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Manhattan **GMAT*** *Prep*

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FRACTIONS, DECIMALS, & PERCENTS

GMAT Preparation Guide

This guide provides an in-depth look at the variety of GMAT questions that test your knowledge of fractions, decimals, and percents. Learn to see the connections among these part-whole relationships and practice implementing strategic shortcuts.

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FRACTIONS, DECIMALS, & PERCENTS

Math Preparation Guide

This guide provides an in-depth look at the variety of GMAT questions that test your knowledge of fractions, decimals, and percents. Learn to see the connections among these part-whole relationships and practice implementing strategic shortcuts.

Fractions, Decimals, and Percents GMAT Preparation Guide

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Chapter 1
of
FRACTIONS, DECIMALS, & PERCENTS

DIGITS & DECIMALS

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- Powers of 10: Shifting the Decimal
- The Last Digit Shortcut
- The Heavy Division Shortcut
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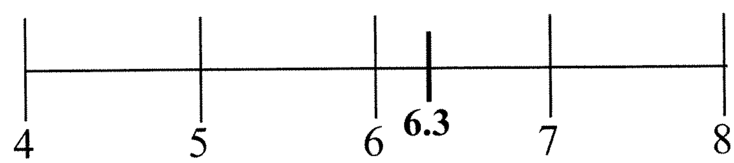
DECIMALS

GMAT math goes beyond an understanding of the properties of integers or whole numbers. The GMAT also tests your ability to understand the numbers that fall in between the whole numbers. These numbers are called decimals.

For example, the decimal 6.3 falls between the integers 6 and 7.



You can use a number line to decide between which whole numbers a decimal falls.

Some other examples of decimals include:

Decimals less than -1:	-3.65, -12.01, -145.9
Decimals between -1 and 0:	$-.65$, $-.8912$, $-.076$
Decimals between 0 and 1:	.65, .8912, .076
Decimals greater than 1:	3.65, 12.01, 145.9

Note that an integer can be expressed as a decimal by adding the decimal point and the number 0. For example:

8 = 8.0 123 = 123.0 400 = 400.0

DIGITS

Every number is composed of digits. There are only ten digits in our number system: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The term digit refers to one building block of a number; it does not refer to a number itself. For example:

356 is a number composed of three digits: 3, 5, and 6.

Numbers are often classified by the number of digits they contain. For example:

2, 7, and -8 are each single-digit numbers (they are each composed of one digit).
43, 63, and -14 are each double-digit numbers (composed of two digits).
100, 765 and -890 are each triple-digit numbers (composed of three digits).
500,000 and $-468,024$ are each six-digit numbers (composed of six digits).
789,526,622 is a nine-digit number (composed of nine digits).

Note that decimals can be classified by the number of digits they contain as well:

3.4 is a number that contains two digits.
7.05 is a number that contains three digits.
106.7559 is a number that contains seven digits.

Place Value

Every digit in a number has a particular place value depending on its location within the number. For example, in the number 452, the digit 2 is in the ones (or units) place, the digit 5 is in the tens place, and the digit 4 is in the hundreds place. The name of each location corresponds to the "value" of that place. Thus:

2 is worth two units, or 2.
5 is worth five tens, or 50.
4 is worth four hundreds, or 400.

You should memorize the names of all the place values.

6	**9**	**2**	**5**	**6**	**7**	**8**	**9**	**1**	**0**	**2**	**3**	**.**	**8**	**3**	**4**	**7**
HUNDRED BILLIONS	TEN BILLIONS	ONE BILLIONS	HUNDRED MILLIONS	TEN MILLIONS	ONE MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	UNITS		TENTHS	HUNDREDTHS	THOUSANDTHS	TEN THOUSANDTHS

The chart to the left analyzes the place value of all the digits in the number:

692,567,891,023.8347

Notice that the place values to the left of the decimal all end in "-s", while the place values to the right of the decimal all end in "-ths." This is because the suffix "-ths" gives these places (to the right of the decimal) a fractional value.

Let's analyze the end of the preceding number: **.8347**

8 is in the tenths place, giving it a value of 8 tenths, or $\frac{8}{10}$.

3 is in the hundredths place, giving it a value of 3 hundredths, or $\frac{3}{100}$.

4 is in the thousandths place, giving it a value of 4 thousandths, or $\frac{4}{1000}$.

7 is in the ten thousandths place, giving it a value of 7 ten thousandths, or $\frac{7}{10,000}$.

To use a concrete example, .8 means eight tenths of one dollar, which would be 8 dimes or 80 cents. Additionally, .03 means three hundredths of one dollar, which would be 3 pennies or 3 cents.

Using Place Value on the GMAT

You will never be asked to find the digit in the tens place on the GMAT. However, you will need to use place value to solve fairly difficult GMAT problems.

Consider the following problem:

A and B are both two-digit numbers. If A and B contain the same digits, but in reverse order, what integer must be a factor of (A + B)?

To solve this problem, assign two variables to be the digits in A and B: x and y. Using your knowledge of place value, you can express A as $10x + y$, where x is the digit in the tens place and y is the digit in the units place. B, therefore, can be expressed as $10y + x$. The sum of A and B can be expressed as follows:

Place value can help you solve tough problems about digits.

$$A + B = 10x + y + 10y + x = 11x + 11y = 11(x + y)$$

Clearly, 11 must be a factor of A + B.

Rounding to the Nearest Place Value

Knowing place value is important because the GMAT occasionally requires you to round a number to a specific place value. For example:

What is 3.681 rounded to the nearest tenth?

First, find the digit located in the specified place value. The digit 6 is in the tenths place.

Second, look at the right-digit-neighbor (the digit immediately to the right) of the digit in question. In this case, 8 is the right-digit-neighbor of 6.

If the right-digit-neighbor is 5 or greater, round the digit in question UP. But if the right-digit-neighbor is 4 or less, the digit in question remains the same. In this case, since 8 is greater than five, the digit in question (6) must be rounded up to 7. Thus, 3.681 rounded to the nearest tenth equals 3.7.

Note that all the digits to the right of the right-digit-neighbor are irrelevant when rounding. In the last example, the digit 1 is not important.

Rounding appears on the GMAT in the form of questions like these:

If x is the decimal 8.1d5, and x rounded to the nearest tenth is equal to 8.1, which integers could not be the value of d?

In order for x to be 8.1 when rounded to the nearest tenth, the right-digit neighbor, d, must be less than 5. Any integer greater than or equal to 5 cannot be the value of d.

Adding Zeroes to Decimals

Adding zeroes to the end of a decimal does not change the value of the decimal. For example: 3.6 = 3.60 = 3.6000 & 65.689 = 65.68900 = 65.68900000

Removing zeroes from the end of a decimal does not change the value of the decimal either: 3.600000 = 3.6 & 65.68900000 = 65.689

Be careful, however, not to add or remove any zeroes from within a number. Doing so will change the value of the number:

$7.01 \neq 7.1$ & $923.01 \neq 923.001$

When you shift the decimal to the right, the number gets bigger. When you shift the decimal to the left, the number gets smaller.

Powers of 10: Shifting the Decimal

Place values continually decrease from left to right by powers of 10. Understanding this can help you understand the following shortcuts for multiplication and division.

In words	thousands	hundreds	tens	ones	tenths	hundredths	thousandths
In numbers	1000	100	10	1	.1	.01	.001
In powers of ten	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

When you multiply any number by a power of ten, move the decimal forward (right) the specified number of places:

$3.9742 \times 10^3 = 3974.2$ (Move the decimal forward 3 spaces.)
$89.507 \times 10 = 895.07$ (Move the decimal forward 1 space.)

When you divide any number by a power of ten, move the decimal backward (left) the specified number of places:

$4169.2 \div 10^2 = 41.692$ (Move the decimal backward 2 spaces.)
$89.507 \div 10 = 8.9507$ (Move the decimal backward 1 space.)

Note that if you need to add zeroes in order to shift a decimal, you should do so:

$2.57 \times 10^6 = 2{,}570{,}000$ (In order to move the decimal forward 6 spaces, add four zeroes to the end of the number.)
$14.29 \div 10^5 = .0001429$ (In order to move the decimal backward 5 spaces, add three zeroes to the beginning of the number.)

Finally, note that negative powers of ten reverse the regular process:

$6782.01 \times 10^{-3} = 6.78201$ (Moving the decimal forward by negative 3 spaces means moving it backward by 3 spaces.)
$53.0447 \div 10^{-2} = 5304.47$ (Moving the decimal backward by negative 2 spaces means moving it forward by 2 spaces.)

The Last Digit Shortcut

Consider this problem:

What is the units digit of $(5)^2(9)^2(4)^3$?

In this problem, you can use the Last Digit Shortcut. Solve the problem step by step. However, only pay attention to the last digit of every intermediate product. Drop any other digits.

STEP 1: $5 \times 5 = 25$	Drop the tens digit and keep only the last digit: 5.
STEP 2: $9 \times 9 = 81$	Drop the tens digit and keep only the last digit: 1.
STEP 3: $4 \times 4 \times 4 = 64$	Drop the tens digit and keep only the last digit: 4.
STEP 4: $5 \times 1 \times 4 = 20$	Multiply the last digits of each of the products.

The units digit of the final product is 0.

Use the Heavy Division Shortcut when you need an approximate answer.

The Heavy Division Shortcut

Some division problems involving decimals can look rather complex. Often on the GMAT, you only need to find an approximate solution in order to answer a question. In these cases, you should save yourself time by using the heavy division shortcut. This shortcut employs the decimal-shifting rules you just learned.

What is $1{,}530{,}794 \div (31.49 \times 10^4)$ to the nearest whole number?

Since we are looking for an estimate, we can use the heavy division shortcut.

Step 1: Set up the division problem in fraction form:

$$\frac{1{,}530{,}794}{31.49 \times 10^4}$$

Step 2: Rewrite the problem, eliminating powers of 10:

$$\frac{1{,}530{,}794}{314{,}900}$$

Step 3: Your goal is to get a single digit to the left of the decimal in the denominator. In this problem, you need to move the decimal point backward 5 spaces. You can do this to the denominator as long as you do the same thing to the numerator.

$$\frac{1{,}530{,}794}{314{,}900} = \frac{15.30794}{3.14900}$$

Now you have the single digit 3 to the left of the decimal in the denominator.

Step 4: Focus only on the whole number parts of the numerator and denominator and solve.

$$\frac{15.30794}{3.14900} \approx \frac{15}{3} = 5$$

An approximate answer to the complex division problem is 5.

Decimal Operations

The rules for decimal operations are different for each operation.

ADDITION AND SUBTRACTION

To add or subtract decimals, make sure to line up the decimal points. Then add zeroes to make the right sides of the decimals the same length.

4.319 + 221.8

Line up the decimal points and add zeroes.

$$\begin{array}{r} 4.319 \\ +\ 221.800 \\ \hline 226.119 \end{array}$$

10 − .063

Line up the decimal points and add zeroes.

$$\begin{array}{r} 10.000 \\ -\ \ .063 \\ \hline 9.937 \end{array}$$

Addition & Subtraction: Line up the decimal points!

MULTIPLICATION

To multiply decimals, ignore the decimal point until the end. Just multiply the numbers as you would if they were whole numbers. Then count the *total* number of digits to the right of the decimal point in the factors. The product should have the same number of digits to the right of the decimal point.

.02 × 1.4

Multiply normally:

$$\begin{array}{r} 14 \\ \times 2 \\ \hline 28 \end{array}$$

There are 3 digits to the right of the decimal point in the factors (0 and 2 in the first factor and 4 in the second factor). Therefore, move the decimal point 3 places to the left in the product: 28 → .028

Multiplication: Conserve digits to the right of the decimal point!

DIVISION

If there is a decimal point in the dividend (the inner number) only, you can simply bring the decimal point up and divide normally.

Ex. **12.42 ÷ 3** = 4.14

```
   4.14
3)12.42
  12
   04
    3
    12
```

However, if there is a decimal point in the divisor (the outer number), you should shift the decimal point in both the divisor and the dividend to make the *divisor* a whole number. Then, bring the decimal point up and divide.

Remember, in order to divide decimals, you must make the OUTER number a whole number by shifting the decimal point.

Ex: **12.42 ÷ .3** → 124.2 ÷ 3 = 41.4

```
   41.4
3)124.2
  12
   04
    3
    12
```

Move the decimal one space to the right to make .3 a whole number. Then, move the decimal one space in 12.42 to make it 124.2.

You can always simplify division problems that involve decimals by shifting the decimal point in both the divisor and the dividend, even when the division problem is expressed as a fraction:

$$\frac{.0045}{.09}=\frac{45}{900}$$

Move the decimal 4 spaces to the right to make both the numerator and the denominator whole numbers.

Note that this is essentially the same process as simplifying a fraction. You are simply multiplying the numerator and denominator of the fraction by a power of ten—in this case, 10^4, or 10,000.

Problem Set

Solve each problem, applying the concepts and rules you learned in this section.

1. What is the units digit of $(2)^5(3)^3(4)^2$?

2. What is the sum of all the possible 3-digit numbers that can be constructed using the digits 3, 4, and 5, if each digit can be used only once in each number?

3. In the decimal, 2.4*d*7, *d* represents a digit from 0-9. If the value of the decimal rounded to the nearest tenth is less than 2.5, what are the possible values of *d*?

4. If k is an integer, and if $.02468 \times 10^k$ is greater than 10,000, what is the least possible value of k?

5. Which integer values of b would give the number $2002 \div 10^{-b}$ a value between 1 and 100?

6. Estimate: $\frac{4{,}509{,}982{,}344}{5.342 \times 10^4}$

7. Simplify: $(4.5 \times 2 + 6.6) \div .003$

8. Simplify: $(4 \times 10^{-2}) - (2.5 \times 10^{-3})$

9. What is $4{,}563{,}021 \div 10^5$, rounded to the nearest whole number?

10. Simplify: $(.08)^2 \div .4$

11. If k is an integer, and if 422.93×10^k is less than 3, what is the greatest possible value of k?

12. Simplify: $[8 - (1.08 + 6.9)]^2$

13. Which integer values of j would give the number $-3{,}712 \times 10^j$ a value between -100 and -1?

14. What is the units digit of $\left(\frac{6^6}{6^5}\right)^6$?

15. Simplify: $\frac{.00081}{.09}$

1. **4:** Use the Last Digit Shortcut, ignoring all digits but the last in any intermediate products:

STEP ONE: $2^5 = 32$	Drop the tens digit and keep only the last digit: 2.
STEP TWO: $3^3 = 27$	Drop the tens digit and keep only the last digit: 7.
STEP THREE: $4^2 = 16$	Drop the tens digit and keep only the last digit: 6.
STEP FOUR: $2 \times 7 \times 6 = 84$	Drop the tens digit and keep only the last digit: 4.

2. **2664:** There are 6 ways in which to arrange these digits: 345, 354, 435, 453, 534, and 543. Notice that each digit appears twice in the hundreds column, twice in the tens column, and twice in the ones column. Therefore, you can use your knowledge of place value to find the sum quickly:

$$100(24) + 10(24) + (24) = 2400 + 240 + 24 = 2664.$$

3. **{0, 1, 2, 3, 4}:** If d is 5 or greater, the decimal rounded to the nearest tenth will be 2.5.

4. **6:** Multiplying .02468 by a positive power of ten will shift the decimal point to the right. Simply shift the decimal point to the right until the result is greater than 10,000. Keep track of how many times you shift the decimal point. Shifting the decimal point 5 times results in 2,468. This is still less than 10,000. Shifting one more place yields 24,680, which is greater than 10,000.

5. **{−2, −3}:** In order to give 2002 a value between 1 and 100, we must shift the decimal point to change the number to 2.002 or 20.02. This requires a shift of either two or three places to the left. Remember that, while multiplication shifts the decimal point to the right, division shifts it to the left. To shift the decimal point 2 places to the left, we would divide by 10^2. To shift it 3 places to the left, we would divide by 10^3. Therefore, the exponent $-b = \{2, 3\}$, and $b = \{-2, -3\}$.

6. **90,000:** Use the Heavy Division Shortcut to estimate:

$$\frac{4{,}509{,}982{,}344}{53{,}420} = \frac{4{,}500{,}000{,}000}{50{,}000} = \frac{450{,}000}{5} = 90{,}000$$

7. **5,200:** Use the order of operations, PEMDAS (Parentheses, Exponents, Multiplication & Division, Addition and Subtraction) to simplify.

$$\frac{9 + 6.6}{.003} = \frac{15.6}{.003} = \frac{15{,}600}{3} = 5{,}200$$

8. **.0375:** First, rewrite the numbers in standard notation by shifting the decimal point. Then, add zeroes, line up the decimal points, and subtract.

$$\begin{array}{r} .0400 \\ -\ .0025 \\ \hline .0375 \end{array}$$

9. **46:** To divide by a positive power of 10, shift the decimal point to the left. This yields 45.63021. To round to the nearest whole number, look at the tenths place. The digit in the tenths place, 6, is more than five. Therefore, the number is closest to 46.

10. **.016:** Use the order of operations, PEMDAS (Parentheses, Exponents, Multiplication & Division, Addition and Subtraction) to simplify.

$$\frac{(0.08)^2}{.4} = \frac{.0064}{.4} = \frac{.064}{4} = .016$$

11. **−3:** Multiplying 422.93 by a negative power of ten will shift the decimal point to the left. Simply shift the decimal point to the left until the result is less than 3. Keep track of how many times you shift the decimal point. Shifting the decimal point 2 times results in 4.2293. This is still more than 3. Shifting one more place yields .42293, which is less than 3.

12. **.0004:** Use the order of operations, PEMDAS (Parentheses, Exponents, Multiplication & Division, Addition and Subtraction) to simplify.

First, add 1.08 + 6.9 by lining up the decimal points:

$$\begin{array}{r} 1.08 \\ +\ 6.9 \\ \hline 7.98 \end{array}$$

Then, subtract 7.98 from 8 by lining up the decimal points, adding zeroes to make the decimals the same length:

$$\begin{array}{r} 8.00 \\ -\ 7.98 \\ \hline .02 \end{array}$$

Finally, square .02, conserving the number of digits to the right of the decimal point.

$$\begin{array}{r} .02 \\ \times\ .02 \\ \hline .0004 \end{array}$$

13. **{−2, −3}:** In order to give −3,712 a value between −100 and −1, we must shift the decimal point to change the number to −37.12 or −3.712. This requires a shift of either two or three places to the left. Remember that, while multiplication shifts the decimal point to the right, division shifts it to the left. To shift the decimal point 2 places to the left, we would multiply by 10^{-2}. To shift it 3 places to the left, we would multiply by 10^{-3}. Therefore, the exponent $j = \{-2, -3\}$.

14. **6:** First, use the rules for combining exponents to simplify the expression $\frac{6^6}{6^5}$. To combine exponents in division, subtract the exponents. Therefore, $\frac{6^6}{6^5} = 6$. Then, raise this to the sixth power:

$$6^6 = 6^2 \times 6^2 \times 6^2 = 36 \times 36 \times 36.$$

Ignore any digits other than the last one: 6 × 6 × 6 = 36 × 6. Again, ignore any digits other than the last one: 6 × 6 = 36. The last digit is 6.

15. **.009:** Shift the decimal point 2 spaces to eliminate the decimal point in the denominator. Then divide, either mentally or using long division.

$$\frac{.00081}{.09} = \frac{.081}{9}$$

$$\begin{array}{r} .009 \\ 9\overline{).081} \\ \underline{0} \\ 08 \\ \underline{0} \\ 81 \\ \underline{81} \\ 0 \end{array}$$

Chapter 2
of
FRACTIONS, DECIMALS, & PERCENTS

FRACTIONS

In This Chapter . . .

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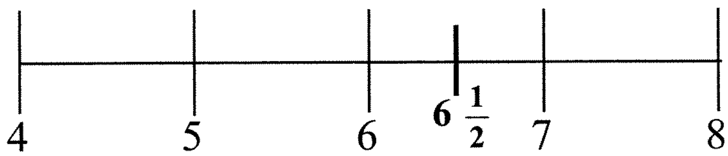
- Numerator and Denominator Rules
- Simplifying Proper Fractions
- Simplifying Improper Fractions
- The Multiplication Shortcut
- No Addition or Subtraction Shortcuts
- Dividing Fractions: Use the Reciprocal
- Division in Disguise
- Fraction Operations: Funky Results
- Comparing Fractions: Cross-Multiply
- Never Split the Denominator
- Benchmark Values
- Smart Numbers: Multiples of the Denominators
- When Not to Use Smart Numbers

FRACTIONS

Decimals are one way of expressing the numbers that fall in between the whole numbers. Another way of expressing these numbers is through fractions.

For example, the fraction $6\frac{1}{2}$ falls between the integers 6 and 7.



Proper and improper fractions behave differently in many cases.

Proper fractions are those that fall between 0 and 1. In proper fractions, the numerator is always smaller than the denominator. For example:

$$\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{7}{10}$$

Improper fractions are those that are greater than 1. In improper fractions, the numerator is greater than the denominator. For example:

$$\frac{5}{4}, \frac{13}{2}, \frac{11}{3}, \frac{101}{10}$$

Improper fractions can be rewritten as mixed numbers. A mixed number is an integer and a proper fraction. For example:

$$\frac{5}{4} = 1\frac{1}{4} \qquad \frac{13}{2} = 6\frac{1}{2} \qquad \frac{11}{3} = 3\frac{2}{3} \qquad \frac{101}{10} = 10\frac{1}{10}$$

Although all the preceding examples use positive fractions, note that fractions can be negative as well.

Numerator and Denominator Rules

Certain key rules govern the relationship between the numerator (the top number) and the denominator (the bottom number) of proper fractions. These rules are important to internalize, but keep in mind that they **only apply to positive proper fractions (fractions between 0 and 1).**

These rules only apply to positive proper fractions!

As the NUMERATOR goes up, the fraction INCREASES. If you increase the numerator of a fraction, while holding the denominator constant, the fraction increases in value as it approaches 1.

$$\frac{1}{8} < \frac{2}{8} < \frac{3}{8} < \frac{4}{8} < \frac{5}{8} < \frac{6}{8} < \frac{7}{8} < \frac{8}{8} = 1$$

As the DENOMINATOR goes up, the fraction DECREASES. If you increase the denominator of a fraction, while holding the numerator constant, the fraction decreases in value as it approaches 0.

$$\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \frac{1}{6} \ldots > \frac{1}{1000} \approx 0$$

Increasing BOTH the numerator and the denominator by THE SAME VALUE brings the fraction closer to 1. If you add the same number to both the numerator and the denominator, the fraction increases in value as it approaches 1.

$$\frac{1}{2} < \frac{1+1}{2+1} = \frac{2}{3} < \frac{2+9}{3+9} = \frac{11}{12} < \frac{11+1000}{12+1000} = \frac{1011}{1012}$$

Thus: $\frac{1}{2} < \frac{2}{3} < \frac{11}{12} < \frac{1011}{1012} \approx 1$

Simplifying Proper Fractions

Simplifying fractions is a process that attempts to express a fraction in its lowest terms. The process of simplifying is governed by one simple rule:

MULTIPLYING or DIVIDING both the numerator and the denominator by the same number does not change the value of the fraction.

$$\frac{\mathbf{4}}{\mathbf{5}} = \frac{4(3)}{5(3)} = \frac{12}{15} = \frac{12(2)}{15(2)} = \frac{24}{30} \qquad \frac{\mathbf{24}}{\mathbf{30}} = \frac{24 \div 2}{30 \div 2} = \frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5}$$

Simplify fractions by multiplying or dividing both the numerator and the denominator by the same number.

Simplifying a fraction means dividing both the numerator and the denominator by a common factor. This must be repeated until no common factors remain.

$\frac{\mathbf{20}}{\mathbf{30}} \rightarrow \frac{20 \div 5}{30 \div 5} = \frac{4}{6}$ (Numerator and denominator are divided by the common factor 5.)

$\frac{\mathbf{4}}{\mathbf{6}} \rightarrow \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$ (Numerator and denominator are divided by the common factor 2.)

There are no more common factors, so $\frac{20}{30}$ has been fully simplified to $\frac{2}{3}$.

Alternatively, we could have simplified $\frac{20}{30}$ in only one step by dividing both the numerator and denominator by their greatest common factor (10) to yield $\frac{2}{3}$.

Simplifying Improper Fractions

Simplifying an improper fraction involves converting it into a mixed number. To do this, divide the numerator by the denominator:

$$\frac{\mathbf{9}}{\mathbf{4}} = 9 \div 4 = 4\overline{)9} \quad \text{quotient } 2,\ \ 9 - 8 = 1$$

Since 9 ÷ 4 equals 2 with a remainder of 1, we can write the improper fraction as the integer 2 with a fractional part of 1 over the original denominator of 4.

Thus, $\frac{9}{4} = 2\frac{1}{4}$.

This process can also work in reverse. In order to convert a mixed number into an improper fraction (something you need to do in order to multiply or divide mixed numbers), use the following procedure:

$\mathbf{2\frac{1}{4}}$ Multiply the denominator (4) by the whole number (2) and add the numerator (1): $4 \times 2 + 1 = 9$.

$\frac{\mathbf{9}}{\mathbf{4}}$ Place the number 9 over the original denominator, 4.

The Multiplication Shortcut

To multiply fractions, first multiply the numerators together, then multiply the denominators together, and finally simplify your resulting product by expressing it in lowest terms. For example:

$$\frac{\mathbf{8}}{\mathbf{15}} \times \frac{\mathbf{35}}{\mathbf{72}} = \frac{8(35)}{15(72)} = \frac{280}{1080} = \frac{28}{108} = \frac{7}{27}$$

There is, however, a shortcut that can make fraction multiplication much less tedious. The shortcut is to simplify your products BEFORE multiplying. This is also known as "cancelling."

This shortcut is known as "cancelling."

Notice that the **8** in the numerator and the **72** in the denominator both have 8 as a factor. Thus, they can be simplified from $\frac{8}{72}$ to $\frac{1}{9}$.

Notice also that **35** in the numerator and **15** in the denominator both have 5 as a factor. Thus, they can be simplified from $\frac{35}{15}$ to $\frac{7}{3}$.

Now the multiplication will be easier and no further simplification will be necessary:

$$\frac{8}{15} \times \frac{35}{72} = \frac{8(35)}{15(72)} = \frac{1(7)}{3(9)} = \frac{7}{27}$$

In order to multiply mixed numbers, you must first convert each mixed number into an improper fraction:

$$\mathbf{2\frac{1}{3}} \times \mathbf{6\frac{3}{5}} = \frac{7}{3} \times \frac{33}{5}$$

You can simplify the problem, using the multiplication shortcut of cancelling, and then convert the result to a mixed number:

$$\frac{7}{3} \times \frac{33}{5} = \frac{7(33)}{3(5)} = \frac{7(11)}{1(5)} = \frac{77}{5} = 15\frac{2}{5}$$

No Addition or Subtraction Shortcuts

While shortcuts are very useful when multiplying fractions, you must be careful NOT to take any shortcuts when adding or subtracting fractions. In order to add or subtract fractions, you must:

(1) find a common denominator
(2) change each fraction so that it is expressed using this common denominator
(3) add up the numerators only

To add and subtract fractions, you must find a common denominator.

$\mathbf{\frac{3}{8} + \frac{7}{12}}$ A common denominator is 24. Thus, $\frac{3}{8} = \frac{9}{24}$ and $\frac{7}{12} = \frac{14}{24}$.

$\frac{9}{24} + \frac{14}{24}$ Express each fraction using the common denominator 24.

$\frac{9}{24} + \frac{14}{24} = \frac{23}{24}$ Finally, add the numerators to find the answer.

Another example:

$\mathbf{\frac{11}{15} - \frac{7}{30}}$ A common denominator is 30. $\frac{11}{15} = \frac{22}{30}$ and $\frac{7}{30}$ stays the same.

$\frac{22}{30} - \frac{7}{30}$ Express each fraction using the common denominator 30.

$\frac{22}{30} - \frac{7}{30} = \frac{15}{30}$ Subtract the numerators.

$\frac{15}{30} = \frac{1}{2}$ Simplify $\frac{15}{30}$ to find the answer.

In order to add or subtract mixed numbers, set up the problem vertically and solve the fraction first and the whole number last.

Addition

$$
\begin{array}{r}
7\frac{1}{3} = 7\frac{2}{6} \\
+\ 4\frac{1}{2} = 4\frac{3}{6} \\
\hline
11\frac{5}{6}
\end{array}
$$

Subtraction

$$
\begin{array}{r}
7\frac{1}{3} = 7\frac{2}{6} = 6\frac{8}{6} \\
-\ 4\frac{1}{2} = 4\frac{3}{6} = 4\frac{3}{6} \\
\hline
2\frac{5}{6}
\end{array}
$$

Notice the reason for starting with the fraction. You may need to borrow from the whole number, as in this example. Here, the top number "borrows" a whole number (worth 6 parts), thus increasing its numerator from 2 to 8 (making it bigger than the 3 numerator of the bottom fraction).

Dividing Fractions: Use the Reciprocal

In order to divide fractions, you must first understand the concept of the reciprocal. You can think of the reciprocal as the fraction flipped upside down.

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

The reciprocal of $\frac{2}{9}$ is $\frac{9}{2}$.

What is the reciprocal of an integer? Think of an integer as a fraction with a denominator of 1. Thus, the integer 5 is really just $\frac{5}{1}$. To find the reciprocal, just flip it.

The reciprocal of **5** or $\frac{5}{1}$ is $\frac{1}{5}$.

The reciprocal of **8** is $\frac{1}{8}$.

To divide fractions, flip the second fraction and multiply.

To check if you have found the reciprocal of a number, use this rule: **The product of a number and its reciprocal always equals 1.** The following examples reveal this to be true:

$$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1 \qquad 5 \times \frac{1}{5} = \frac{5}{1} \times \frac{1}{5} = \frac{5}{5} = 1 \qquad \sqrt{7} \times \frac{\sqrt{7}}{7} = \frac{\sqrt{7}}{1} \times \frac{\sqrt{7}}{7} = \frac{7}{7} = 1$$

In order to divide fractions,

(1) change the divisor into its reciprocal, and then

(2) multiply the fractions. Note that the divisor is the second number:

$\frac{1}{2} \div \frac{3}{4}$ First, change the divisor $\frac{3}{4}$ into its reciprocal $\frac{4}{3}$.

$\frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$ Then, multiply the fractions and simplify to lowest terms.

In order to divide mixed numbers, first change them into improper fractions:

$5\frac{2}{3} \div 8\frac{1}{2} = \frac{17}{3} \div \frac{17}{2}$ Then, change the divisor $\frac{17}{2}$ into its reciprocal $\frac{2}{17}$.

$\frac{17}{3} \times \frac{2}{17} = \frac{2}{3}$ Multiply the fractions, cancelling where you can.

Division in Disguise

Sometimes, dividing fractions can be written in a confusing way. Consider one of the previous examples:

$\frac{1}{2} \div \frac{3}{4}$ can also be written this way: $\dfrac{\frac{1}{2}}{\frac{3}{4}}$

Do not be confused. When a fraction is written with four terms, just rewrite it as the top fraction divided by the bottom fraction, and solve it normally (by using the reciprocal of the divisor and then multiplying).

$$\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$$

Multiplying and dividing positive proper fractions yields unexpected results.

Fraction Operations: Funky Results

Performing basic operations—addition, subtraction, multiplication, and division—on proper fractions (those between 0 and 1) yields some unexpected results. Consider the following chart:

OPERATION	EXAMPLE	INCREASE OR DECREASE
Adding Fractions	$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$	INCREASE: Similar to adding positive integers, adding fractions increases their value.
Subtracting Fractions	$\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$	DECREASE: Similar to subtracting positive integers, subtracting fractions decreases their value.
Multiplying Fractions	$\frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$	**DECREASE:** Unlike multiplying positive integers, multiplying fractions decreases their value.
Dividing Fractions	$\frac{3}{5} \div \frac{1}{5} = \frac{3}{5} \times \frac{5}{1} = 3$	**INCREASE:** Unlike dividing positive integers, dividing fractions increases their value.

Note that multiplying proper fractions decreases their value, while dividing proper fractions increases their value—exactly the opposite of what happens with integers. The GMAT sometimes tests whether you remember this important difference between fractions and integers.

Comparing Fractions: Cross-Multiply

Which fraction is greater, $\frac{7}{9}$ or $\frac{4}{5}$?

The traditional method of comparing fractions involves finding a common denominator and comparing the two fractions. The common denominator of 9 and 5 is 45.

Thus, $\frac{7}{9} = \frac{35}{45}$ and $\frac{4}{5} = \frac{36}{45}$. We can see that $\frac{4}{5}$ is slightly bigger than $\frac{7}{9}$.

However, there is a shortcut to comparing fractions, called cross-multiplication. This is a process that involves multiplying the numerator of one fraction with the denominator of the other fraction, and vice versa:

Cross-multiplication can help you compare sets of several fractions.

$\frac{7}{9}$ ⨉ $\frac{4}{5}$		Set up the fractions next to each other.
(7×5) 35	(4×9) 36	Cross-multiply the fractions and put each answer under the corresponding numerator.
$\frac{7}{9}$ <	$\frac{4}{5}$	Since 35 is less than 36, the first fraction must be less than the second one.

This process can save you a lot of time when comparing fractions (usually more than two!) on the GMAT.

Never Split the Denominator

One final rule, perhaps the most important one, is one that you must always remember when working with complex fractions. A complex fraction is a fraction in which there is a sum or a difference in the numerator or the denominator. Three examples of complex fractions are:

a) $\frac{15+10}{5}$ b) $\frac{5}{15+10}$ c) $\frac{15+10}{5+2}$

In example a), the numerator is expressed as a sum.
In example b), the denominator is expressed as a sum.
In example c), both the numerator and the denominator are expressed as sums.

When simplifying fractions that incorporate sums or differences, remember this rule: You may split up the terms of the numerator, but you may NEVER split the terms of the DENOMINATOR.

Thus, the terms in example a) may be split:

$$\frac{15+10}{5} = \frac{15}{5} + \frac{10}{5} = 3 + 2 = 5$$

But the terms in example b) may not be split:

$$\frac{5}{15+10} \neq \frac{5}{15} + \frac{5}{10} \quad \textbf{NO!}$$

Instead, simplify the denominator first:

$$\frac{5}{15+10} = \frac{5}{25} = \frac{1}{5}$$

The terms in example c) may not be split either:

$$\frac{15+10}{5+2} \neq \frac{15}{5} + \frac{10}{2} \quad \textbf{NO!}$$

You can NEVER split the denominator!

Instead, simplify both parts of the fraction:

$$\frac{15+10}{5+2} = \frac{25}{7} = 3\frac{4}{7}$$

Often, GMAT problems will involve complex fractions with variables. On these problems, it is tempting to split the denominator. Do not fall for it!

It is tempting to perform the following simplification:

$$\frac{\mathbf{5x - 2y}}{\mathbf{x - y}} = \frac{5x}{x} - \frac{2y}{y} = 5 - 2 = 3$$

But this is **WRONG** because you cannot split a difference in the denominator.

The reality is that $\frac{5x - 2y}{x - y}$ cannot be simplified further.

On the other hand, the expression $\frac{\mathbf{5x^2y - 2xy}}{\mathbf{xy}}$ can be simplified by splitting the difference, because this difference appears in the numerator.

Thus: $\frac{5x^2y - 2xy}{xy} = \frac{5x^2y}{xy} - \frac{2xy}{xy} = 5x - 2$ (assuming $xy \neq 0$)

Benchmark Values

You will use a variety of estimating strategies on the GMAT. One important strategy for estimating with FDP's is to use Benchmark Values. These are simple fractions with which you are already familiar:

$$\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \text{ and } \frac{3}{4}$$

You can use Benchmark Values to compare fractions:

When you find seemingly complicated fractions on the GMAT, use Benchmark Values to make sense of them.

Which is greater: $\frac{127}{255}$ or $\frac{162}{320}$?

If you recognize that 127 is less than half of 255, and 162 is more than half of 320, you will save yourself a lot of cumbersome computation.

You can also use Benchmark Values to compute with fractions:

What is $\frac{10}{22}$ of $\frac{5}{18}$ of 2000?

If you recognize that these fractions are very close to the Benchmark Values $\frac{1}{2}$ and $\frac{1}{4}$, you can estimate:

$\frac{1}{2}$ of $\frac{1}{4}$ of $2000 = 250$. Therefore, $\frac{10}{22}$ of $\frac{5}{18}$ of $2000 \approx 250$.

Smart Numbers: Multiples of the Denominators

Sometimes, fraction problems on the GMAT include unspecified numerical amounts; often these unspecified amounts are described by variables. In these cases, pick real numbers to stand in for the variables. To make the computation easier, choose **Smart Numbers** equal to common multiples of the denominators of the fractions in the problem.

For example, consider this problem:

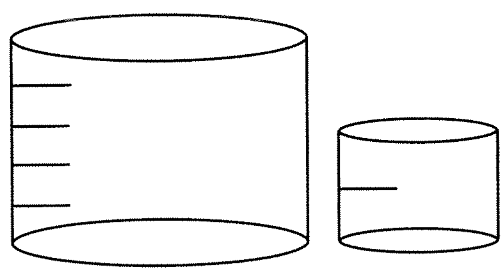
> **The Crandalls' hot tub is half filled. Their swimming pool, which has a capacity four times that of the tub, is filled to four-fifths of its capacity. If the hot tub is drained into the swimming pool, to what fraction of its capacity will the pool be filled?**

You can often use Smart Numbers to help you solve problems with unspecified amounts.

The denominators in this problem are 2 and 5. The Smart Number is the least common denominator, which is 10. Therefore, assign the hot tub a capacity of 10. Since the swimming pool has a capacity 4 times that of the pool, the swimming pool has a capacity of 40. We know that the hot tub is only half filled; therefore, it has 5 units of water in it. The swimming pool is four-fifths of the way filled, so it has 32 units of water in it.

Let's add the 5 units of water from the hot tub to the 32 units of water that are already in the swimming pool: 32 + 5 = 37.

With 37 units of water and a total capacity of 40, the pool will be filled to $\frac{37}{40}$ of its total capacity.



swimming pool
capacity: 40
filled: 32

hot tub
capacity: 10
filled: 5

When Not to Use Smart Numbers

In some problems, even though an amount might be unknown to you, it is actually specified in the problem in another way. In these cases, you cannot use Smart Numbers to assign real numbers to the variables. For example, consider this problem:

> **Mark's comic book collection contains 1/3 Killer Fish comics and 3/8 Shazaam Woman comics. The remainder of his collection consists of Boom! comics. If Mark has 70 Boom! comics, how many comics does he have in his entire collection?**

If there is even 1 specified amount in a problem, you cannot use Smart Numbers to solve it.

Even though you do not know the number of comics in Mark's collection, you can see that the total is not completely unspecified. You know a piece of the total: 70 Boom! comics. You can use this information to find the total. Do not use Smart Numbers here. Instead, solve similar problems by figuring out how big the known piece is; then, use that knowledge to find the size of the whole:

$\frac{1}{3}$ Killer Fish + $\frac{3}{8}$ Shazaam Woman = $\frac{17}{24}$ comics that are not Boom!

Therefore, $\frac{7}{24}$ of the comics are Boom! comics.

$$\frac{7}{24}x = 70$$

$$x = 70 \times \frac{24}{7}$$

$$x = 240$$

Mark has 240 comics.

Problem Set

For problems #1-5, decide whether the given operation will yield an INCREASE, a DECREASE, or a result that will STAY THE SAME.

1. Multiply the numerator of a positive, proper fraction by $\frac{3}{2}$.

2. Add 1 to the numerator of a positive, proper fraction and subtract 1 from its denominator.

3. Multiply both the numerator and denominator of a positive, proper fraction by $3\frac{1}{2}$.

4. Multiply a positive, proper fraction by $\frac{3}{8}$.

5. Divide a positive, proper fraction by $\frac{3}{13}$.

Solve problems #6-15.

6. Simplify: $\frac{10x}{5+x}$

7. Simplify: $\frac{8(3)(x)^2(3)}{6x}$

8. Simplify: $\dfrac{\frac{3}{5}+\frac{1}{3}}{\frac{2}{3}+\frac{2}{5}}$

9. Simplify: $\frac{12ab^3 - 6a^2b}{3ab}$ (given that $ab \neq 0$)

10. Put these fractions in order from least to greatest: $\frac{9}{17}$ $\frac{3}{16}$ $\frac{19}{20}$ $\frac{7}{15}$

11. Put these fractions in order from least to greatest: $\frac{2}{3}$ $\frac{3}{13}$ $\frac{5}{7}$ $\frac{2}{9}$

12. LuAnn spends 3/8 of her monthly paycheck on rent and 5/12 on food. Her roommate, Carrie, who earns twice as much as LuAnn, spends 1/4 of her monthly paycheck on rent and 1/2 on food. If the two women decide to donate the remainder of their money to charity each month, what fraction of their combined monthly income will they donate?

13. Dujuan spends 1/2 of his monthly paycheck, after taxes, on rent. He spends 1/3 on food and 1/8 on entertainment. If he donates the entire remainder, $500.00, to charity, what is Dujuan's ***annual*** income, after taxes?

14. Are $\frac{\sqrt{3}}{2}$ and $\frac{2\sqrt{3}}{3}$ reciprocals?

15. Estimate: What is $\frac{11}{30}$ of $\frac{6}{20}$ of 120?

1. **INCREASE:** Multiplying the numerator of a positive fraction increases the numerator. As the numerator of a positive, proper fraction increases, its value increases.

2. **INCREASE:** As the numerator of a positive, proper fraction increases, the value of the fraction increases. As the denominator of a positive, proper fraction decreases, the value of the fraction also increases. Both actions will work to increase the value of the fraction.

3. **STAY THE SAME:** Multiplying or dividing the numerator and denominator of a fraction by the same number will not change the value of the fraction.

4. **DECREASE:** Multiplying a positive number by a proper fraction decreases the number.

5. **INCREASE:** Dividing a positive number by a positive, proper fraction increases the number.

6. **CANNOT SIMPLIFY:** There is no way to simplify this fraction; it is already in simplest form. Remember, you cannot split the denominator!

7. **12*x*:** First, combine terms in the numerator. Then, cancel terms in both the numerator and denominator.

$$\frac{72x^2}{6x} = 12x$$

8. **7/8:** To save time, multiply each of the small fractions by 15, which is the common denominator of all the fractions in the problem. Because we are multiplying the numerator *and* the denominator of the whole complex fraction by 15, we are not changing its value.

$$\frac{9+5}{10+6} = \frac{14}{16} = \frac{7}{8}$$

9. **$2(2b^2 - a)$ or $4b^2 - 2a$:** First, factor out common terms in the numerator. Then, cancel terms in both the numerator and denominator.

$$\frac{6ab(2b^2 - a)}{3ab} = 2(2b^2 - a) \text{ or } 4b^2 - 2a$$

10. **3/16 < 7/15 < 9/17 < 19/20:** Use Benchmark Values to compare these fractions.

9/17 is slightly more than 1/2. (8.5/17 would be exactly 1/2.)
3/16 is slightly less than 1/4. (4/16 would be exactly 1/4.)
19/20 is slightly less than 1. (20/20 would be exactly 1.)
7/15 is slightly less than 1/2. (7.5/15 would be exactly 1/2.)

This makes it easy to order the fractions: 3/16 < 7/15 < 9/17 < 19/20.

11. **2/9 < 3/13 < 2/3 < 5/7:** Using Benchmark Values, you should notice that 3/13 and 2/9 are both less than 1/2. 2/3 and 5/7 are both more than 1/2. Use cross-multiplication to compare each pair of fractions:

$$3 \times 9 = 27 \quad \frac{3}{13} \quad \frac{2}{9} \quad 2 \times 13 = 26 \qquad \frac{3}{13} > \frac{2}{9}$$

$$2 \times 7 = 14 \quad \frac{2}{3} \quad \frac{5}{7} \quad 5 \times 3 = 15 \qquad \frac{2}{3} < \frac{5}{7}$$

This makes it easy to order the fractions: $\frac{2}{9} < \frac{3}{13} < \frac{2}{3} < \frac{5}{7}$.

12. **17/72:** Use Smart Numbers to solve this problem. Since the denominators in the problem are 8, 12, 4, and 2, assign LuAnn a monthly paycheck of $24. Assign her roommate, who earns twice as much, a monthly paycheck of $48. The two women's monthly expenses break down as follows:

	Rent	Food	Leftover
LuAnn	3/8 of 24 = 9	5/12 of 24 = 10	24 − (9 + 10) = 5
Carrie	1/4 of 48 = 12	1/2 of 48 = 24	48 − (12 + 24) = 12

The women will donate a total of $17, out of their combined monthly income of $72.

13. **$144,000.00:** You cannot use Smart Numbers in this problem, because the total amount is specified. Even though the exact figure is not given in the problem, a portion of the total is specified. This means that the total is a certain number, although you do not know what it is. In fact, the total is exactly what you are being asked to find. Clearly, if you assign a number to represent the total, you will not be able to accurately find the total.

First, use addition to find the fraction of Dujuan's money that he spends on rent, food, and entertainment: 1/2 + 1/3 + 1/8 = 12/24 + 8/24 + 3/24 = 23/24. Therefore, the $500.00 that he donates to charity represents 1/24 of his total monthly paycheck. Dujuan's monthly income is $500.00 × 24, or $12,000.00. However, the question asks for the annual income, which is 12 × $12,000.00, or $144,000.00.

14. **YES:** The product of a number and its reciprocal must always equal 1. To test whether or not two numbers are reciprocals, multiply them. If the product is 1, they are reciprocals; if it is not, they are not.

$$\frac{\sqrt{3}}{2} \times \frac{2\sqrt{3}}{3} = \frac{2(\sqrt{3})^2}{2(3)} = \frac{6}{6} = 1$$

The numbers are indeed reciprocals.

15. **13:** Use Benchmark Values to estimate: 11/30 is slightly more than 1/3. 6/20 is slightly less than 1/3. Therefore, 11/30 of 6/20 of 120 should be approximately 1/3 of 1/3 of 120, or slightly more than 13. (The exact answer is 13.2.)