deceived*:** *A Nation Deceived: How Schools Hold Back America's Brightest Students* is a 2004 publication that provided comprehensive

coverage about the effectiveness of academic acceleration as an intervention for highly capable students.

National Council of Teachers of Mathematics: The national organization for teachers of mathematics.

National Mathematics Advisory Panel: The President of the U.S. established the panel via Executive Order in 2006. The panel's charge included multiple elements, but there was a clear emphasis on the preparation of students for entry into, and success in, algebra. Over a 20-month period, the panel heard public testimony as a whole, but also worked in task groups and subcommittees to address the President's charge. Their work resulted in the 2008 publication, *Foundations for Success*.

Norm group: As a part of the test-development process, a test is administered to a large group of students among whom the scores will be compared; thus, the group is called the "normative" or "comparison" group. Later, when individual students are tested, their scores can be compared to those in the norm group. The student's performance might be average, above average, or below average when compared to that norm group.

Optimal match: In its most basic form, this term means that the learning task at hand matches the individual's readiness to learn that material. This means that there is enough background knowledge upon which to build new information.

Parallel forms of a test (alternate forms): Two or more tests containing different items that cover the same material at the same level of difficulty. The measurement specialists who develop the tests have used statistical techniques that make the tests equivalent, but not identical. Parallel forms of a test are useful for pre- and posttesting.

Percentile rank: Allows the student's score on a test to be compared to other children his or her age. An average score on a test yields a

percentile ranking of 50, meaning that the child scored higher than 50% of the children in the comparison group. Talented children tend to score at the 97th percentile or above when compared to their own age group.

Perfectionism: When a student is focused on producing products that are without fault. Often, when gifted students are underchallenged, they develop tendencies to try to produce "perfect" products as a means to remain challenged and engaged.

Precocious: Showing exceptionally early development and achievement. For example, a precocious mathematics student may multiply and divide at the age of 4.

Pretest: Before instruction begins, students are given a test on the material. The purpose of pretesting is to determine what students know, as well as what they are ready to learn. Instruction can then focus on teaching the student new material, rather than reviewing material already mastered.

Psychological assessment: Can include intelligence testing, achievement testing, personality profiles, and information about affective development, including social and emotional maturity. This assessment provides information about a student when compared to others of the same age.

Pull-out/resource program: A model of gifted education service delivery that provides enrichment to the regular core curriculum. The enrichment options are provided outside of the regular classroom setting.

Radical acceleration: Academic advancement that is at least two grades beyond the student's age peers.

Raw score: The number of items answered correctly on a test.

Reliability: A term used in educational testing to describe how consistent test results are from one administration to another.

Residential summer program: Academic courses that are either enrichment or acceleration focused, typically offered on university or college campuses during the summer months, and students live on the campus.

Response to Intervention (RtI): A model of academic intervention that was originally developed to address the needs of learners who experienced difficulty in reading. The main focus is on early, effective intervention that is based upon frequent measurement of progress and increasingly intensive instruction. Of note, RtI is considered by some educators to be an effective alternative to the traditional ability-achievement discrepancy model used to diagnose learning disabilities.

Spiral approach to mathematics curriculum: Mathematics is presented to students a little bit at a time. Students then return to the topic year after year, each time at a slightly more advanced level. For example, in first grade, students learn one-digit addition; in second grade, they learn two-digit addition; and, in third grade, they learn three-digit addition. We do not advocate this approach for talented youth because it is usually not challenging to bright students.

Standardized test: An ability, aptitude, or achievement test developed by testing specialists such as Educational Testing Service, the Iowa Testing Program, and ACT, Inc. These tests are developed so that scores earned by students can be compared to each other. Teacher-made tests assess the students' understanding of the material that was studied. Standardized tests assess the students' performance compared to a norm group.

STEM: An acronym for science, technology, engineering, and mathematics fields.

Study of Mathematically Precocious Youth (SMPY): Founded in 1971 by Julian C. Stanley at Johns Hopkins University. The purpose of SMPY was to conduct research and offer programs and services for mathematically talented students. Some participants in SMPY take part in a longitudinal study housed at Peabody College, Vanderbilt University (codirected by Camilla Benbow and David Lubinski). SMPY has evolved to include the Study of Exceptional Talent (SET) at Johns Hopkins University. SET is directed by Linda Brody and housed in the Center for Talented Youth (CTY).

Talent search: A model of above-level testing used to discover academically talented precollege students (elementary, middle, and high school students).

Telescoping: A system for delivering instruction that uses less time than normal (e.g., 2 years instead of 3). A telescoped curriculum is an accelerated delivery system.

Test: A method designed to measure knowledge, aptitude, or ability. Tests may be designed by teachers to measure classroom learning, or by testing organizations, which usually develop standardized tests that yield scores that can be compared with large groups of individuals.

Twice-exceptional student: Student who has talent in one or more areas, yet also has a diagnosed, or diagnosable, disability.

Validity: A term used in educational testing to describe how well a test measures what it sets out to measure.

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Printed in the USA

Cover design by Marjorie Parker
Layout design by Raquel Trevino